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# A Simulation Model on an Artificial Society with Multiple Adaptive Agents for Financing Public Goods with Lotteries

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## ABSTRACT

Recently, effectiveness of lotteries for financing public goods has been shown by developing a mathematical model of equilibrium and conducting experiments with human subjects, compared with voluntary contributions. In this paper, by using an agent-based simulation model in which artificial adaptive agents have a mechanism of decisions and learning based on neural networks and genetic algorithms, we show effectiveness of lotteries for financing, and examine validity of the mathematical equilibrium model and the experiments with human subjects.

**Keywords**: agent-based simulations, financing public goods with lotteries, neural networks, genetic algorithms.

## 1. INTRODUCTION

Morgan [1] develops a mathematical equilibrium model of lotteries for financing public goods. Morgan and Sefton [2] conduct experiments with human subjects and focus on the following three points. First, when it is efficient to provide positive amount of a public good, the provision of the public good through the lottery mechanism is more than the provision through the voluntary contribution mechanism. Second, the provision of the public good increases with the size of the lottery prize. Third, wagers of the lottery mechanism vary with the return from the public good. On the whole the results of the experiments support the above three predictions from the mathematical equilibrium model.

Simulation analysis is advantageous for implementing a model of a certain social system and examining effectiveness of the social systems, and then in this paper we employ simulation analysis in order to show effectiveness of lotteries for financing and to examine validity of the mathematical equilibrium model and the experiments with human subject. While mathematical models are based on optimization such as maximization of an individual payoff or utility, our agent-based simulation model employs adaptive behavior models in which agents evaluate results of their decisions and revise policies to choose one of alternatives as actual decision makers do so. From this sense we can expect a reasonable interpretation of the gaps between the results of the mathematical equilibrium model and the experiments with human subjects.

As concerns approaches based on adaptive behavior models, Holland and Miller [3] interpret most of economic systems as complex adaptive systems, and point out that simulations using artificial societies with adaptive agents is effective for analysis of such economic systems. Axelrod [4] insists on the need for simulation analysis in social sciences, and states that purposes of the simulation analysis include prediction, performance, training, entertainment, education, proof and discovery. The number of researches related to adaptive behavior models and agent-based simulation analyses have been reported [5, 6, 7, 8, 9, 10, 11, 12], and then effectiveness of simulation analysis with artificial adaptive agents has been recognized. In this paper, to examine the effectiveness of lotteries for financing public goods, we conduct agent-based simulations with a decision making and learning mechanism based on neural networks and genetic algorithms by extensively varying values of the parameters of the mathematical equilibrium model [1] which is also the basis of the experiments with human subjects by Morgan and Sefton [2].

In the simulations, we deal with three parameters: the exogenous contribution which becomes the prize in the lottery game and utilizes for funding the public fund directly in the voluntary contribution game, the marginal per capita return of the public good provision, and the group size which is the number of players in the games. Furthermore, providing a simple learning mechanism and a more elaborate one, we examine which of agent bahavior with those two learning mechanisms approaches closer to the prediction of the mathematical equilibrium model.

## 2. THE MODEL AND THE EXPERIMENTS

A mathematical equilibrium model for financing public goods by lotteries is developed by Morgan [1]. Let a set of players be denoted $N = \{1, \dots, n\}$, where a player is a contributor in a voluntary contribution game or a bettor in a lottery game. In general, player $i$ has the following payoff function.

$$U_i = w_i + h_i(G), \quad (1)$$

where $w_i$ is the wealth of player $i$ and $G \in \mathbb{R}_+$ denotes the level of the public good provided; $\mathbb{R}_+$ is the set of non-negative real numbers; player $i$ has diminishing marginal payoff from the provision of the public good, i.e., $h_i'(\cdot) > 0$ and $h_i''(\cdot) < 0$; and $U_i$ is assumed to be quasi-linear.

For a voluntary contribution game, player $i$ chooses $x_i \in [0, w_i]$ so as to maximize the payoff

$$U_i = w_i - x_i + h_i(x(N)), \quad (2)$$

where $x_i$ is the amount of wealth contributed by player $i$, and $x(N) \equiv \sum_{i \in N} x_i$ denotes the total contribution of all the players.

