Linear modal logic and Linear temporal logic

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Systematic research of modal logic was started by Alistotle, and it was arranged from a modern viewpoint by C.I.Lewis and C.H.Langford, present base was built. Moreover, the Kripke semantics introduced by S.A. Kripke, contributions to the study of modal logic very much.

modal logic is logic that treats the proposition related to "Necessity" and "Possibility", but propositional logic is not treat.To express the necessity and the possibility, modal operator $\Box$ and $\Diamond$ are introduced. They are favorably accepted though there are various interpretations in $\Box$ and $\Diamond$.

we pays attention when $\Box$A means "henceforth A" and $\Diamond$A means "eventually A".It is called temporal logic, which is a system for which the truth value of the logical expression depended at time. because of character to treat the concept of time, temporal logic is applied in the wide field of information science,natural language, artificial intelligence, Specification verification and description of parallel program, etc.Therefore, the research of the time logic is very useful in the information science whole. It is known, The temporal logics have the following logics. The linear temporal logic thought that flow of time is number line. And the branching temporal logic thought the branching point exists in flow of time.Especially, the result of the research concerning the completeness of the linear time logic was announced in the master's thesis examination association that had been held by this learning in February, 2006.

In the master's thesis of Yonemori, two problems were pointed out, and proof was corrected for Completeness of logic of time to have used concept of cluster by Goldblatt's proof method. One is p-morphism is not exist from integer frame $(Z, <)$ to Dumbell D and the other is axiom $Z_F and Z_P$ become false. Now, it is possible to think about the frame that is called a circle as a frame that fills all axioms of LinDisc. in the circle, p-morphism is exist from integer frame $(Z, <)$, and axiom $Z_F and Z_P$ become valid, too. However, the system is known to be different in the system and LinDisc characterized by the circle.Therefore, it aims to clarify the axiom of the circle that has not been known in this research yet.

The circle is thought as a frame that makes all axioms of discrete linear temporal logic LinDisc is valid. However, because the reflexive law consists in the circle, it becomes

a different logoc from LinDisc. then, it is expected that logic L_C characterized by the circle is a logic to which axiom $T_F and T_P$ corresponding to reflective R besides all axiom $L(= C_F, C_P, 4_F, 4_P, D_F, D_P, Z_F, Z_P, and linear axiom)$of LinDisc are added.

Theorem 1
Logic L_C characterized in the circle becomes equal with logic $L'(= L + T_F + T_P)$.

Firstly,it is shown that $T_F and T_P$ become valid by arbitrary circle C'. secondly, $\nvdash_{L'} A$ implies in a certain circleC'',$C'' \not\models A$ is shown based on Goldblatt ' s proof method.Then, the theorem is proven.

Moreover, the following can be said by reviewing these axioms.

1. If 5_F and T_F are taken as an axiom, Z_F is led.
   If 5_P and T_P are taken as an axiom, Z_P is led.
2. If 5_F and T_F are taken as an axiom, 4_F is led.
   If 5_P and T_P are taken as an axiom, 4_P is led.
3. If T_F are taken as an axiom, D_F is led.
   If T_P are taken as an axiom, D_P is led.
4. If 5_F and T_F, 5_p and T_P are taken as an axiom, axiom of linear is led.
5. If C_F are taken as an axiom and use "$[F] \leftrightarrow [P]$ is valid" ,C_P is led.
6. If C_F and T_F are taken as an axiom, T_P is led.
7. If 5_F are taken as an axiom and use "$[F] \leftrightarrow [P]$ is valid" ,5_P is led.

From 1.∼7.,more easily,we can guess axioms of logic L_C given dy axioms $C_F, 5_F, \text{and} T_F$.

Theorem 2
Logic L_C characterized in the circle becomes equal with logic $L^*(= C_F + 5_F + T_F)$.

Firstly,it is shown that $C_F, 5_F and T_F$ become valid by arbitrary circle C'. secondly, $\nvdash_{L^*} A$ implies in a certain circleC'',$C'' \not\models A$ is shown based on Goldblatt ' s proof method.Then, the theorem is proven.