The Theoretical Foundation of Potential Utility

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ABSTRACT

We discuss the situations in which an agent makes a decision by maximizing *incorrect* utility. In our setting, an agent chooses an alternative that maximizes utility function parameterized by the state variables with incorrect estimates. However, the agent's evaluation of the consequences of the choice ex-post is represented by the utility function parameterized by the correct state variables, which we call *potential* utility. We present a simple model to analyze the welfare loss caused by the incorrect choice. As an application, we analyze the problem of education (or giving information). An authority aims to support an agent to make a correct choice by giving her correct information. We carry out comparative statics on the solution of the government's decision problem to gain some insights regarding the effectiveness of environment education. Our framework enables comparison of utility the agent feels before and after the education. Thereby, our framework helps the design of an education system that makes the agent happier most effectively. We carry out comparative statics on the solution of the government's decision problem to gain some insights regarding the effectiveness of environment education.

keywords: education, expected utility, incomplete information, value of information

1. INTRODUCTION

In this paper, we present a simple framework with which an analyst can discuss the true welfare level of an agent making a decision based on incorrect information. As an application, we present a model of education.

First, consider the following two simple examples of the situations we deal with.

1) Child's decisions and education: A child (she) is clearly not well informed. However, she also makes decisions based on her incomplete information. A parent (he) wants to educate the child, so that she can be happier. However, he cannot control his child fully. The child changes her decisions after being educated, but she continues to make autonomous decisions based on the updated information. How should the parent educate his child?

2) Electric bill and education: These days, the population of environment-conscious people has been increasing. As an example, how can one reduce her electric bill to reduce emission of global warming gas? She does not necessarily know what appliance consumes how much electricity. She may not know the impact of 1 kWh of electricity to global warming either. In such a case, even an environment-conscious person cannot make adequate environment-friendly decisions. How should the informed authority educate the citizens?

In the above two examples, a single decision maker (an agent) has to make decisions based on incorrect information. The agent is not necessarily aware of the incorrectness. However, she is always purposeful so that she tries to do the best within her informational constraints.

An analyst is interested in dealing with how well-off the agent is from the decision she made. The analyst is not the agent herself. Standard rational choice models have ignored the situations in which agents make decisions based on incorrect information (see for instance [1], [2], [3]). We shed light on such cases.

In our setting, the agent maximizes incorrect utility function when she believes that she has correct information at hand and in fact the information is incorrect. At other times, even when the agent is aware of the incompleteness of the information she has at hand, she may still use an incorrect estimate, whose value she believes is not too far away from the true value. We claim that in the above two examples, a child or a citizen in general follows such a decision procedure quite often.

We describe the situation of our interest in a little more detail below.

For each consequence of the choice made, the agent acquires some subjective evaluation ex-post. Following standard ordinal utility theory, we assume that it is possible to represent the evaluation by a number on the set of real numbers. The better the evaluation, the bigger the number.

How is the evaluation determined? The evaluation depends on the choice and the state of the world. We assume that there is a complete set of state variables, such that once the state variables are determined, the agent acquires a unique evaluation from the consequence of each choice she made. State variables may represent various factors such as market size, medical information or anything. However, state variables represent some objective data describing some aspect of the world. Once the agent knows the correct state variables and her utility function, she faces a choice problem without uncertainty. We call *potential* utility function this deterministic ordinal utility function parameterized by the state variables of true value.

We assume that the agent knows that the decision situation has the above stated structure. However, the agent may not necessarily know the true value of state variables at the time of decision making. Nevertheless, no matter whether the agent is aware of the fact that she has only incomplete information regarding the world, the agent makes a guess or an estimate on the state variables and maximizes the utility function parameterized by the state variables with wrong estimates.

To sum up, an agent maximizes subjective utility determined with her estimate *ex-ante* at the time of decision making but experiences potential utility from the consequence *ex-post*.

An analyst who knows all the relevant data of the decision situation and the agent's mind described above may analyze the situation in which the agent makes a wrong decision. In the next section, we discuss the justification of our approach from epistemological and procedural perspective.

2. JUSTIFICTATION OF POTENTIAL UTILITY APPROACH

2.1. Use of potential utility models The most basic assumption regarding potential utility approach is that the decision maker (she) does not know the potential utility *facing the decision problem*. Then, who is an analyst (he) using potential utility models and for what purpose does he use the models?

One possible application is ex-post analysis, in which the agent is the analyst herself *after a decision*. After a decision, the agent may know the correct information regarding the world. The agent may calculate the welfare loss caused by the wrong choice due to incorrect information she had at the time of decision making. This calculation corresponds to the calculation of the value of information in decision analysis. Decision analysis compares the utility brought about by expected utility maximization and utility resulting from maximization of the utility of complete information.