For a lottery game, player $i$ chooses a wager $x_i \in [0, w_i]$ so as to maximize the expected payoff

$$U_i = w_i - x_i + \frac{x_i}{x(N)} R + h_i(x(N) - R), \quad (3)$$

where $R$ is a prize of some fixed amount.

The results of the mathematical equilibrium model [1] are summarized as follows.

1. Voluntary contributions underprovide the public good relative to first-best best levels.
2. The lottery with a fixed prize has a unique equilibrium.
3. The lottery with a fixed prize provides more of the public good than the voluntary contributions.

In the experiments by Morgan and Sefton [2], a linear-homogeneous version of the above-mentioned model [1] is treated. For the voluntary contribution game, each player has the same endowment $e$, and an exogenous contribution $R$ is used to fund the public good together with the total contribution of all the players. Thus, the payoff of player $i$ is represented by

$$U_i = e - x_i + \beta(x(N) + R), \quad (4)$$

where $\beta$ is the constant marginal per capita return of the provision of the public good, and player $i$ chooses a contribution $x_i \in [0, e]$ so as to maximize the payoff (4).

Assuming $\beta < 1$, for all $i$, the predicted equilibrium contribution of the voluntary contribution game is $x_i^{VC} = 0$.

For the lottery game, the whole sum of wagers is assigned to the public good provision, and the exogenous contribution $R$ is used to fund a prize. Therefore, the expected payoff of player $i$ is represented by

$$U_i = e - x_i + R\frac{x_i}{x(N)} + \beta x(N), \quad (5)$$

player $i$ chooses a wager $x_i \in [0, e]$ so as to maximize the payoff (5). Then, the predicted equilibrium contribution of the lottery game is $x_i^L = R(n-1)/\{n^2(1-\beta)\}$.

In the experiments, the payoff (4) is given to a subject in the voluntary contribution game or the payoff (5) in the lottery game. We focus on one of the two experiments conducted by Morgan and Sefton [2] , and the values of the primary parameters of the experiment are given as: the number of players $n = 4$, the initial endowment $e = 20$, the exogenous contribution $R = 8$, and the marginal per capita return $\beta = 0.75$. The game is iterated 20 times each treatment. The results are summarized as follows.

1. In the voluntary contribution game, the average contribution in the initial round was about 10.5, it decreased as rounds proceeded, and finally it became 8.075 in the final 20th round. The final average contribution 8.075 was considerably larger than the equilibrium contribution $x_i^{VC} = 0$.
2. In the lottery game, the average wager in the initial round was about 10, it was almost changeless as rounds proceeded, and finally it became 10.35 in the final 20th round. The final average wager 10.35 was larger than the equilibrium wager $x_i^L = \frac{8(4-1)}{4^2(1-0.75)} = 6$ and the final average contribution of 8.075 in the voluntary contribution game of the experiment.
3. In the treatment of the lottery game with the exogenous contribution $R = 16$ which was twice as large as that of the baseline treatment, the average wager in the initial round was about 13, it was almost changeless as rounds proceeded, and finally it became 13.825 in the final 20th round. This result implies that large prize lotteries will be more successful fund-raising devices than smaller scale endeavors.
4. In the treatment of the lottery game without the marginal per capita return, i.e., $\beta = 0$, the average wager in the initial round was about 8, it extremely decreased as rounds proceeded, and finally it became 2.425 in the final 20th round. This result implies that wagers are substantially reduced when the link between public good provision and lottery proceeds is broken.

## 3. THE SIMULATIONS

In most of mathematical models, it is assumed that players are rational and then they maximize their payoffs. Such optimization approaches are not always appropriate for analyzing human behaviors and social phenomena. Models based on adaptive behavior can be alternatives to the optimization models, and it is natural that behavior of agents in simulation models is described by using adaptive behavioral rules. Especially, we employ a learning model of agents taking account of not only payoff of self but also those of the others from a viewpoint that observation of other players' actions and payoffs may affect learning of agents [13].

