

Title	Finding Efficiency and Inefficiency : Multi-Viewpoint Approach
Author(s)	AOKI, Shingo; MINAMI, Kyousei; TSUJI, Hiroshi
Citation	
Issue Date	2005-11
Type	Conference Paper
Text version	publisher
URL	<a href="http://hdl.handle.net/10119/3885">http://hdl.handle.net/10119/3885</a>
Rights	2005 JAIST Press
Description	The original publication is available at JAIST Press <a href="http://www.jaist.ac.jp/library/jaist-press/index.html">http://www.jaist.ac.jp/library/jaist-press/index.html</a> , IFSR 2005 : Proceedings of the First World Congress of the International Federation for Systems Research : The New Roles of Systems Sciences For a Knowledge-based Society : Nov. 14-17, 2005, Kobe, Japan, Symposium 3, Session 5 : Intelligent Information Technology and Applications Evaluation and Standard

# Finding Efficiency and Inefficiency: Multi-Viewpoint Approach

Shingo AOKI, Kyousei MINAMI, Hiroshi TSUJI

Graduate School of Engineering, Osaka Prefecture University  
1-1 Gakuen-Cho, Sakai, Osaka 599-8531, Japan  
{aoki, tsuji}@cs.osakafu-u.ac.jp

## ABSTRACT

In order to evaluate the performance of DMU (Decision Making Unit), this paper proposes a new decision support technique referred to as Multi-Viewpoint DEA model. The proposed model is an extended model which integrates the DEA-efficiency model and the DEA-inefficiency model into one mathematical formulation, and allows us to analyze the performance of DMU between the strong points and weak points. A case study shows that the proposed model has desirable two features: (1) robustness of the evaluation value, and (2) unification between DEA-efficiency model and DEA-inefficiency model.

**Keywords:** Data Envelopment Analysis, Efficiency Approach, Inefficiency Approach.

## 1. Introduction

As initiated and developed by Charnes et al. [1], DEA is a nonparametric method for finding the relative efficiency of DMUs, each of which is a company responsible for converting multiple inputs into multiple outputs. DEA has been applied to a variety of managerial and economic problem situations in both public and private sectors [5, 9, 13, 14]. DEA assumes each DMU uses multiple inputs to yield multiple outputs, and defines the process which changes multiple inputs into multiple outputs as one evaluation value.

From the analysis concept, the decision method based on such evaluation value induces two kinds of approaches: One is the efficiency approach based on the Pareto optimal solution for the aspect only of the strong points [1, 5]. The other is the inefficiency approach based on the Pareto optimal solution for the aspect only of the weak points [7]. Then, the evaluation values in two approaches are inconsistent [8]. However, analysts have evaluated DMUs only by extreme aspect: either strong points or weak points. Thus, the traditional two approaches lack flexibility and robustness [17].

In fact, while there are many inputs and outputs in DEA framework, these items are not fully used in the previous

approaches. This type of DEA problem has been usually tackled by multiplier restriction approaches [15] and cone ratio approaches [16]. While such multiplier restrictions usually reduce the number of zero weight, they often produce an infeasible solution in DEA. Therefore, new DEA model which has robustness on the evaluation values is required.

This paper proposes a new decision support technique referred to as Multi-Viewpoint DEA model. The remaining structure of this paper is organized as follows: the next section reviews the traditional DEA models. Section 3 proposes a new model. The proposed model integrates the DEA-efficiency model and the DEA-inefficiency model into one mathematical formulation, and allows us to analyze the performance of DMU by multi-viewpoint between the strong points and weak points. Section 4 verifies the proposed model through a case study. A case study shows that the proposed model has two desirable features: (1) robustness of the evaluation value, and (2) unification between DEA-efficiency model and DEA-inefficiency model. Finally, conclusion and future study are summarized in section 5.

