Kripke completeness for some distributive substructural logics

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Substructural logics are obtained by deleting some or all of structural rules from formal systems LK, LJ introduced by Gentzen. Through researching substructural logics, we can consider the relation between logical properties and structural rules. Moreover, substructural logics are including well researched logics, like many-valued logics, fuzzy logics, relevance logics, etc. A semantics for substructural logics is usually based on algebras and a lot of logical properties are proved by algebraic methods.

One of main reasons we introduce algebraic semantics is the Lindenbaum-Tarski technique, yielding almost immediate soundness and completeness results, although some authors are not satisfied with this kind of completeness. On the other hand, relational semantics introduced by Kripke are recently subject of intensive research because of their intuitive character and connection with applicative structures like automata or transition systems in computer science. They are particularly popular in modal logic and intuitionistic logic.

Although it may seem these two types of semantics have nothing in common, Stone's representation theorem provides a bridge between algebraic semantics and relational semantics. For example, it is known that relational completeness results for *canonical* modal logics can be immediately proved using Stone's duality.

In recent years, several relational semantics for substructural logics were introduced. Their results were mostly based on Priestley duality. On the other hand, relevance logics which form a subclass of substructural logics possess a relational semantics, called Routley-Meyer semantics. Urquhart studied the duality between relevance algebras and Routley-Meyer semantics. In addition, Seki defined a relational semantics for relavance modal logics and proved Sahlqvist theorem.

But, there are relatively few results for distributive substructural logics. Distinct points of our approach are as follows:

- Since our relational semantics based not on Pristley's duality but Stone's one, our relational semantics consist of just one underlying set and just one ternary relation.
- Moreover, the single ternary relation provides an interpretation for almost all connectives, that is, $\vee$, $\wedge$, $\circ$, $\backslash$ and $/$.

- Because of its simplicity (one set & one relation), our semantics resembles Kripke semantics for modal logics, which allows for easier transfer of methods and techniques from the well-developed metatheory of those systems.

The main results we obtained can be summed up as follows:

- For all basic extensions of DFL, we identified corresponding frame conditions and proved completeness results.
- We extended Stone's duality to duality between DFL algebras and DFL frames.
- Finally, we have obtained general completeness result: every DFL logic is complete with respect to a class of descriptive frames.