修　士　論　文

Reducing the redundancy in association rule by frequent closed itemsets

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Chapter 1. Introduction

1.1 Data mining and association rule mining

Knowledge Discovery in Database (KDD) the rapidly growing interdisciplinary field that merges together database, statistics and machine learning-aims to extract useful and understandable knowledge from large volumes of data. Data mining is the main step of the KDD process that performs the extraction of unknown knowledge in data.

Association rule mining finds interesting associations and correlation relationships among a large set of items. With massive amounts of data continuously being collected and stored, many industries are becoming interested in mining association rules from their databases. The discovery of interesting association relationships among huge amounts of business transaction records can help in many business decision making process, such as catalog design, cross-marketing, and loss-leader analysis.

A typical example of association rule mining is market basket analysis. This process analyzes customer-buying habits by finding associations between the different items that customer place in their “shopping baskets”. The discovery of such association can helps retailers develop marketing strategies by gaining insight into which items are frequency purchased together by customers.

The purpose of association rule extraction, introduced [1], is to discover significant relations between binary attributes extracted from databases. An example of association rules extracted from a database of supermarket sales is: “cereal $\wedge$ sugar $\rightarrow$ milk (support 7%, confidence 50%)”. This rule states that the customer who buy cereals and sugar also tend to buy milk.

The support defines the range of the rule, i.e. the proportion of customers who bought the three items among all customers, and the confidence defines the precision of the rule, i.e. the proportion of customers who bought milk among those who bought cereals and sugar. An association rule is considered relevant for decision making if it has support and confidence at least equal to some minimal support and minimal confidence thresholds, min_support and min_confidence, defined by user.

1.2 Research objectives

Association rule mining produced many rules. Therefore we have two problems:

- First: the cost of calculation to find association rules is high.
- Second: evaluate of association rules is difficult.

Many researchers consider some kinds of solutions to above problems. We divide three categories of these researches.

Category 1: Efficient algorithm for mining frequent itemsets [5]. This category's research objects to enumerate all frequent itemsets.

Category 2: Mining interesting association rules [2]. This category's research objects to incorporates user-specified constrains on the kind of rules generated or to define objects metrics of interesting.

Category 3: Non-redundant association rules by Zaki [7], Lakhal [4]. This category's research objects to generate non-redundant association rules.

We focus on non-redundant association rules, because association rules are evaluated by users, if we have many rules; The cost of evaluating them will be very high. So we try to reduce the non-informative association rules, by generating only non-redundant association rules.

We have three objectives in trying to reduce the non-informative association rules.

1. To investigate the problem of non-redundant association rules.
2. To try to formulate another form of non-redundant association rules.
3. To develop an algorithm that finds non-redundant association rules.
4. To try to improve the algorithm.

1.3 Contribution of the work

In the rest of paper, two kind of association rules are distinguished:

- 100% confidence rules
- under 100% confidence rules

The solution proposed in this paper consists of generating bases, or reduced covers for association rules. These bases contain non-redundant rules, being thus of smaller size. Our goal is to limit the extraction to the most informative association rules from the point of view of the user with respect to strong association rules.

Using the semantic for the extraction of association rules based on the closure of the Galois connection [10], the generic bases for 100% confidence association rules and the informative basis for under 100% confidence association rules are defined. Rules are constructed using the frequent closed itemsets and their generators, and they minimize the number of association rules generated while maximizing the quality of the information conveyed. They allow us to do

1. The generation of only the most informative non-redundant association rules, i.e. of the most useful and relevant rules: those having a minimal antecedent and maximal consequent. Thus redundant rules, which represent in certain databases the majority of extracted rules, particularly in the case of deuce or correlated data for which the total number of valid rules is very large, will be pruned.

2. The presentation to the user of a set of rules conveying all the attributes of the databases, i.e. containing rules where the union of the antecedents is equal to the unions of the antecedents of all the association rules valid in the context. This is necessary in order to discover rules that are “surprising” to the user, which constitute important information that it is necessary to consider [11, 12, 13].

3. The extraction of a set of rules without any loss of information, i.e. conveying all the information conveyed by the set of all valid association rules. It is possible to deduce efficiently, without access to the datasets, all valid association rules with their supports and confidence from databases.

The union of these two bases thus constitutes a small non-redundant generating set for all valid association rules, their supports and confidences.

1.4 Organization

In section 2, we recall the basic notions in association rule mining. In section 3, we present frequent closed itemsets that is key concept of our research. In section 4, we try to develop a new definition on non-redundant association rules. In section 5 we improve the algorithm given Zaki. Chapter 6 we present datasets and our experiments for evaluation, and section 7 concludes this paper.

Chapter 2. Association rule mining

In this chapter we present a framework of association rule mining and the most popular and traditional algorithm “apriori”.

2.1 A framework of association rule mining

The association rule extraction is performed from a data-mining context. This framework of association rule mining has two steps, to find frequency items, to generate rules based on frequency items.

Definition 1 (Data mining context)
A data-mining context is defined as $D=(T,I,\delta)$, where T and I are finite sets of transactions and items, respectively, and $\delta \subseteq T \times I$ is a binary relation. Each couple $(t,i) \in \delta$ denotes the fact that transaction $t \in T$ is related to the item $i \in I$.

Example 1. A data-mining context D constructed of six transactions (each one identified by its *TID*) and five items is represented in the fig. 1. This context is used as support for the example in the rest of the paper.

TID	Item
1	ACTW
2	CDW
3	ACTW
4	ACDW
5	ACDTW
6	CDT

Table 2.1. Data mining context D

2.1.1 Find frequency items in data-mining context

In this part, we present how to discover the frequency items in data mining context.

Definition 2 (Galois connection)

Let $D=(T,I,\delta)$ be a data-mining context, 2^T the set of subsets of *T*, 2^I the set of subsets of I, and

$\phi: 2^T \to 2^I, \phi(O)=\{i\in I \mid \forall t\in T,(t,i)\in\delta\}, O\subseteq T$

ϕ associates with O items common to all transactions $t\in T$

$\varphi: 2^I \to 2^T, \varphi(A)=\{t\in T \mid \forall i\in I,(t,i)\in\delta\}, A\subseteq I$

φ associates with an itemset A the transactions related to all items $i\in l$

The couple of applications (ϕ,φ) is a *Galois connection* between the power set of T (2^T) and power set of I (2^I). The following properties hold for all $I,I_1,I_2\subseteq I$ and $T,T_1,T_2\subseteq T$:

(1) $I_1 \subseteq I_2 \Rightarrow \varphi(I_1) \subseteq \varphi(I_2)$
(2) $T_1 \subseteq T_2 \Rightarrow \phi(T_1) \subseteq \phi(T_2)$
(3) $T \subseteq \varphi(I) \Leftrightarrow I \subseteq \phi(T)$

Definition 3 (frequent itemsets)
A set of items $l \subseteq I$ is called an *itemset*. The *support* of an itemset l is the percentage of transactions in D containing l:

$\text{Support}(l) = |\varphi(l)| / |O|$

l is a *frequent itemset* if $\text{support}(l) \geq \text{min\_support}$.

Lattice structure

The set of all itemsets has the *lattice structure*. It is easy to represent the itemsets to use lattice. A lattice L_k has two features:

1. There exist a partial order on the lattice elements.

2. All subsets of L_k have one greatest lower bound, the join element and one lowest upper bound, the meet element.

Given $|I| = m$, there are possibly 2^m frequent itemsets, which form a lattice of subsets over I with height equal to m.

Consider this example, m = 5, we can calculate the possibly 32 itemsets and the lattice with height 5 as shown in Figure 1.

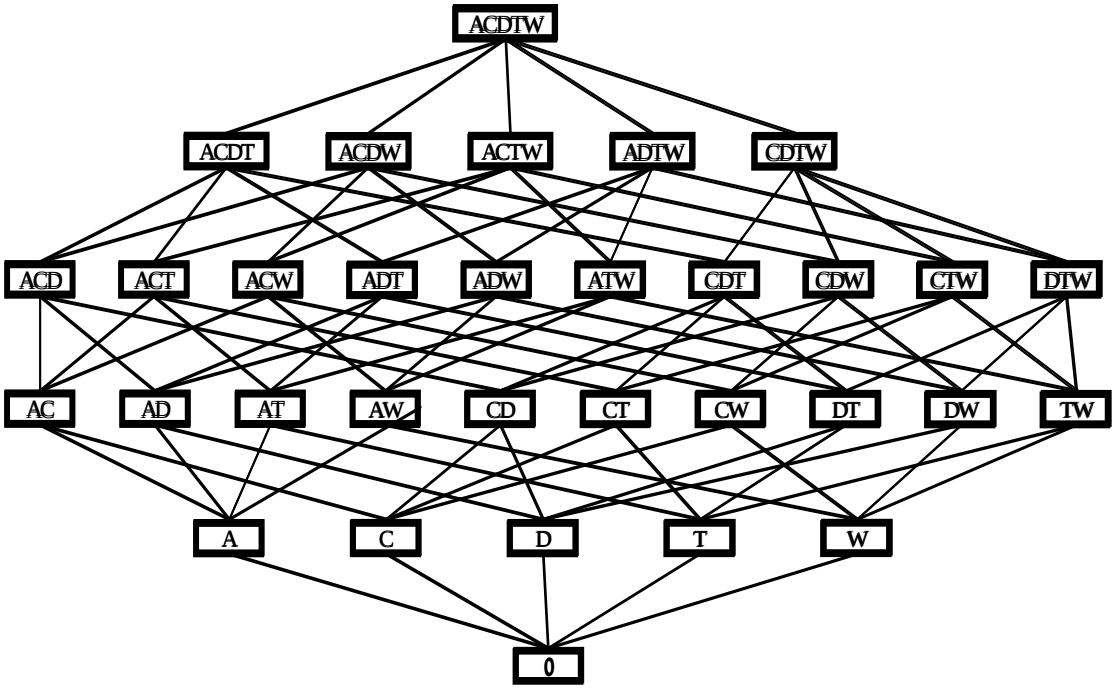


Figure 2.1. Lattice structure

2.1.2 Generate the rules based on frequency items

In this part, we present how to generate the rules based on frequency itemsets.