An artificial adaptive agent in our simulation model has a decision making and learning mechanism based on neural networks and genetic algorithms. An agent corresponds to a neural network which is characterized by synaptic weights between two nodes in the neural network, a threshold which is a parameter for output of a node, and a learning rate concerned with learning by error-correction. Because a structure of neural networks is determined by the number of layers and the number of nodes in each layer, an agent is prescribed by the fixed number of parameters. By forming a chromosome consisting of these parameters which is identified with an agent, each of the artificial agents can be evaluated through the fitness computed from the payoff obtained by playing the voluntary contribution or the lottery game and they evolve in our artificial genetic system. From this sense, in our simulation model, a player of the game is referred to as an agent. The structure of a neural network and a chromosome in the genetic algorithm are depicted in Figure 1.

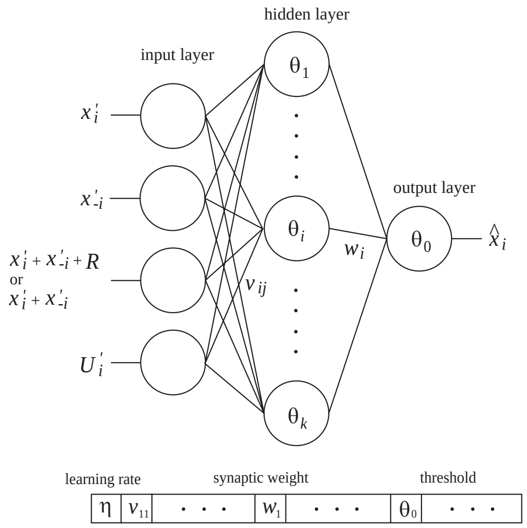


Figure 1: The structure of a neural network and a chromosome in the genetic algorithm

We design a simulation model related to the experimental model by Morgan and Sefton [2] to compare with their results. In our simulation model, the population of agents is divided into a certain number of groups, and each group has the same number of agents. Each agent in a group determines the amount of contribution or wager by an output of the neural network and plays the voluntary contribution or the lottery games. In the voluntary contribution game, an agent obtains a payoff defined by (4). In the lottery game, an agent obtains a payoff

$$U_i = \begin{cases} e - x_i + R + \beta x(N) & \text{if winning} \\ e - x_i + \beta x(N) & \text{otherwise.} \end{cases} \tag{6}$$

The payoff (6) differs from (5) of the mathematical model in the third term, which is an expected payoff $R\frac{x_i}{x(N)}$.

We provide two learning mechanisms. One is a simple learning mechanism based only on genetic algorithms, and in this mechanism agents evolve through the fitness which is computed by payoffs obtained in the games. The other is a learning mechanism based on both genetic algorithms and neural networks, and in addition to learning by genetic algorithms, after finishing games, synaptic weights of the neural network corresponding to the agent are revised by the error back propagation algorithm with teacher signals (target outputs) obtained by computing optimal contributions or wagers for the given contributions or wagers of the other agents. For simplicity, let the simple learning mechanism and the more complicated one be denoted by GA and GABP, respectively. By providing the two learning mechanisms, we can verify whether actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

The procedures of simulations with GA and GABP are summarized as follows.

Step 1 (Generating the initial population) Let the number of agents in a group and the number of groups in the population of the simulations be $n$ and 10, respectively. Then, the initial population of $10n$ agents is formed.

Step 2 (Dividing the population into groups) From the population, $n$ agents are randomly chosen and then one group is formed, and this procedure is repeated 10 times. Eventually, 10 groups are made in all.

Step 3 (Playing games) For each group, the voluntary contribution game or the lottery game is played by $n$ agents.

*The voluntary contribution game.*

Step 3-1-VC (Determining the amount of a contribution) As input values to the neural network, the contribution $x'_i$ of agent $i$, the sum $x'_{-i}$ of the contributions of the other agents, the total fund $x'_i + x'_{-i} + R$ for the public good, and the payoff $U'_i$ of agent $i$ in the previous generation are normalized into $[0,1]$. The four normalized values are inputted to the neural network, and an output $\hat{x}_i$ is obtained. Especially, for the first generation, the input values are randomly determined from $[0,1]$. The contribution of agent $i$ of the present generation is determined as $x_i = \lfloor (e+1)\hat{x}_i \rfloor$, where $\lfloor \cdot \rfloor$ denotes rounding off fractions.