Theory of hypergames[4] and the related frameworks such as model of interperception[5] and intelligent polyagent learning model (I-PALM) [6] treating multi-agent decision situations belong to this category. Their frameworks enable us to represent the agent's misperception regarding the perception of other agents in concern. In hypergames, the state parameters are the other agents' choices. However, they do not explicitly treat incorrectness of understanding of the world in general. By introducing potential utility perspective to those frameworks, it may be possible to obtain deeper understanding on the situations in which multiple agents make decisions on incorrect information.

Another possible application we treat in the next section is education problem. While a child (an agent) may not know the correct information, her parent (an analyst) may know both what his child has in mind and the correct information. Then the parent can predict the consequence of his child's incorrect decision and the consequent welfare loss. The parent wants his child to live a smart life and be happy. Potential utility model may support the parent to design the optimal education that makes his child happier.

How natural is the assumption that the authority knows the structure of the utility function of the learner? We have two responses to this criticism. First, it is not unnatural at all to assume that the authority knows the structure of the parameterized utility function. Even general equilibrium models, whose main value is that the model works without the institution modeler knowing the specific utility functions of the market participants, assumes some mild structure on the participants' utility functions. Secondly, the authority does not necessarily have to know what particular information the learner has. It may very well be that the decision problem for the authority is with uncertainty expressed by a standard probabilistic model.

2.2. Decision procedural issue The second important assumption of our approach is that the agent maximizes some incorrect utility function. How can we justify the assumption?

First, it is obvious that there are many cases in which we are confident before the consequence that we made a correct choice and in fact it turned out wrong. Standard rational choice models do not treat such a situation while our approach can.

Secondly, even when the decision maker herself is aware that she may not have the correct information, she may still adopt utility maximization based on some estimation. Clearly in public policy and management, forecasts and estimates have large impacts and they are used as parameters composing the utility function, instead of expected utility with a probabilistic distribution. At least, we the authors often make decisions on a good guess of relevant information rather than expected utility maximization.

Suppose the decision maker does not know the correct probabilistic distribution or there is no objective quantitative model describing uncertainty in concern, does it make sense to maximize expected utility using almost arbitrarily determined subjective probability[7]? We do not know any evidence. Likewise, decision criteria like max-min, min-max, or Hurwitz are also meta-chosen subjectively and there is no objective reason why one decision criterion is better than another. If the agent is aware of the possibility of error in the estimate, choice by estimation-parameterized utility can be considered as a decision criterion.

3. MODEL

An agent faces a choice-level decision problem. The agent chooses from a set of alternatives A. When a chioce is made, the agent acquires subjective evaluation ex-post represented by utility (real number in set $\mathbf{R}$). The causal relationship between the alternative variable and utility is objective and deterministic with respect to the state of the world represented by the state variables K. That is, there exists a function $U_{\cdot}(\cdot) : A \times K \to \mathbf{R}$ that corresponds choice $e \in A$ and the state of the world $k \in K$ to the evaluation of the consequence of choice $U_k(e)$. When the agent makes a choice in the world with state $k^* \in K$, her evaluation is given by the potential utility $U_{k^*}(\cdot)$. The agent knows that she is facing a decision context characterized by the family of utility functions $U_{\cdot}(\cdot)$, but she does not know the correct value of state variables K. Denote $k \in K$ the value of state variables that the agent estimates and denote $e_k \in A$ the solution of the decision problem $\max_{e \in A} U_k(e)$. The potential utility the agent acquires when she chooses the optimal (in terms of her incorrect model) alternative is given by $U_{k^*}(e_k)$. If the agent had the correct information of the state variables $k^* \in K$, she would solve the decision problem $\max_{e \in A} U_{k^*}(e)$, and her choice will make her feel $U_{k^*}(e_{k^*})$. Thus, the welfare loss of the agent due to incorrect choice is $U_{k^*}(e_{k^*}) - U_{k^*}(e_k)$.

An analyst can use the understanding of the welfare loss to design the institution on which the agent makes decisions. We see an application of the model to education problem in the next section.

4. APPLICATIONS TO EDUCATION OF ENVIRONMENTAL ISSUES

A government or a nonprofit wants to educate people to know more about the environmental issues. The aim of the education is to change the people's behavior so that they can be happiest. However, the government wants to minimize the education cost as well. The task of the government is to look for the best trade-off. In this section, we first describe the decision problem of the people (agents) regarding environment-friendly actions. Then, we describe the decision problem of the government (analyst) to determine the optimal education level. We carry out comparative statics on the solution of the government's decision problem to gain some insights regarding the effectiveness of environment education.