Once the frequent itemsets from transaction database D have been found, it is straightforward to generate strong association rules from them. This can be done using the following equation for confidence, where the conditional probably is expressed in terms of itemset support count;

$$\text{Confidence}(\mathrm{I}_1 \rightarrow I_2 - \mathrm{I}_1) = P(I_2 - \mathrm{I}_1 \mid \mathrm{I}_1) = \frac{\text{sup\_}count(I_2 - \mathrm{I}_1 \cup \mathrm{I}_1)}{\text{sup\_count}(\mathrm{I}_1)}$$

Where $\text{sup\_}count(I_2 - \mathrm{I}_1 \cup \mathrm{I}_1)$ is the number of transaction containing the itemsets $(I_2 - \mathrm{I}_1 \cup \mathrm{I}_1)$ and $\text{sup\_count}(\mathrm{I}_1)$ is the number of transaction containing the itemsets (I_1). Based on this equation, association rules can be generated as follows.

-For each frequent itemsets l, generated nonempty subsets of l.
-For every nonempty subsets of l, out put the rule "$l_1 \rightarrow (l_2 \setminus l_1)$" if this rule's confidence satisfy the minimum confidence, minimum confidence is a threshold defined by user.

Definition 5 (association rules)

An association rule r is an implication between two frequent itemsets $l_1, l_2 \subseteq I$ of the form $l_1 \rightarrow (l_2 \setminus l_1)$ where $l_1 \subset l_2$. The support and the confidence of r are defined as: $\text{support}(r) = \text{support}(l_2)$ and $\text{confidence}(r) = \text{support}(l_2)/\text{support}(l_1)$. If association rules satisfy the min_support and min_confidence, then we called theses association rules "Strong".

Example2

These examples are association rules based on Fig1 dataset. We called itemsets on the left side "antecedent", right side "consequent".

1) $A \rightarrow CW$ (support = 4, confidence = 100%
2) $D \rightarrow C$ (support = 4, confidence = 100%)
3) $C \rightarrow W$ (support = 5, confidence = 75.0%)
4) $AW \rightarrow T$ (support = 3, confidence = 83.3%)

These rules means if we find antecedent, we can find consequent on confidence percentage, and these rules occur support count in the analyzing database.

2.2 Apriori Algorithm

Apriori is an influential algorithm for mining frequent itemsets for Boolean association rules. The name of the algorithm is based on the fact that algorithm uses priori knowledge of frequent itemset property. Apriori employs an iterative approach known as a level-wise search, where k-itemsets are used to explore (k+1)-itemsets. First, the set of frequent 1-itemsets is found. This set is denoted L_1. L_1 is used to find L_2, the set of frequent 2-itemsets, which is used to find L_3, and so on, until no more frequent itemsets can be found. The finding of each L_k requires one full scan of the database.

The Apriori algorithm has two steps. 1st is join step and 2nd is prune step.

2.2.1 Join step in Apriori algorithm

To find L_k a set C_k of candidates k-itemsets is generating by joining L_{k-1} with itself. This set of candidates is denoted C_k. Let l_1 and l_2 be itemsets in L_{k-1}. The notation $l_i[j]$ refer to the j^{th} item in l_i. By convention, Apriori assumes that items within a transaction or itemset are sorted in lexicographic order. The join $L_{k-1} >< L_{k-1}$, is performed, where members of L_{k-1} are joinable if their first (k-2) items are in common.

That is, members l_1 and l_2 of L_{k-1} are joined if $(l_1[1]=l_2[1]) \wedge (l_1[2]=l_2[2]) \wedge ... \wedge (l_1[k-2]=l_2[k-2]) \wedge (l_1[k-1]<l_2[k-1])$. The condition $l_1[k-1]<l_2[k-1]$ simply ensure that no duplicates are generated. The resulting itemset formed by joining l_1 and l_2 is $l_1[1]l_1[2]..l_1[k-1]l_2[k-1]$.

2.2.2 Prune step in Apriori algorithm

C_k is superset of L_k, that is, its member may or may not be frequent, but all frequent k-itemsets are include in C_k. A scan of the database to determine the count of each candidate in C_k would result in the determination of L_k. C_k, however, can be huge, and so this could involve heavy computation. To reduce the size of C_k, the Apriori property is used as follows. Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset. Hence, if any(k-1)-subset of a candidate k-itemset is not in L_{k-1}, then the candidate cannot be frequent either and so can be removed from C_k. This subset testing can be done quickly by maintaining a hash tree of all frequent itemsets.

2.2.3 Algorithm of Apriori

Apriori algorithm

Input: Database D of transactions, minimum support threshold min_sup

Output: L, frequent itemsets in D

1.L_1 = find_frequent_1-itemsets(D);

2.for (k=2; L_{k-1}¹ Æ; k++) {

3. C_k = apriori_gen(L_{k-1}, min_sup) ;

4. for each transaction t Î D { /* scan D for count */

5. C_t = subset(C_k,t) ; /* get subsets of t that are candidate */

6. for each candidate c Î C_t

7. c.count++ ; }

8. L_k = {c Î C_k | c.count ³ min_sup}

9.}

10.return L = È$_k$ L_k**Procedure apriori_gen(L_{k-1}: frequent (k-1) itemsets; min_sup)**

1. for each itemset l_1 Î L_{k-1}2. for each itemset l_2 Î L_{k-1}3. if (l_1[1]= l_2[1]) Ù (l_1[2]= l_2[2])Ù..Ù (l_1[k-2]= l_2[k-2]) Ù (l_1[k-1]<l_2[k-1]) then {

4. c = l_1 z l_2 /* join step: generate candidates */

5. if has_infrequent_subset(c, L_{k-1}) then

6. delete c; /* prune step: remove unfruitful candidate */

7. else add c to C_k; }

8. return C_k;

Procedure has_infrequent_subset(c: candidate k-itemset; L_{k-1}: frequent (k-1) itemsets)

1. for each (k-1)-subset s of c
2. if s Ï L_{k-1} then
3. return TRUE
4. return FALSE

Step1 of Apriori finds the frequent 1-itemsets, L_1. In step 2 to 10, L_{k-1} is used to generate candidate Ck in order to find Lk. The **apriori_gen** procedure generate candidate and then uses the Apriori property to eliminate those having a subsets that is not frequent (step3). This procedure is described below. Once all the candidates have been generated, the database scanned (step4). For each transaction, a subset function is used to find all subsets of the transaction that are candidates (step5). And the count for each of these candidates is accumulated (step6 and 7). Finally, all those candidates satisfying minimum support form the set of frequent itemsets, L. A procedure can then be called to generate association rules from the frequent itemsets.

The **apriori_gen** procedure performs two kinds of actions, namely, join and prune, as described above. In the join component L_{k-1} is joined with L_{k-1} to generate potential candidates (step1 to 4). The prune component (step 5 to7) employs the Aprioiri property to remove candidates that have subsets that is not frequent. The test for infrequent subsets in shown in procedure **has_infrequent_subsets**.

We show the road map of to discover the frequent itemsets by Aprioir algorithm.

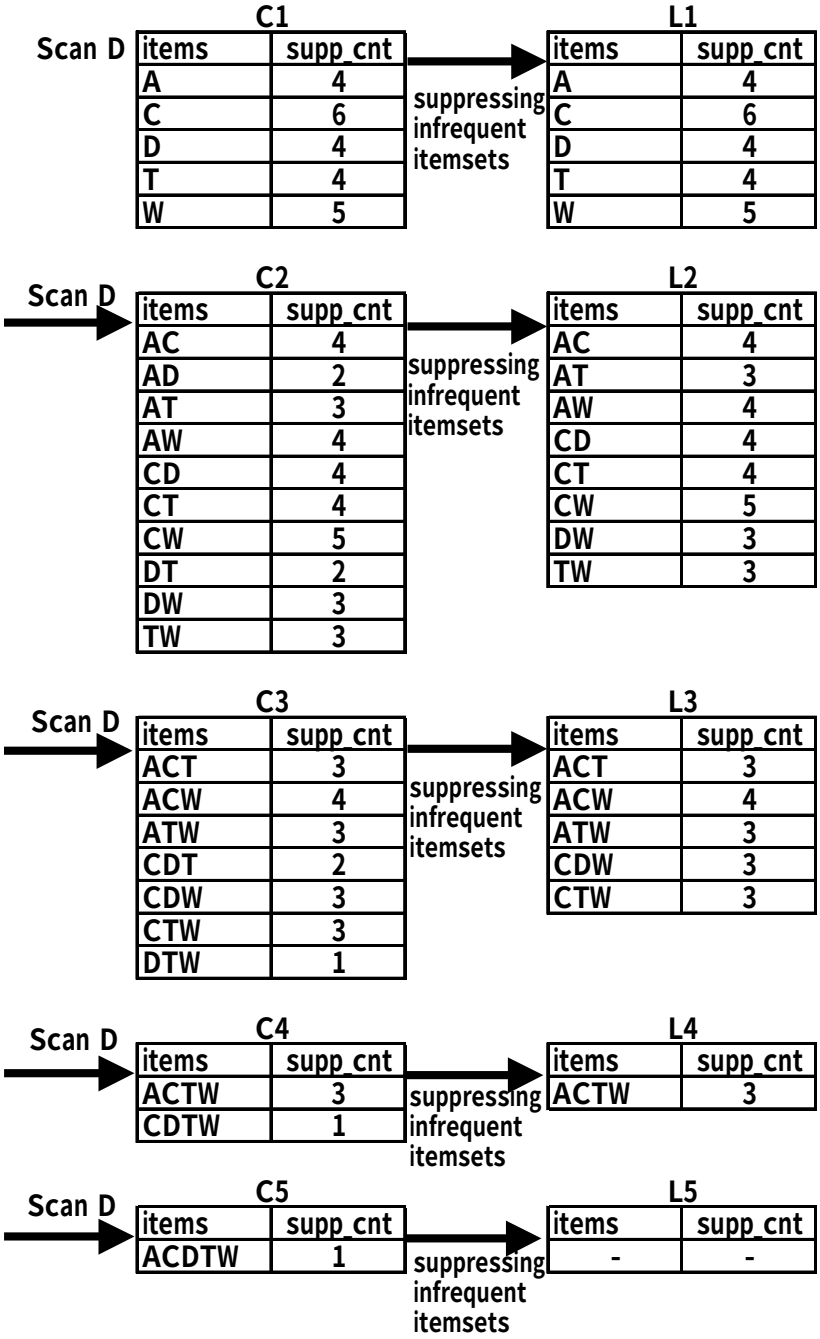


Figure.2.2 Extracting frequent itemset from D by Apriori algorithm.
With min_sup = 2

2.3 Problems of Redundant rules in association rules

This part, we present problem of redundant rules in association rule. It is widely recognized that the set of association rules can rapidly grow to be unwieldy, especially as we lower the frequency requirements. The larger set of frequent itemsets the number of rules presented to the user, many of which are redundant. This is true even for sparse datasets, but for dense datasets it is simply not fesible to mine all possible frequent itemsets, let alone to generate rules between itemsets. In such datasets one typically finds an exponential number of frequent itemsets.