Step 3-2-VC (Informing about the contributions of the others) Agent $i$ is informed about the sum $x_{-i}$ of the contributions of the other agents in the present generation.

Step 3-3-VC (Computing the payoff) The payoff $U_i$ of agent $i$ is calculated by (4).

*The lottery game.*

Step 3-1-L (Determining the amount of a wager) As input values to the neural networks, the wager $x'_i$ of agent $i$, the sum $x'_{-i}$ of the wagers of the other agents, the total fund $x'_i + x'_{-i}$ for the public good, and the payoff $U'_i$ of agent $i$ in the previous generation are normalized into $[0,1]$. The four normalized values are inputted to the neural network, and an output $\hat{x}_i$ is obtained. Especially, for the first generation, input values are randomly determined from $[0,1]$. The wager of agent $i$ is determined as $x_i = \lfloor (e+1)\hat{x}_i \rfloor$.

Step 3-2-L (Drawing lotteries) After wagers of all the agents are determined, winners are selected by a roulette wheel in which agent $i$ draws a winning with the probability $p_i = x_i / \sum_{j=1}^{n} x_j$.

Step 3-3-L (Informing about the contributions of the others) Agent $i$ is informed about the result of the lottery and the sum $x_{-i}$ of the wagers of the other agents in the present generation.

Step 3-4-L (Computing the payoff) The payoff $U_i$ of agent $i$ is calculated by (6).

Step 4 (Learning by the error back propagation algorithm) *This step is executed only for GABP.* Synaptic weights of the neural network for agent $i$ are revised by teacher signals obtained by computing the optimal wagers for the given wagers of the other agents. For agent $i$, the wagers of self and the sums of the wagers of the other agents for the last $k$ games are recorded and they are used as training data for revising the synaptic weights.

If the number of rounds does not reach the given maximal round, return to Step 3.

Step 5 (Performing genetic operations) The following genetic operations are performed to each of the chromosomes for all the agents, and then the population of the next generation is formed.

Step 5-1 (Reproduction) The fitness $f_i$ of each agent is obtained by appropriately scaling the payoff $x_i$ obtained in the present generation. Especially for simulations with GABP, the payoff only at the first round is used in order to exclude the effect of learning by the error back propagation algorithm. As a reproduction operator, the elitist roulette wheel selection is adopted. The elitist roulette wheel selection is a combination of the elitism and the roulette wheel selection. The elitist means that a chromosome with the largest fitness is preserved into the next generation. By a roulette wheel with slots sized by the probability $p_i^{selection} = \frac{f_i}{\sum_{i=1}^{10n} f_i}$, each chromosome is selected into the next generation.

Step 5-2 (Crossover) A single-point crossover operator is applied to any pair of chromosomes with the probability of crossover $p^c$. Namely, a point of crossing over on the chromosomes is randomly selected and then two new chromosomes are created by swapping subchromosomes which are the right side parts of the selected point of crossing over on the original chromosomes.

Step 5-2 (Mutation) With a given small probability of mutation $p^m$, each gene, which represents a synaptic weight, a threshold or a learning rate, in a chromosome is randomly changed. The selected gene is replaced by a random number in $[-1,1]$ for a synaptic weight, or in $[0,1]$ for a threshold and a learning rate.

If the number of generation does not reach the given final generation, return to Step 2.

## 4. THE RESULTS OF THE SIMULATIONS

### 4.1. Treatments of the Simulations

In the simulations, the voluntary contribution and the lottery games are played by agents, and there are three important parameters in the model: the exogenous contribution $R$ which is used to fund the public good or a prize, the marginal per capita return $\beta$ of the public good provision, and the group size $n$ which is the number of agents in a group. Then, we conduct the three simulations: the exogenous contribution simulation, the marginal per capita return simulation, and the group size simulation. Furthermore, providing two learning mechanisms, GA and GABP, we verify whether actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model. In this paper, we give only the result of the exogenous contribution simulation because of limited space.