4.1. Decision problem of an agent

An agent wants to know how environment-friendly she should act. She feels good when the environment improves by her action, but environment-friendly action burdens her. She is looking for the best tradeoff.

Let e be the effort level of environment-conscious actions such as purchasing environment-friendly goods, and reduction and separation of garbage. Denote k the marginal environmental improvement brought about by a unit effort level. For instance, reduction of 1 kg kWh of electricity consumption may cause reduction of k kg of carbon-dioxide. k represents objective data determined by the external factors. The agent does not always know the correct value of k. Denote k^* the correct value of k.

When the agent makes effort e, she feels ak^*e of positive utility. On the other hand, environment-friendly effort e burdens the agent by de^2. This cost function is a simple example of cost functions in which the marginal cost increases with effort. For example, it is impossible to reduce CO_2 consumption to 0 kg. However, it is much easier for an ordinary citizen in OECD countries to reduce 1 kg a day of CO_2 consumption from their current consumption level. a and d are environment sensitivity and cost sensitivity respectively, subjective for each agent.

The information relevant to the agent's decision is only k. Therefore, the agent understands the causal relationship between her action and how she feels as a result by a parameterized utility function $U_k(e) = ake - de^2$. The causal model (parameterized utility) is correct in that, when the substituted information parameter is correct, that is $(k = k^*)$, U_{k^*} gives correct potential utility for each effort level e.

The agent chooses the effort level e that maximizes $U_k(e)$. Denote e_k the solution of the decision problem $\max_k U_k(e)$. The potential utility the agent feels when she chooses the optimal (in terms of her incorrect model) effort level is $U_{k^*}(e_k)$.

4.2. Education problem of the authority The decision problem of the authority is how much it should invest to enlighten wrongly informed agents. We assume that when an agent is enlightened, she knows the correct value k^*. Thus when an agent is enlightened, utility function of her decision model changes from U_k to U_{k^*}. education requires communication such as commercials or pamphlets for which the authority has to pay the cost. Let p^2/a be the cost required to enlighten citizens with environment sensitivity parameter a, where $p(0 \le p \le 1)$ is the probability that an agent is enlightened. This cost function is again a simple function whose marginal cost increases regarding p. Cost is decreasing in a since it is easier to enlighten more sensitive agents. Particularly, it is extremely costly to educate completely indifferent agent for whom $a = 0$.

The authority evaluates the effect of education by the rise in the welfare of agents. The utility of the agent before and after the education are $U_{k^*}(e_k)$ and $U_{k^*}(e_{k^*})$ respectively. Thus the rise in the welfare is $U_{k^*}(e_k^*) - U_{k^*}(e_k)$.

The decision problem of the authority is to determine the best education level p. That is $\max_p p(U_{k^*}(e_k^*) - U_{k^*}(e_k)) - p^2/a$.

4.3. Analyses The solution of the agent's decision problem is $e_k = ak/(2d)$.

The solution of the education problem is

$$p_{opt} = \begin{cases} a^2(k-k^*)^2/(8d) & (0 \le a^2(k-k^*)^2/(8d) \le 1) \\ 1 & (a^2(k-k^*)^2/(8d) \ge 1) \end{cases}$$

In the subsequent analyses, $p_{opt} \le 1$.

First, as is well known in the field of institution design, education does not always lead to more environment-friendly behavior. If an agent is environment maniac, for whom $k > k^*$ before education, she will make less effort after education. Notice however that the agent's utility always rises by education.

Comparative statics on the solution of the authority implies the following features.

1. The higher the environment sensitivity, the higher the optimal level of education.

2. The higher the cost sensitivity, the lower the optimal level of education.

3. The larger the difference between the information the agent has before and after the education, the higher the optimal level of education.

The implications are backed up by intuition.

First, regarding 1., it is obvious that education works more effectively in a society with conscious citizens. It is not effective to educate indifferent people. Notice that giving correct information and raising people's interest are different issues.

Secondly, regarding 2., if environment-friendly products are too expensive or reduction of energy consumption is too costly for daily life for instance, it is clearly difficult to change the behavior of people by education. Both education and technological innovation regarding environment should be implemented to make a more environment-friendly society.

Finally, regarding 3., if the agents are already well-informed, it is no use educating them more. The insight is similar to value of information in decision analysis.

In the case in which $p_{opt} = 1$, notice that function in the classification condition $(a^2(k-k^*)^2/(8d) \ge 1)$ retains the comparative statics described above. Therefore, we know in what intuitive cases we want to educate agents for sure. Particularly, when correct knowledge seems vital for the welfare of the agent, that is $|k - k^*|$ is large, perfect education seems essential. A familiar example may be the education of the danger of drugs to young people.

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