In generating non-redundant association rules research, there are two definitions of non-redundant association rules, now we introduce two concepts.
One concept is minimal antecedent and maximal consequent rules with same support and same consequent. This concept indicates to generate only most informative rules. The other concept is minimal antecedent and minimal consequent with same support and same confidence. This concept indicates to generate minimal rules. Therefore we can easily understand rules, later selectively derive other rules of interest.

We discuss carefully after that chapter. About first concept in chapter 4, second concept in chapter 5.

Chapter 3. Frequent closed itemsets

In this chapter we describe the frequent closed itemsets, and show that these sets are necessary and sufficient to capture all the information about frequent itemsets, and has a small cardinality for the set of all frequent itemsets.

3.1 Frequent closed itemsets

3.1.1 Frequent closed itemsets thesis

The closure operator γ of the Galois connection [14] is the composition of the application ϕ, that associates with $O \subseteq T$ the item common to all transactions $t \subseteq T$, and the application φ, that associates with an itemset $l \subseteq I$ the transaction related to all items $i \in l$ (the transaction "containing" l).

The closure operator $\gamma = \phi \circ \varphi$ associates with an itemset l the maximal set of items common to all transactions containing l, i.e. the intersection of these transactions. Using this closure operator, we define the frequent closed itemsets that constitute a minimal non-redundant generating set for all frequent itemsets and their supports, and thus for all association rules, their supports and their confidences. This property comes from the facts that the supports of a frequent itemsets is equal to the support of its closure and that the maximal frequent itemsets are maximal frequent closed itemsets [10].

Definition 6 (Frequent closed itemsets)
A frequent itemset $l \subseteq I$ is a frequent closed itemset, iff $\gamma(l) = l$, and It's satisfy the min_supp. The smallest (minimal) closed itemset containing an itemset l is $\gamma(l)$, i.e. the closure of l.

Lemma 1
The set of maximal frequent itemsets $M = \{I \in L \mid \neg \exists I' \in L where I \subset I'\}$ is initical to the set of maximal frequent closed itemsets $MC = \{I \in FC \mid \neg \exists I' \in FC where I \subset I'\}$.

3.1.2 Algorithm Deriving Frequent Closed Itemsets from frequent itemsets

We show algorithm that objects to find frequent closed itemsets based on frequent itemsets.

Algorithm 2 Deriving Frequent Closed Itemsets from frequent itemsets

Input: frequent itemsets

Output: frequent closed itemsets

1) $FC_K \leftarrow L_K$

2) **For** $(i \leftarrow k-1; i \neq 0; i--)$ **do begin**

3) $FC_i \leftarrow \{\ \}$

4) **forall itemsets** $l \in L_i$ **do begin**

5) **isclosed** $\leftarrow$ **true;**

6) **forall itemsets** $l' \in L_{i+1}$ **do begin**

7) **if** $(l \subset l')$ **and** $(l.\sup port = l'.\sup port)$ **then isclosed** $\leftarrow$ **false;**

8) **end**

9) **if (isclosed = true) then** $FC_i \leftarrow FC_i \cup \{l\}$;

10) **end**

11) **end**

12) $FC_0 \leftarrow \{0\}$;

13) **forall itemsets** $l \in L_1$ **do begin**

14) **if** $(l.\sup port = \|O\|)$ **then** $FC_0 \leftarrow \{\ \}$;

15) **end**

First, the set of FC_k is initialized with the set of largest frequent itemsets L_k (step1). Then, the algorithm iteratively determines which i-itemsets in L_i are closed from L_{k-1} to L_1(step2 to 11). At the beginning of the i^{th} iteration the set FC_i of frequent closed itemsets is empty (step3). In steps 4 to 10, for each frequent itemset l in L_i. We verify that l has the same support as a frequent

$(i+1)$-itemset l' in L_{i+1} in which it is includes. If so, we have $l' \subseteq h(l)$ and then $l \neq f(l)$: l is not closed (step7). Otherwise, l is a frequent closed itemset and inserted in FC_i (step9). During the last phase, the algorithm determines if the empty itemset is closed by initializing FC_0 with the empty itemset (step12) and then considering all frequent 1-itemsets in L_1(step13 to 15). If a 1-itemset l has a support equal to the number of transactions in the context, meaning that l is common to all transactions, then the itemset 0 cannot be closed (we have $\sup p(\{0\} = \| o \| = \sup p(l))$) and is removed from FC_0 (step14). Thus, at the end of the algorithm, each set FC_i contains all frequent closed i-itemsets.

Example 3

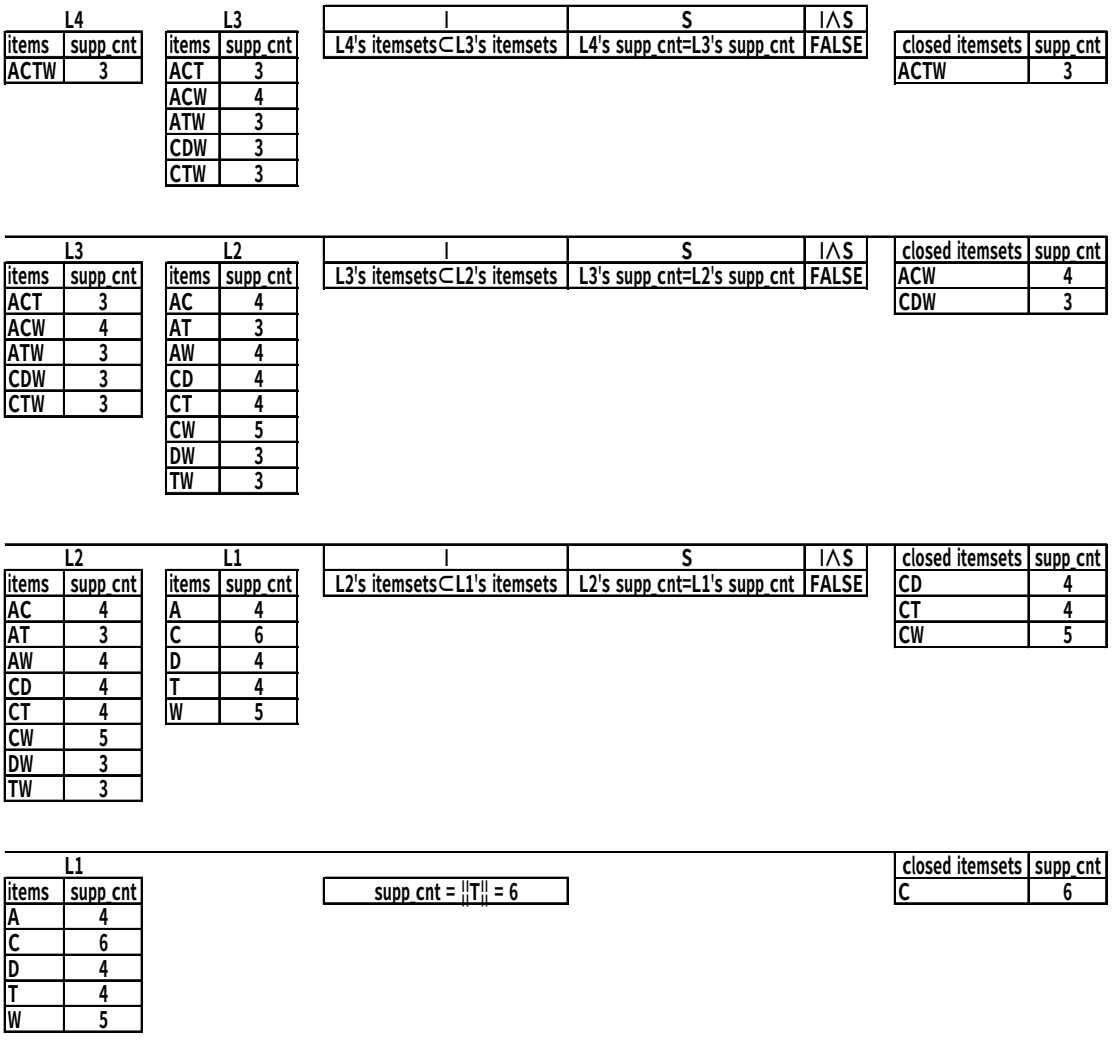


Figure 3.1 Deriving frequent closed itemsets from frequent otemsets

Correctness

Since all maximal frequent itemsets are maximal frequent closed itemsets(Lemma 1), the computation of the set FC_k containing the largest frequent closed itemsets is correct. The correctness of the computation of sets FC_i for $i < k$ relies on proposition1. this proposition enables to determine if a

frequent i-itemsets l is closed by comparing its support and the supports of the frequent $(i+1)$-itemsets in which l is included. If one of them has the same support as l, then l cannot be closed.

3.2 Generate generators based on frequent closed itemsets

In this chapter we show how to generate the generators. Generators can generate the frequent closed itemsets.

Basically, generators has two properties

generator $\subset$ *FrequentCloseditemsets*

generator.sup *port* = *FrequentCloseditemset*.sup *port*

Definition 7 (generators)
An itemset $g \subseteq I$ is a *generator* of a closed itemset l iff $\gamma(g) = l$ and $\neg\exists g' \subseteq I$. With $g' \subset g$ such that $\gamma(g') = l$. A generator of cardinality k is a k-generator.