We summarize the general settings of the simulations and the parameters of the neural networks and the genetic algorithm as follows. The initial endowment is usually set at $e = 20$, and only for the case where the equilibrium wager is larger than 20, it is set at $e = 40$. Let $n$ denote a group size, and because 10 groups are provided for each treatment, the population size of each generation becomes $10n$. Each treatment of the simulations is performed 10 runs. There is 6 units in the hidden layer of the neural networks. Each of the output functions of units in the hidden and the output layers is a logistic function $f(x) = \frac{1}{1+\exp(-x)}$. For the GABP treatments, the game is repeated 10 rounds in each generation. After the game finishes in each round, the error back propagation algorithm is performed using 10 sets of the training data. To do so, each agent records the results of the games, i.e., $x_i$ and $x_{-i}$, for the last 10 games. Each of the initial values of synaptic weights and thresholds is set at 1 so that a contribution or a wager in the first generation becomes the maximal values, i.e., 20 or 40, and the initial value of the learning rate is set at a random number in $[0,1]$. For simulations with GABP, the fitness is computed by using the payoff only at the first round in each generation in order to exclude the effect of learning by the error back propagation algorithm. The probabilities of crossover and mutation are specified at $p_c = 0.6$ and $p_m = 0.01$, respectively. When a certain gene is selected for mutation, the gene is replaced by a random number in $[-1,1]$ if it is for a synaptic weight, and the gene is replaced by a random number in $[0,1]$ if it is for a threshold or a learning rate. The simulations last to generation 1000 which is the final generation for treatments with the group size $n = 2,4,10$, or to generation 2000 which is the final generation for treatments with $n = 50,100$.

### 4.2. The Exogenous Contribution Simulation

In the exogenous contribution simulation, the group size and the marginal per capita return are fixed at $n = 4$ and $\beta = 0.75$, respectively, related to the experiment by Morgan and Sefton, and each treatment consists of four cases with $R = 2,4,8,16$. Each of the treatments is repeated 10 times, and then numerical data given in the tables and the figures of this subsection are averages of the 10 runs.

*The voluntary contribution games:* The result of the voluntary contribution games is summarized in Table 1 where the average contributions of the last 100 generations in the GA treatments are shown in the column of GA, the average contributions of the 10 rounds in the final generation in the GABP treatment are shown in the column of GABP, and the result of the experiment by Morgan and Sefton is also given in the rightmost column. As seen in the table, the average contributions of both the GA and the GABP treatments are close to the equilibrium of zero, and the contributions of the GABP treatments are closer to the equilibrium than those of the GA treatments. Thus, the result supports the predictions of the mathematical equilibrium model that the equilibria are zero regardless of the value of $R$, and it is found that actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

Table 1: The voluntary contribution games

| $R$ | equilibrium | GABP | GA | subjects |
|---|---|---|---|---|
| 2 | 0 | 0.033 | 0.220 | — |
| 4 | 0 | 0.022 | 0.199 | — |
| 8 | 0 | 0.023 | 0.175 | $10.5 \to 8.075$ |
| 16 | 0 | 0.031 | 0.228 | — |

Transitions of contributions of the GA treatments are depicted in Figure 2. The equilibrium of contribution is shown at the both sides of the vertical axis. As seen in Figure 2 and Table 1, the average contributions of all the treatments with $R = 2,4,8,16$ approach 0.2 up to around generation 200, and for convergence of the sequence of the average contributions, an obvious difference among the four treatments is not found.