Algorithm3. To generate generator by frequent closed itemsets

Input: FI: frequent itemsets, FCI: frequent closed itemsets.

Output: generator

1) **For all** FI_k **and** FCI_k

2) **For** $(i \leftarrow 0; i \leq k; i++)$ **do begin**

3) **Forall itemsets** $l \in FCI_i$

4) **For** $(j \leftarrow 0;\ j \leq k;\ j++)$

5) **Forall itemsets** $l' \in FI_j$

6) **if** $(l' \subset l)$ **and** $(l.\sup port = l'.\sup port)$

then $Can\_g_i \leftarrow \cup l'$

7) **and**

8) end

9) For $(i \leftarrow 0; i \leq k; i++)$

10) **For** $(l \leftarrow 0; l \leq Can\_g > count; l++)$

11) **forall item** $a \in Can\_g_{ij}$

for $(m \leftarrow l+1; l \leq Can\_g > count; m++)$

12) forall item $a' \in Can\_g_{ij}$

13) if $(a' \supset a)$ and $(a'.\sup port = a'.\sup port)$

14) then delete

15) else $\text{generator}_i \leftarrow \text{generator}_i \cup \{a\}$

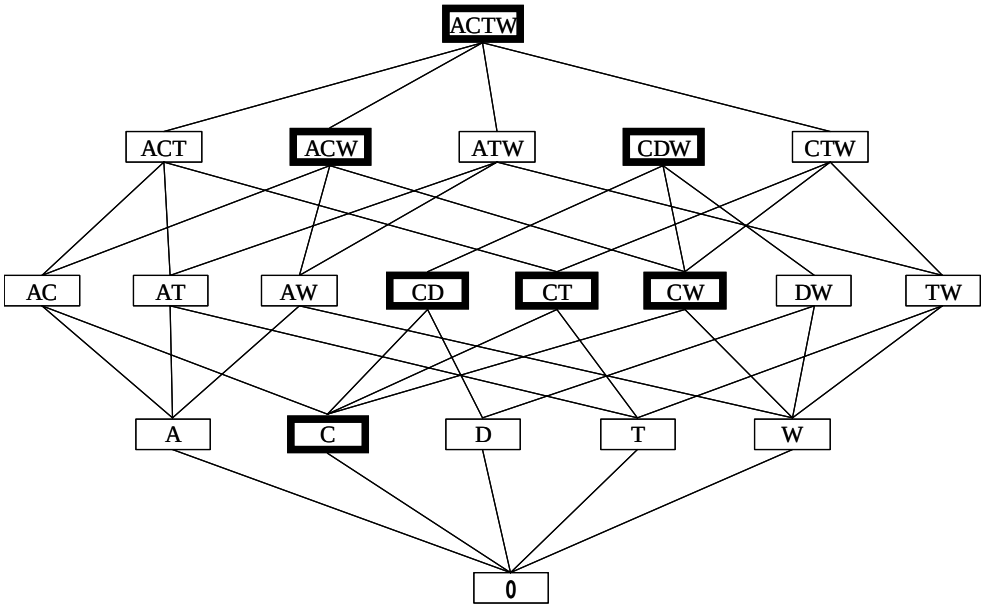
16) end

17) end

18) end

3.3 Frequent itemsets vs. frequent closed itemsets

We show that how effective we use frequent closed itemsets. For example, Figure 4 describes all frequent itemsets and frequent closed itemsets on the lattice structure in this paper's instance. If we focus on the item "A", including "A" frequent itemsets are {A}, {AC}, {AT}, {AW}, {ACW}, {ACT}. {ATW}, {ACTW}, the total number of frequent itemsets containing "A" is 8. But frequent closed itemsets are only two {ACW}, {ACTW} The meaning of this fact is that if A exists in database and min_supp = 3, if we find transactions {1, 3, 4, 5}, we can discover the frequent closed itemsets "ACW". Therefore we can obtain "ACW" all together with support count 4, i.e. maximal set of transaction {1, 3, 4, 5} is "ACW". And we can find other items "CTW". if we check transactions {1, 3, 5}, we can find the frequent closed itemsets "ACTW". Therefore we can obtain "ACTW" with support count 3, i.e. maximal set of transaction {1, 3, 5} is "ACTW". Totally apriori algorithm produced 19 itemsets, but if we use frequent closed itemsets only 7 itemsets. Therefore, We can reduce the number of frequency itemsets and we don't lose frequency information in their datasets.



Frequent itemsets □;

Frequent closed itemsets ☐;

Figure3.2.Frequent itemsets VS Frequent closed itemsets on the lattice structure.

condition	frequent itemsets	support_count	frequent closed itemsets	support_count
itemsets	{A}	4		
	{C}	6	{C}	
	{D}	4		
	{T}	4		
	{W}	5		
	{AC}	4		
	{AT}	3		
	{AW}	4		
	{CD}	4	{CD}	4
	{CT}	4	{CT}	4
	{CW}	5	{CW}	5
	{DW}	3		
	{TW}	3		
	{ACW}	4	{ACW}	4
	{CDW}	3	{CDW}	3
	{ACT}	3		
	{ATW}	3		
	{CTW}	3		
	{ACTW}	3	{ACTW}	3
total itemsets	19 itemsets		7 itemsets	

Table 3.1 frequent itemsets vs. frequent closed itmsets

Chapter 4. Strong non-redundant association rules

In this chapter we analyze definition of non-redundant association rules given by Lakhal, and based on this analysis we try to propose another form definition of non-redundant association rules. This definition called “strong non-redundant association rules”. Recall non-redundant association rules. As pointed out in example 4.1, it is desirable that only the non-redundant association rules with minimal antecedent and maximal consequent, i.e. the most useful and relevant rules, are extracted and presented to the user. Such rules are called non-redundant association rules.

4.1 Lakhal’s definition vs. Strong definition

Lakhal [4] considered, an association rule is redundant if it conveys the same information or less general information – than the information conveyed by another rule of the same usefulness and the same relevance. An association rule $r \in E$ is non-redundant and minimal if there is no other association rule $r' \in E$ having the same support and same confidence, of which the antecedent is a subset of the antecedent of r and the consequent is a superset of the consequent of r.

Definition 8 (Lakhal’s definition of non-redundant association rules)
An association rule $r : l_1 \rightarrow l_2$ is a non-redundant rule, iff there does not exist an association rule $r' : l_1' \rightarrow l_2'$ with support(r) = support (r’), confidence(r) = confidence (r’), and $l_1' \subseteq l_1, l_2 \subseteq l_2'$

Example 4.1

We present an example of non-redundant association rules based on Lakhal's definition [4]. Because our new definition basis his definition, so we present our basic concept of non-redundant association rules. This example extracted from UCI KDD's archive's datasets Mushroom [4]. These 9 rules have same support and same confidence.

1) Free gills →eatable
2) Free gills →eatable, partial veil
3) Free gills →eatable, white veil
4) Free gills, white veil →eatable
5) Free gills, partial veil →eatable
6) Free gills →eatable, partial veil, white veil
7) Free gills, partial veil →eatable, white veil
8) Free gills, white veil →eatable, partial veil
9) Free gills, partial veil, white veil →eatable

Obviously, give rule 6, rule 1 to 5 and 7 to 9 are redundant, since they do not convey any additional information to the user. Rule 6 has minimal antecedent and maximal consequent and it is the most informative among these nine rules.

But, our idea of non-redundant association rule is: "strong non-redundant association rules". This leads to a definition of non-redundant association rules in strong, i.e. those satisfy the min_supp and min_conf. This definition says, "Compare all strong rules". Noticing that Lakhal's definition uses same support and same confidence, but our definition do not use same support and same confidence, we consider those with large or small support and confidence. Therefore our definition is

Definition 9 (Strong non-redundant association rules)

An association rule $r: l_1 \to l_2$ is a non-redundant rule, iff there does not exist an association rule $r': l_1' \to l_2'$ with support(r) ≦ support(r'), confidence(r) ≦ confidence(r'), and $l_1' \subseteq l_1, l_2 \subseteq l_2'$

The meaning of this definition, we consider that why people decrease min_supp, so we think everyone want to discover other rules. Therefore if we want to know the new rules in the strong space, then we do not need same information rules, in the strong space. So our new definition helps to discover other rules in the strong space.
Our new definition can reduce non-informative rules, i.e. to reduce the non-informative rules in strong.

4.2 Rules with confidence 100%

The rules with confidence 100% of the form $r: l_1 \rightarrow (l_2 \setminus l_1)$, are rules between two frequent itemsets l_1 and l_2 whose closures are identical: $\gamma(l_1) = \gamma(l_2)$. Indeed, from $\gamma(l_1) = \gamma(l_2)$ we deduced that $l_1 \subset l_2$ and support (l_1) = support (l_2), and thus confidence (r) = 1. Since the maximum itemset among these itemsets is the itemset $\gamma(l_2)$, all supersets of l_1 that are subsets of $\gamma(l_2)$ have same support, and the rules between these two itemsets are rules with confidence 100%.

Definition 10 (confidence with 100% rules based on Lakhal)
Let FC be the set of frequent closed itemsets extracted from the context and, for each frequent closed itemset f, let denote G_f the set of generators of f.

Confidence with 100% rules = $\{r: g \rightarrow (f \setminus g) \mid f \in FC \wedge g \in G_f \wedge g \neq f\}$

Definition 11 (confidence with 100% rules based on strong)
Let FC be the set of frequent closed itemsets extracted from the context and, for each frequent closed itemset f, let denote G_f the set of generators of f.

Confidence with 100% rules =

$\{r: g \rightarrow (f \setminus g) \mid f \in FC \wedge g \in G_f \wedge g \neq f\}$

$\{r$ does not have $r' \mid g \supseteq g' \wedge (f \setminus g) \subseteq (f \setminus g)' \wedge f.\text{sup}\,port > f'.\text{sup}\,port\}$

The condition $g \neq f$ ensures that rules of the form $g \rightarrow 0$, which are non-informative, are discarded. The following proposition states that this definition.

Proposition 1.

(i) All valid confidence with 100% association rules, their supports and their confidences (that are equals to 100%) can be deduced from the rules of the generator, frequent closed itemsets and theirs supports.

(ii) The generator and frequent closed itemsets basis for exact association rules contains only minimal non-redundant rules.