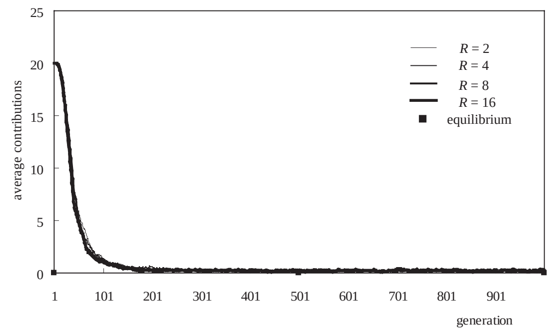


Figure 2: Transitions of the GA treatments of the voluntary contribution games

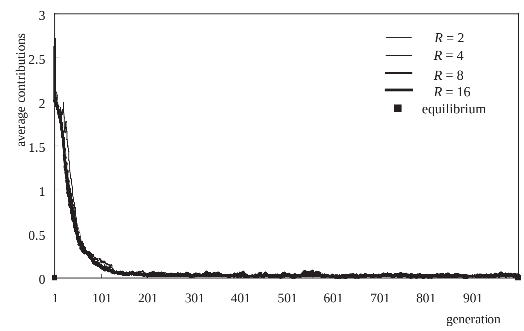


Figure 3: Transitions of the GABP treatments of the voluntary contribution games

Transitions of contributions of the GABP treatments are depicted in Figure 3. As seen in Figure 3 and Table 1, the average contributions of all the treatments with $R = 2, 4, 8, 16$ approach zero up to about generation 200, and for convergence of the sequence, an obvious difference among the four treatments is not also found. By the learning by the error back propagation algorithm, average contributions approach almost zero before the fourth round of the final generation in all the four treatments.

We compare the result of the voluntary contribution games in the exogenous contribution simulation with the corresponding result of the experiment by Morgan and Sefton. In the experiment, the voluntary contribution game with $R = 8$ is played. The average contribution at the initial round of the game is about 10.5, it decreases as the round proceeds, and it finally becomes 8.075 at the final 20th round of the game. This contribution is considerably larger than the equilibrium of zero, but the contribution slightly decreases as subjects gain experiences. For the result of the simulation, the average contributions decrease from 20 to almost zero until around generation 200 in the both GA and GABP treatments.

We summarize the result of the voluntary contribution games as follows. The contributions of both the simulation and the experiment decrease as the learning develops. While the contribution of the experiment is larger than the equilibrium, that of the simulation approaches the equilibrium. Because the repetition of the game in the simulation is vary large compared with that of the experiment, it suggests that human subjects with rich experience of the game may make contributions close to the equilibrium. The contributions of the experiment correspond to the contributions of the simulation from generation 39 to generation 43. Although this correspondence depends on the initial arrangement of the simulation, in general it would be expected to exist some correspondence between the result of the experiment and a part of the whole transition of the simulation with a larger process of the learning.

*The lottery games:* The result of the lottery games is summarized in Table 2. The equilibria of wagers are shown in the second column of the table, and they increase with growing the exogenous contribution $R$, i.e., the size of the prize. As seen in the table, the average wagers of both the GA and the GABP treatments are close to the equilibria, and the wagers of the GABP treatments are closer to the equilibria than those of the GA treatments. Thus, the result supports the predictions of the mathematical equilibrium model that the equilibria of wagers increase as the value of $R$ becomes larger, and it is found that actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.

Table 2: The lottery games

| $R$ | equilibrium | GABP | GA | subjects |
|---|---|---|---|---|
| 2 | 1.5 | 1.471 | 2.222 | — |
| 4 | 3 | 2.975 | 3.735 | — |
| 8 | 6 | 5.954 | 5.868 | $10 \to 10.35$ |
| 16 | 12 | 11.835 | 10.560 | $13 \to 13.825$ |

Transitions of wagers of the GA treatments are depicted in Figure 4. As seen in Figure 4 and Table 2, the differences among the average wagers of the treatments can be seen from around generation 40, and the average contributions of the treatments with $R = 2, 4, 8, 16$ converge at about 2.2, 3.7, 5.9, 10.6, respectively, after around generation 150. Compared with the equilibria of wagers, the average wagers of the treatments with $R = 2, 4$ are slightly larger than the equilibria, and those of the treatments with $R = 8, 16$ are slightly smaller than the equilibria. For convergence of the sequences, an obvious difference among the four treatments is not found.

Transitions of wagers of the GABP treatments are depicted in Figure 5. After around generation 70, each of the average wagers clearly converges at the corresponding equilibrium. Compared with the GA treatments, the transitions of the GABP treatments converge at the equilibria more exactly and earlier, and variances of the wagers are obviously smaller than those of the GA treatments. By the learning of the error back propagation algorithm, the average wagers of the treatments with $R = 2, 4, 8$ converge almost at the equilibria after the third round, and even for the treatment with $R = 16$, although there exists an oscillation around the equilibrium, the average wagers after the sixth round stably converge at the equilibrium.

To examine the relation between the average wagers of the simulations and the equilibria of the mathematical model, we perform the treatments with $R = 1.4$, 2.7, 5.4, 6.7, 9.4, 10.7, 13.4 in addition to the original treatments with $R = 2, 4, 8, 16$. In Figure 6, given the equilibria in the horizontal axis, we show the differences between the average wagers of the simulations and the equilibria in the vertical axis. An seen in Figure 6 for the GA treatments, the average wagers of the simulations are higher than the corresponding equilibria in the games whose equilibria are smaller than 6, and the average wagers of the simulations are lower than the equilibria in the games whose equilibria are larger than 8. In contrast, for the GABP

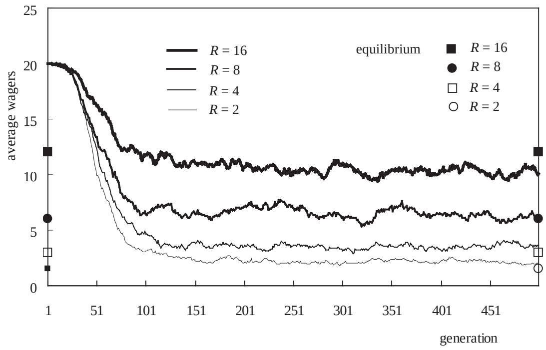


Figure 4: Transitions of the GA treatments of the lottery games

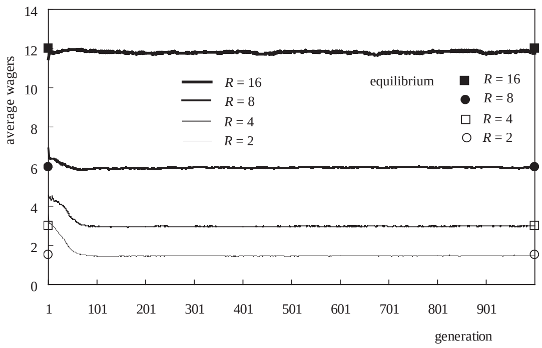


Figure 5: Transitions of the GABP treatments of the lottery games

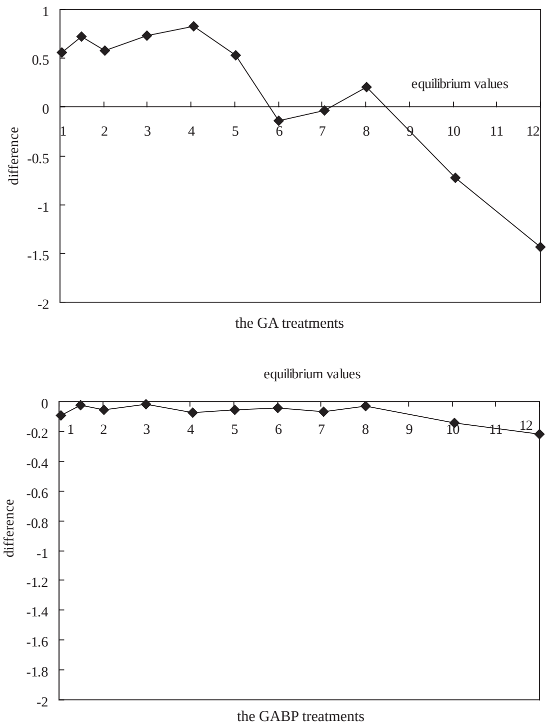


Figure 6: Differences between the wagers of the simulation and the equilibria

treatments, the average wagers of the simulation are close to the equilibria regardless of the sizes of the equilibria. The learning mechanism of GABP is complicated and requires a heavy load of computation. Because the learning of people does not have a complicated mechanism such as the GABP, we can conceive that the learning mechanism of GA is closer to the learning of people, compared with the GABP. This suggestion might give some reason for the fact that the average wagers by human subjects shown in Table 2 from the experiments by Morgan and Sefton [2] are larger than the equilibria.