Proof

Let $r : l_1 \rightarrow (l_2 \setminus l_1)$ be a valid confidence with 100% association rule between two frequent itemsets with $l_1 \subset l_2$. Since confidence(r) = 100% we have support (l_1) = support (l_2) . Given the property that the support of an itemset is equal to the support of its closure, we deduce that support ($\gamma(l_1)$) = support $(\gamma(l_2)) \rightarrow \gamma(l_1) \rightarrow \gamma(l_2) = f$

The itemset f is a frequent closed itemset $f \in FC$ and, obviously, there exists a rule $r' : g \rightarrow (f \setminus g)$ such that g is a generator of f for which $g \subset l_1$ and $g \subset l_2$. We show that the rule r and its support can be deduced from the rule r' and its support. Since $g \subset l_1$ and $g \subset l_2$, the rule r can be derived from the rule r'. From $\gamma(l_1) = \gamma(l_2) = f$, we deduce that support (r) = support (l_2) = support $\gamma(l_2)$ = support (f) = support (r').

□

Algorithm for constructing the generator and frequent closed itemsets basis

---

Algorithm 4. Constructing the generator and frequent closed itemsets basis.

Input: sets FC_k of k-groups of frequent k-generators;
Output: set GB of confidence with 100% association rules.

1) GB ~ {}
2) forall set $FC_k \in FC$ do begin
3) forall k-generator $g \in FC_k$ such that $g \neq \gamma(g)$ do begin
4) GB ~ GB U {(r : g $\rightarrow$ (,$\gamma(g) \setminus g$), γ (g)·support)};
5) end
6) end
7) return GB; _____Lakhal's definition stop
8) forall itemsets $g \in GB$ and $\gamma(g) \setminus g \in GB$
9) If { $\neg\exists(g \supseteq g') \wedge \neg\exists((\gamma(g) \setminus g) \subseteq (\gamma(g') \setminus g'))$
$\wedge$ support $(\gamma(g) \setminus g) \geq$ support $(\gamma(g') \setminus g')$}
then Strong{} $\leftarrow$ g and $(\gamma(g) \setminus g)$
11) end _____Strong definition stop

The algorithm starts by initializing the set GB as the empty set (step 1). Each set FC_k of frequent k-groups is then examined successively (steps 2 to 6). For each k-generator $g \in FC_k$ of the frequent closed itemset $\gamma(g)$ for which g is different from its closure $\gamma(g)$ (steps 3 to 5), the rule $r : g \Rightarrow (\gamma(g) \setminus g)$, whose support is equal to the support of g and $\gamma(g)$, is inserted into **G***B* (step 4). The algorithm returns the set GB containing non-redundant confidence with 100% association rules between generators and their closures in Lakhal's definition (step 7). For all frequent itemsets generator and frequent closed itemsets in GB (step8). Step (9), if antecedent g does not have subset g' in GB and consequent does not have superset $(\gamma(g) \setminus g)'$ in GB and these itemsets support are larger, input g and $\gamma(g)$ to Strong {}.

Example 4. Rules based on Lakhal's definition

Confidence with 100% association rules extracted from the context D for a minimal support threshold of 3/6 is presented in Table 1. It contains 8 rules whereas 18 confidence with 100% association rules are valid on the whole.

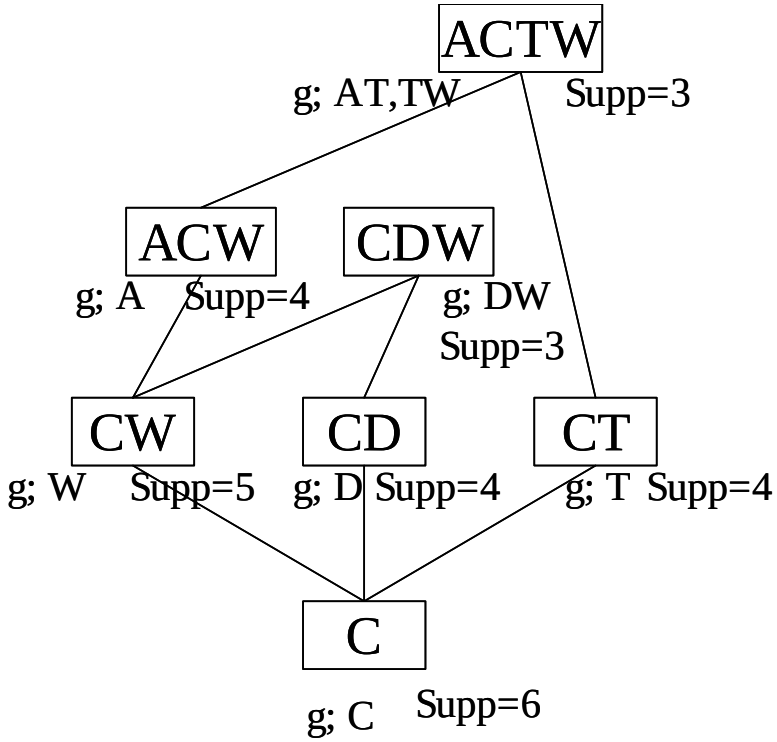


Figure4.1 generate rules on the lattice

FC_1

generator	closure	Sup_count
{A}	{ACW}	4
{C}	{C}	6
{D}	{CD}	4
{T}	{CT}	4
{W}	{CW}	5

FC_2

generator	closure	Sup_count
{AT}	{ACTW}	3
{DW}	{CDW}	3
{TW}	{ACTW}	3

generator	closure	Lakhal's definition	support_count
A	ACW	A→CW	4
C	C		
D	CD	D→C	4
T	CT	T→C	4
W	CW	W→C	5
AT	ACTW	AT→CW	3
DW	CDW	DW→C	3
TW	ACTW	TW→AC	3

Figure.4.2 Rules based on Lakhal's definition

Example 5. Strong definition

Our definition recalculate the rules about antecedent and consequent, Lakhal's definition produced 8 rules, but our Strong definition contain 5 rules.

generator	closure	Lakhal's definition	support_count
A	ACW	A→CW	4
C	C		
D	CD	D→C	4
T	CT	T→C	4
W	CW	W→C	5
AT	ACTW	AT→CW	3
DW	CDW	DW→C	3
TW	ACTW	TW→AC	3

Strong definition	support_count
A→CW	4
D→C	4
T→C	4
W→C	5
TW→AC	3

Figure.4.3 lakhal definition vs. strong definition

For example, DW→C in Lakhal's definition, our definition does not produce it. Because compare to D→C, DW has subset D and C is equal to C and support count is lower, so our definition delete this rule.

4.3 confidence with lower than 100% rules

Each confidence with under100% association rule $r: l_1 \rightarrow (l_2 \setminus l_1)$, is a rule between two frequent itemsets l_1 and l_2 such that the closure of l_1 is a subset of the closure of l_2: $\gamma(l_1) \subset \gamma(l_2)$. The non-redundant confidence with under 100% association rules with minimal antecedent l_1 and maximal consequent $(l_2 \setminus l_1)$ are deduced from this characterization.

Let f_1 be the frequent closed itemset which is the closure of l_1 and g_1 a generator of f_1 such as $g_1 \subset l_1 \subset f_1$. Let f_2 be the frequent closed itemset which is the closure of l_2 and g_2 a generator of f_2 such as $g_2 \subset l_2 \subset f_2$.The rule $g_1 \rightarrow (f_2 \setminus g_1)$ between the generator g_1 and the frequent closed itemset f_2 is the non-redundant rule among the rules between an itemset of the interval* $[g_1\ f_1]$ and an itemset of the interval $[g_2, f_2]$. Indeed, the generator g_1 is the minimal

itemset whose closure is f_1 which means that the antecedent g_1 is minimal and that the consequent $(f_2 \setminus g_1)$ is maximal since f_2 is the maximal itemset of the interval $[g_2, f_2]$'. The generalization of this property to the set of all rules between two itemsets l_1 and l_2 defines the informative basis which thus consists of all the non-redundant confidence with under 100% association rules of minimal antecedents and maximal consequents characterized.

*The interval $[l_1\ l_2]$ contains all the supersets of l_1 that are subsets of l_2.

Definition 12 (Informative basis for confidence with under100% association rules).

Let FC be the set of frequent closed itemsets and let denote G the set of their generators extracted from the context. The informative basis for confidence with under100% association rules is

$$IB = \{r : g \rightarrow (f \setminus g) \mid f \in FC \wedge g \in G \wedge \gamma(g) \subset f\}.$$

Proposition 2.

(i) All valid confidence with under 100% association rules, their supports and confidences, can be deduced from the rules of the informative basis, their supports and theirs confidences.

(ii) All rules in the informative basis are non-redundant confidence with under 100% association rules.

Proof.

Let $r : l_1 \rightarrow (l_2 \setminus l_1)$ be a valid confidence with under100% association rule between two frequent itemsets with $l_1 \subset l_2$. Since confidence(r) < 1 we also have $\gamma(l_1) \subset \gamma(l_2)$. For any frequent itemsets l_1 and l_2, there is a generator g_1 such that $g_1 \subset l_1 \subset \gamma(l_1) = \gamma(g_1)$ and a generator g_2 such that $g_2 \subset l_2 \subset \gamma(l_2) = \gamma(g_2)$. Since $l_1 \subset l_2$, we have $l_1 \subseteq \gamma(g_1) \subset l_2 \subseteq \gamma(g_2)$ and the rule $r' : g_1 \rightarrow (\gamma(g_2) \setminus g_1)$ belongs to the informative basis IB. We show that the rule r, its support and its confidence can be deduced from the rule r', its support and its confidence. Since $g_1 \subset l_1 \subset \gamma(g_1) \subset g_2 \subset l_2 \subset \gamma(g_2)$, the antecedent and the consequent of r can be

rebuilt starting from the rule r'. Moreover, we have $\gamma(l_2)=\gamma(g_2)$ and thus support(r) = support (l_2) = support ($\gamma(g_2)$) = support (r'r'). Since $g_1 \subset l_1 \subset \gamma(g_1)$, we have support ($g_1$) = support ($l_1$) and we thus deduce that: confidence(r) = support (l_2) / support (l_1) = support ($\gamma(g_2)$) / support (g_1) = confidence (r') .

From the definition of the informative basis we deduce the definition of the transitive reduction of the informative basis that is itself a basis for all confidence with under 100% association rules. We note $l_1 < l_2$ if the itemset l_1 is an immediate predecessor of the itemset l_2, i.e. $\neg\exists l_3$ such that $l_1 \subset l_3 \subset l_2$

The transitive rules of the informative basis are of the form $r: g \rightarrow (f \setminus g)$ for a frequent closed itemset f and a frequent generator g such that $\gamma(g) \subset f$ and $\gamma(g)$ is not an immediate predecessor of *fin*FC FC: $\gamma(g) \neg < f$. The transitive reduction of the informative basis thus contains the rules with the form $r: g \rightarrow (f \setminus g)$ for a frequent closed itemset f and a frequent generator g such as $\gamma(g) < f$.

Definition 14 (Transitive reduction of the informative basis based on Lakhal).
Let FC be the set of frequent closed itemsets and let denote G the set of their generators extracted from the context. The transitive reduction of the informative basis for confidence with under 100% association rules is:

$$RI = \{r : g \rightarrow (f \setminus g) \mid f \in FC \wedge g \in G \wedge \gamma(g) < f\}$$

Definition 15 (Informative basis based on Strong).
Let FC be the set of frequent closed itemsets and let denote G the set of their generators extracted from the context. The transitive reduction of the informative basis for confidence with under 100% association rules is:

$$RI = \left\{ \begin{array}{l} r: g \rightarrow (f \setminus g) \mid \\ f \in FC \wedge g \in G \wedge \gamma(g) < f \wedge g \supseteq g' \wedge (f \setminus g) \subseteq (f \setminus g)' \wedge f.\text{sup}\,port > f'.\text{sup}\,port \end{array} \right\}$$

Obviously, it is possible to deduce all the association rules of the informative basis with their supports and their confidences, and thus all the valid confidence with under100% rules, from the rules of this transitive reduction, their supports and their confidences. This reduction makes it possible to decrease the number of

confidence with under 100% rules extracted by preserving the rules which confidences are the highest (since the transitive rules have confidences lower than the non-transitive rules by construction) without losing any information.

Algorithm 5. Generating the transitive reduction of the informative basis.

Input: sets FC_k of k-groups of frequent k-generators;
min_confidence threshold

Output: Transitive reduction of the informative RI

1) RI ~ {}
2) for $(k \leftarrow 1; k \leq \mu - 1; k++)$ do begin
3) forall k-generator g$\in$ G do begin
4) $Succ_g \leftarrow \{\}$;
5) for$(j = |\gamma(g)|; j \leq \mu; j++)$ do begin
6) $S_j \leftarrow \{f \in FC \mid f \supset \gamma(g) \wedge |f| = j\}$;
7) end
8) for$(j = |\gamma(g)); j \leq \mu; j++)$ do begin
9) forall frequent closed itemsets $f \in S_j$ do begin
10) if$(\neg\exists s \in Succ_g \mid s \subset f)$ then begin
11) $Succ_g \leftarrow Succ_g \cup f$;
12) $r.confidence \leftarrow f.\text{sup } port / g.\text{sup } port$;
13) if$(r.confidence \geq \min_confidence)$;
14) then $RI \leftarrow RI \cup \{r : g \rightarrow (f \setminus g), r.confidence, f.\text{sup } port\}$;
15) endif
16) end
17) end
18) end
19) end
20) return RI; _____ Lakhal's definition stop
21) forall itemsets g $\in RI$ and γ(g)\g) $\in RI$
21) If { $\neg\exists$(g $\supseteq$g')$\wedge$ $\neg\exists((\gamma(g) \setminus g) \subseteq (\gamma(g') \setminus g'))$
$\wedge$ support $(\gamma(g) \setminus g) \geq$ support $(\gamma(g') \setminus g')$}
then Strong{}$\leftarrow$ g and $(\gamma(g) \setminus g)$
22)end _____Strong definition stop

The pseudo code of the Gen-RI algorithm for constructing the transitive reduction of the informative basis for the confidence with under 100% association rules using the set of frequent closed itemsets and their generators is presented in algorithm 5.

Each element of a set FC_k consists of three fields: *generator*, *closure* and *support*. The algorithm constructs for each generator g considered a set *Succg* containing the frequent closed itemsets that are immediate successors of the closure of g.

The algorithm starts by initializing the set *RI* with the empty set (step l). Each set FC_k of frequent k-groups is then examined successively in the increasing order of the values of k (steps 2 to 14). For each k-generator $g \in FC_k$ of the frequent closed itemset $\gamma(g)$. (steps 3 to 18), the set *Succg* of the successors of the closure of $\gamma(g)$ is initialized with the empty set (step 4) and the sets S_j of frequent closed j-itemsets that are supersets of $\gamma(g)$ for $|\gamma(g)| < j < \mu$ *3 are constructed (steps 5 to 7). The sets S_j are then considered in the ascending order of the values of j (steps 8 to 17). For each itemset $f \in S_j$ that is not a superset of an immediate successor of $\gamma(g)$ in Succg (step 10), f is inserted in *Succg* (step 11) and the confidence of the rule $r : g \rightarrow (f \setminus g)$ is computed (step 12). If the confidence of r is greater or equal to the minimal confidence threshold minconfidence, the rule r is inserted in *RI* (steps 13 to 15). When all the generators of size lower than μ have been considered, the algorithm returns the set *RI* (step 20).

*3 We denote, μ the size of the longest maximal frequent closed itemsets.

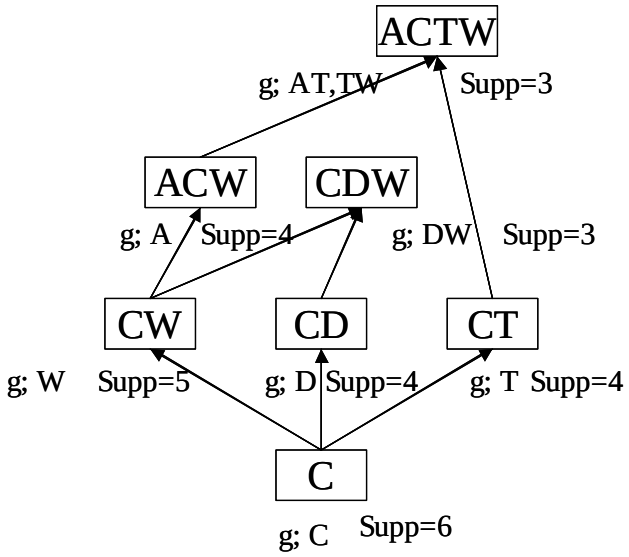


Figure 4.4 generate rules confidence with lower than 100%

generator	closure	closed superset	rules	support	confidence
A	ACW	ACTW	A→CTW	75	75
A	ACW	ACDW	A→CDW	33.3	50
C	C	CD	C→D	66.7	66.7
C	C	CT	C→T	66.7	66.7
C	C	CW	C→W	83.3	83.3
C	C	CDW			
C	C	ACW			
C	C	ACTW			
D	CD	CDW	D→CW	50	75
D	CD	CDT	D→CT	33.3	50
T	CT	ACTW	T→ACW	50	75
T	CT	CDT	T→CD	33.3	50
W	CW	CDW	W→CD	50	60
W	CW	CWA	W→AC	66.7	80
W	CW	ACTW			
AT	ACTW	ACDTW	AT→CDW	16.7	33.3
AD	ACDW	ACDTW	AD→CTW	16.7	50
DW	CDW	ACDW	DW→AC	33.3	66.7
DT	CDT	ACDTW	DT→ACW	16.7	50
TW	ACTW	ACDTW	TW→ACD	16.7	33.3

Table 4.1 generated rules confidence with lower than 100%

Example 7 To compare Lakhal's definition vs. Strong definition

Lakhal definition

rules	support	confidence
A→CTW	75	75
A→CDW	33.3	50
C→D	66.7	66.7
C→T	66.7	66.7
C→W	83.3	83.3
D→CW	50	75
D→CT	33.3	50
T→ACW	50	75
T→CD	33.3	50
W→CD	50	60
W→AC	66.7	80
AT→CDW	16.7	33.3
AD→CTW	16.7	50
DW→AC	33.3	66.7
DT→ACW	16.7	50
TW→ACD	16.7	33.3

total number of rules = 16

Strong definition

rules	support	confidence
A→CTW	75	75
A→CDW	33.3	50
C→D	66.7	66.7
C→T	66.7	66.7
C→W	83.3	83.3
D→CW	50	75
D→CT	33.3	50
T→ACW	50	75
T→CD	33.3	50
W→CD	50	60
W→AC	66.7	80
TW→ACD	16.7	33.3

total number of rules = 12

Figure4.5 Lakhal's definition vs. strong definition

Chapter5 Efficient algorithm to generate non-redundant association rules

In this chapter we investigate the definition of non-redundant association rules given by Zaki. We present an efficient algorithm to generate non-redundant association rules that definition.

5.1 Minimal antecedent and minimal consequent

An association rule is form of the $l_1 \to l_2$, where $l_1, l_2 \in I$. Its support equals $\gamma(l_1 \cup l_2)$, and its confidence is given as $conf = P(l_1 \mid l_2) = \mid \gamma(l_1 \text{ Y } l_2) \mid / \mid \gamma(l_1) \mid$. We are interested in finding all high support and confidence rules, i.e. rules satisfy the min_supp and min_conf.

It is widely recognized that the set of such association rules can rapidly grow to e unwieldy. In this chapter we will show the frequent closed itemsets help us form a generating set of rules, from which all other association rules can be inferred. Thus, only a small and understandable set of rules can be presented user, who can later selettively derive other rules of interest.

Definition (Zaki's definition of non-redundant association rules)
An association rule $r: l_1 \to l_2$, We say that a rule r is more general than a rule $r': l_1' \to l_2'$, denoted $r \le r'$ provided that r' can be generated by additional items to either the antecedent or consequent of r. Let $R = \{R_1 \Lambda, R_n\}$ be a set of rules, such that all their confidence are equal. Then the non-redundant rules in the collection R are those that are most general, with support(r) = support (r'), confidence(r) = confidence (r'), and $l_1' \subseteq l_1, l_2 \subseteq l_2'$

This definition indicates that minimal antecedent and minimal consequent association rules are non-redundant association rules.