We compare the result of the lottery games in the exogenous contribution simulation with the corresponding result of the experiment by Morgan and Sefton. In the experiment, by comparing two lottery games with $R = 8, 16$, they examine how the size of the prize influences the wagers of subjects. The average wager at the initial round of the game in the treatment with $R = 8$ is about 10, the round goes on but it rarely changes, and it finally becomes 10.35 at the 20th round of the game. The average wager at the initial round of the game in the treatment with $R = 16$ is about 13, it is almost changeless even though the round proceeds, and it finally becomes 13.825 at the 20th round of the game. Although the change by acquiring experience is not found in each of the treatments with $R = 8, 16$, the experiment supports the equilibrium prediction that the wagers increase as the value of $R$ becomes larger.

For the corresponding results of the simulation, in the GA treatment with $R = 8$, the average wager starts at 20, it decreases as the generation goes on, and after around generation 150 it converges at almost 6. In the GA treatment with $R = 16$, after around generation 150, the average wager finally oscillates in the interval between 10 and 11. In the GABP treatments, the average wagers converge sooner and closer to the equilibria than those in the GA treatments. Especially, the wagers of the human subjects in the experiments $R = 8$ and $R = 16$ correspond to parts of the transition of the wagers of the simulation. Namely, for the treatment with $R = 8$, the transition of wagers from 10 to 10.35 in the experiment corresponds to the transition from a wager at generation 68 to a wager at generation 70 in the simulation, and for the treatment with $R = 16$, the transition from 13 to 13.825 in the experiment corresponds to the transition from a wager at generation 69 to a wager at generation 73 in the simulation. Finally, as seen in Table 2, Figures 4 and 5, the results of the simulation including the results of the treatments not only with $R = 8, 16$ but also with $R = 2, 4$ more clearly support the equilibrium prediction that the wagers increase as the value of $R$ grows larger.

*Summary of the exogenous contribution simulation:* To conclude this subsection, we summarize the results of the simulation for the exogenous contribution $R$.

1. Although it can be found that there exists a clear difference between the equilibria of the mathematical model and the average contributions of the experiment with human subjects in the voluntary contribution games, in the simulation, we observe that the average contributions of the simulation are sufficiently close to the equilibria with the passage of time or with enough learning of agents.
2. While the result of the experiment by Morgan and Sefton supports the equilibrium prediction that the wagers increase as the value of $R$ grows larger, the result of the simulation supports it more obviously.
3. In both of the voluntary contribution games and the lottery games, the contributions and the wagers of the GABP treatments are closer to the equilibria of the mathematical model than those of the GA treatments. Thus, it is found that actions of agents with more elaborate learning mechanism are closer to the predictions of the mathematical equilibrium model.
4. From comparing Tables 1 and 2, we observe that the lottery mechanism provides more of the public good than the voluntary contributions mechanism.

## 5. CONCLUSIONS

In this paper, we have constructed an agent-based simulation model in which artificial adaptive agents have a mechanism of decision making and learning based on neural networks and genetic algorithms. Dealing with three parameters: the exogenous contribution, the marginal per capita return, and the group size, we have performed simulations and examined the effectiveness of the lottery mechanism compared with the voluntary contribution mechanism. As a result of the simulations, we have observed that the transitions of the average contributions and wagers approach almost the corresponding the predictions of the mathematical equilibrium model, and actions of agents with more elaborate learning mechanism are closer to the equilibria. Moreover, from the simulation, it is also found that the lottery mechanism provides more of the public good than the voluntary contributions mechanism. Thus, the results of the simulation support the equilibrium prediction more obviously compared with the experiments with human subjects, and with the results of the simulations, we have given some interpretation on the differences between the equilibrium of the mathematical model and the result of the experiments with human subjects.

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