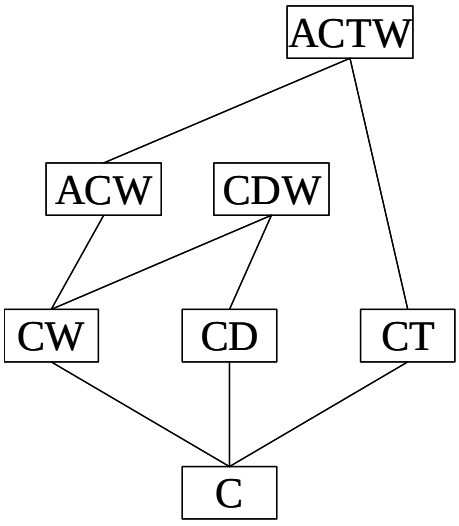


Figure 5.1 frequent closed itemsets lattice

We show how to eliminate the redundant association rules, i.e. rules having the same support and confidence as some more general rules. In this section, we showed that the support of an itemsets l equals the support of its closure $\gamma(l)$.

Thus it suffices to consider rules only among the frequent closure itemsets. In other words the rule $r: l_1 \to l_2$ is exactly the same as rule $r: \gamma(l_1) \to \gamma(l_2)$.

Another observation that follows from the frequent closed itemsets lattice is that sufficient to consider rules among adjacent frequent closed itemsets, since other rules can be inferred be transitivity, that is

Lemma transitivity

Let $l_1\, l_2\, l_3$ be frequent closed itemsets, with $l_1 \subseteq l_2 \subseteq l_3$.

If $l_1 \to l_2 (conf = p)$, and $l_2 \to l_3 (conf = q)$ then $l_1 \to l_3 (conf = p * q)$

5.2 Rules with confidence 100%

In this section, we consider the how to generate confidence with 100% rules.

Lemma (confidence with 100% rule)

An association rule $l_1 \to l_2$ confidence =100%, if and only if $\gamma(l_1) \subseteq \gamma(l_2)$.

This theorem says that all 100% confidence rules are those that are directed from super frequent closed itemsets to a sub frequent closed itemsets.

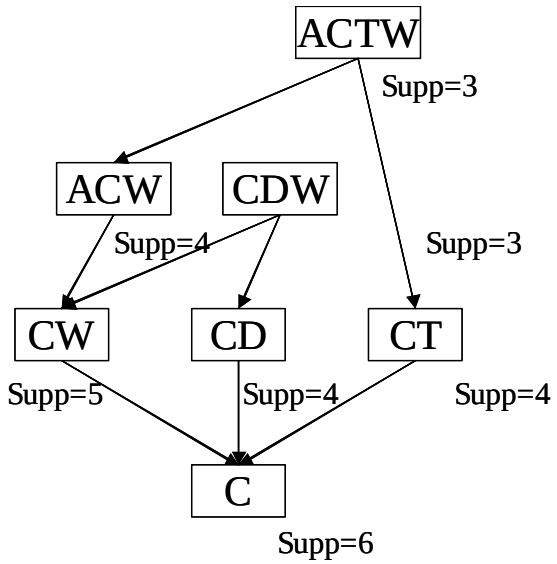


Figure 5.2 rules based on original theorem

For example frequent closed itemsets CW and C, the rule W→C is a 100% confidence rule. Note that if we take the closure on both sides of rule, we obtain CW→C, i.e. a rule between closed itemsets, but since the antecedent and consequent are not disjoint in this case, we prefer to write the rule as W→C, although both

Theorem 1

Let $R = \{R_1 \Lambda , R_n\}$ be a set of rules with 100% confidence rules, such that $I_1 = \gamma(l_1 \cup l_2)$ and $I_2 = \gamma(l_2)$ for all the rules $R_i \neq R_I$ are more special than R_I, and thus are redundant.

But we noticed what mean of this theorem, in chapter 3, we showed that the support of an itemset l_1 equals to the support of its closure $\gamma(l_1)$ and its generator g. Therefore it suffices to consider rules only among the frequent closed itemsets.

Thus most minimal itemsets is generator, so we can generate rules directly generator to generator, instead of closure items.

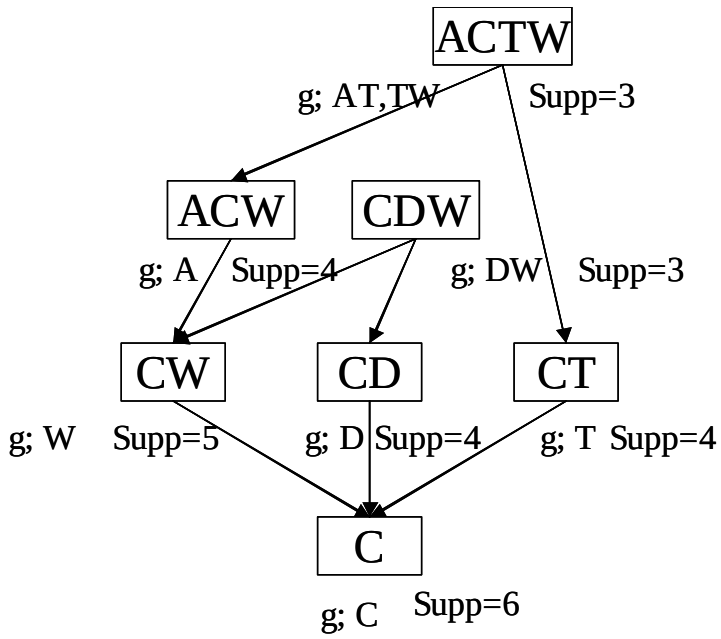


Figure 5.3 rules based generator

5.3 confidence with lower than 100% rules

We now turn to problem of finding a generating set for association rules with confidence less than 100%. As befor, we need to consider only the rules between adjacent frequent closed itemsets.

Theorem

Let $R = \{R_1 \Lambda , R_n\}$ be a set of rules with confidence $p < 1.0$, such that $I_1 = \gamma(l_1)$ and $I_2 = \gamma(l_1 \Upsilon l_2)$ for all rules R_i. Let R_I denote the rule $I_1 \rightarrow I_2$. Then all the rules $R_i \neq R_I$ are more specific than R_I, and thus are redundant.

This theorem differs from that of the 100% confidence rules to account for the up arc.

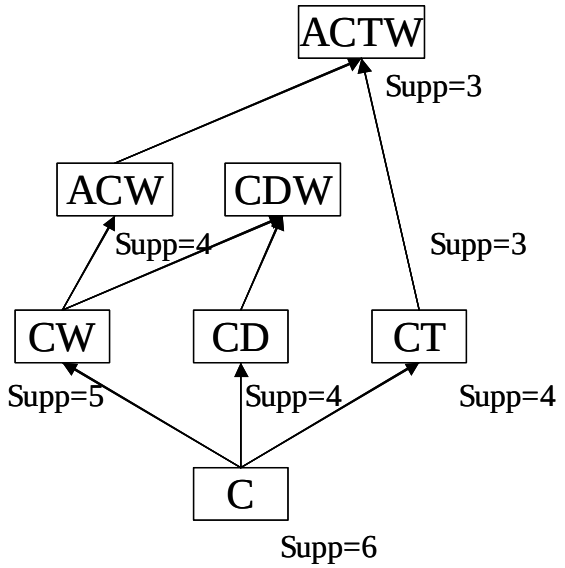


Figure 5.4 Rules based on original theorem

For example frequent closed itemsets C and CW, the rule C→W is a rule with less than 100% confidence. Note that if we take the closure on both sides of rule, we

obtain CW→C, i.e. a rule between closed itemsets,

But we noticed what mean of this theorem, in chapter 3, we showed that the support of an itemset l_1 equals to the support of its closure $\gamma(l_1)$ and its generator g. Therefore it suffices to consider rules only among the frequent closed itemsets.

Thus most minimal itemsets is generator, so we can generate rules directly generator to generator, instead of closure items. In this case, i.e. confidence less than 100% we can generate the rules, generator W comes from frequent closed itemsets CW to generator C comes from frequent closed itemsets C.

So we can generate rule directly. Thus our method is effective.

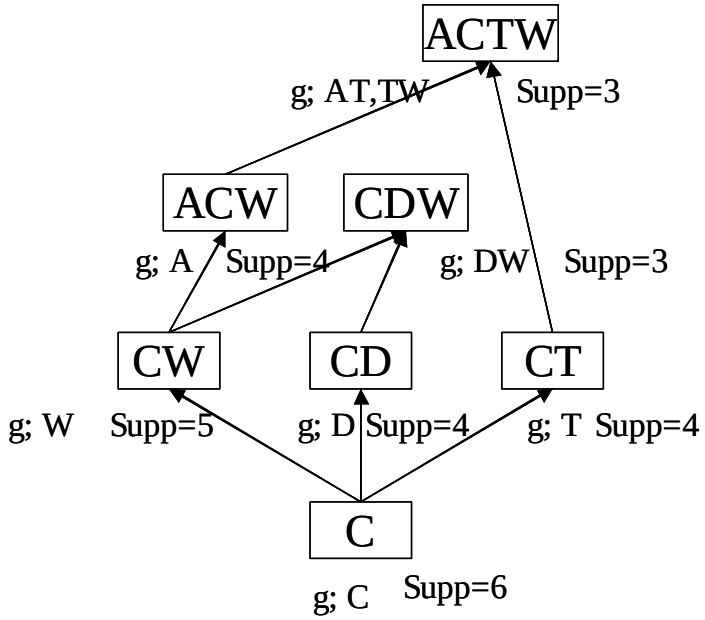


Figure 5.5 Rules based generator

Chapter 6 Experimentals

In this section we describe experimental environments and results. So we try to visualize my definition of non-redundant association rule is more effective.

6.1 Experimental design

Our experiments are three categories. First we try to compare how effective Lakhal's definition, so we try to compare Apriori vs. Lakhal's algorithm. Second we try to how effective our new definition, so we try to compare Lakhal's algorithm vs. Strong algorithm. Finally we compare three algorithms.

Datasets

We used the 10 datasets during these experiments comes from UCI datasets.

	name	*number of items*	*number of transactions*
1	sample	5	6
2	corral	7	31
3	muxf6	7	63
4	party	11	101
5	tutrial	11	10
6	lenses	12	16
7	golf	14	14
8	Co2	14	31
9	mux	14	63
10	led7	16	200
11	hungarian	17	196
12	Monk1	19	124
13	monk2	21	169

Table 6.1 datasets

6.2 Experimental results

6.2.1 apriori vs. Lakhal's algorithm

In this part we present the result of experimental to compare Apriori to Lakhal's definition. Generated rules of Lakhal's definition are all times smaller than Arioiri. Specially if threshold of support i.e. min_supp is small, then generated rules are very small, Lakhal's definition is more effective in this case, i.e. if min_supp is small then Apriori produced many redundant rules.

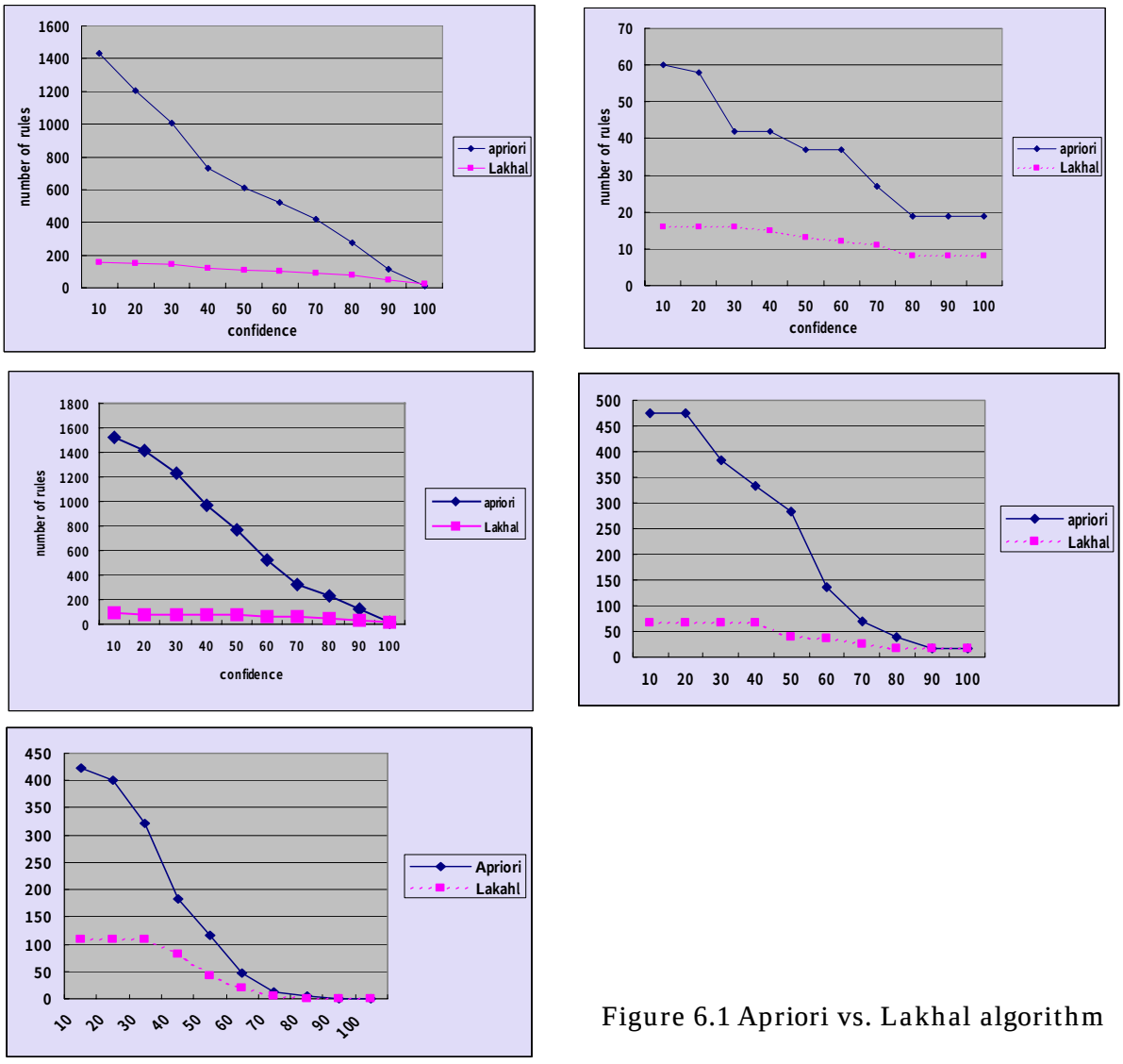


Figure 6.1 Apriori vs. Lakhal algorithm

6.2.2 Lakhal's algorithm vs. strong algorithm.

In this part, we present the result of experimental to compare Lakhal's definition to new definition "Strong non-redundant association rule". We present two cases result, our new definition is more effective case and the other is same result of total number of generated rules.

Almost experiments, our strong definition can reduced the number of rules, but some time the result is equal to previous work..

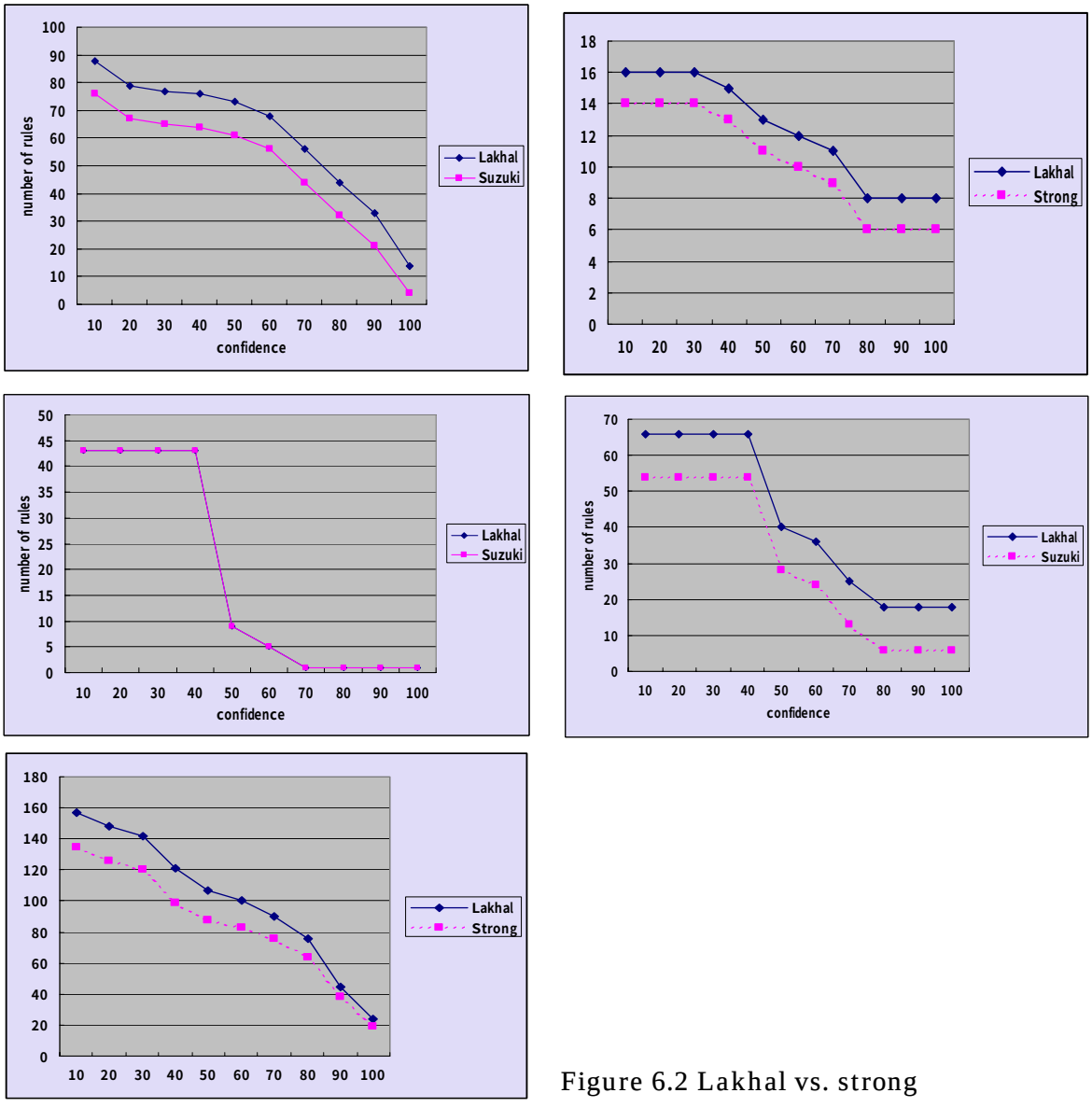


Figure 6.2 Lakhal vs. strong

Chapter 7 Conclusion

Finally we conclude this paper.

Recall, Our research has four objectives.

1. To investigate the problem of non-redundant association rules.
2. To try to formulate another form of non-redundant association rules.
3. To develop an algorithm that finds non-redundant association rules.
4. To try to improve the algorithm.

The frequent closed itemsets helps us to produce non-redundant association rules in chapter 3. If we use frequent closed itemsets, so we can reduce redundant itemsets.

About first objective, We discus chapter 3,4 and 5, two definition exist in previous works, to generate minimal antecedent and maximal consequent rules can deduce the other rules.

About second objective and third objective, we try to develop new definition of non-redundant association rule in chapter4. We experiments in chapter6This definition generate smaller size rules than Lakhal's definition; we do some experiments in chapter 6.

About fourth objective, we did not experiment, but obviously my algorithm is efficiently, because Zaki, produced all candidate rules, the after to select most general rule, but my algorithm can generate most general rule directly. So it may the cost of calculation is lower than original algorithm.

Finally we cannot say when or which time to use which algorithm. But we investigate all definition of previous and our non-redundant association rules, so

if we analyze some datasets, our research help people who want to discover new knowledge by association rule mining.

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