## 2. DEA: Data Envelopment Analysis

### 2.1. Evaluation value

In order to describe the mathematical structure of the evaluation value, this paper assumes that there are  $n$  DMUs ( $DMU_1, \dots, DMU_k, \dots, DMU_n$ ), where each DMU is characterized by  $m$  inputs ( $x_{1k}, \dots, x_{ik}, \dots, x_{mk}$ ) and  $s$  outputs ( $y_{1k}, \dots, y_{rk}, \dots, y_{sk}$ ). Evaluation value of  $DMU_k$  is mathematically formulated by

$$\text{Evaluation value of } DMU_k = \frac{u_1 y_{1k} + u_2 y_{2k} + \dots + u_s y_{sk}}{v_1 x_{1k} + v_2 x_{2k} + \dots + v_m x_{mk}} \quad (1)$$

Here  $u_r$  is multiplier weight given to the  $r^{\text{th}}$  output, and  $v_i$  is multiplier weight given to the  $i^{\text{th}}$  input. From the analysis concept, there are two decision methods for

calculating these weights. One is the efficiency approach based on the Pareto optimal solution for the aspect only of the strong points [1, 5]. The other is the inefficiency approach based on the Pareto optimal solution for the aspect only of the weak points [7, 8].

Figure 1 visually represents the difference of two methods. Suppose that there are nine DMUs which have one input and two outputs where X-axis is output 1 over input and Y-axis is output 2 over input. So, if a DMU is located in upper-right region, it shows that the DMU has high productivity. DEA-efficiency model finds out the efficiency frontier which indicates the best practice line (B-C-D-E-F in Figure 1) and evaluates the relative evaluation value by the aspect only of the strong points. On the other hand, DEA-inefficiency model finds out the inefficiency frontier which indicates the worst practice line (B-I-H-G-F in Figure 1) and evaluates the relative evaluation value by the aspect only of the weak points.

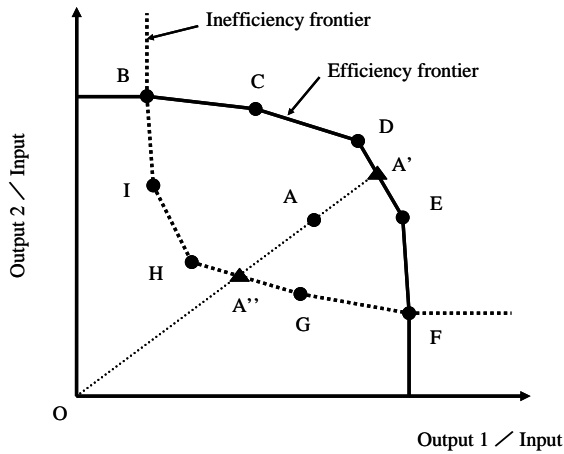


Figure 1. Efficiency model and Inefficiency model

## 2. 2. DEA-efficiency model

The efficiency approach measures the efficiency level of a specific DMU<sub>k</sub> by relatively comparing its performance to the efficiency frontier. This paper is based on CCR model [1] while there are other models [5, 11]. The efficiency approach can be mathematically formulated by

$$\text{Max} \quad \sum_{r=1}^s u_r y_{rk} (= \theta_k^E) \quad (2-1)$$

$$\text{s.t.} \quad -\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} \leq 0 \quad (2-2)$$

$$(j=1, 2, \dots, n) \quad (2)$$

$$\sum_{i=1}^m v_i x_{ik} = 1 \quad (2-3)$$

$$v_i \geq 0, u_r \geq 0$$

Here formula (2-2) is a restriction condition because the productivity of all DMUs (formula (1)) becomes 100% or less. And the objective function (2-1) represents the maximization of the sum of virtual outputs of DMU<sub>k</sub>, setting that the virtual inputs of DMU<sub>k</sub> is equal to 1 (formula (2-3)). Therefore, the optimal solution of (v<sub>i</sub>, u<sub>r</sub>) represents the convenient weight for DMU<sub>k</sub>. Especially, the optimal objective function value indicates the evaluation value (θ<sub>k</sub><sup>E</sup>) for DMU<sub>k</sub>. This evaluation value by the convenient weight is called “efficiency score” in the manner that θ<sub>k</sub><sup>E</sup>=1(100%) means the state of efficiency, while θ<sub>k</sub><sup>E</sup><1(100%) means the state of inefficiency.

## 2. 3. DEA-inefficiency model

There is another approach which measures the inefficiency level of a specific DMU<sub>k</sub> based on Inversed DEA model [7]. The inefficiency approach can be mathematically formulated by

$$\text{Min} \quad \sum_{r=1}^s u_r y_{rk} = \left(\frac{1}{\theta_k^{IE}}\right) \quad (3-1)$$

$$\text{s.t.} \quad -\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} \geq 0 \quad (3-2)$$

$$(j=1, 2, \dots, n) \quad (3)$$

$$\sum_{i=1}^m v_i x_{ik} = 1 \quad (3-3)$$

$$v_i \geq 0, u_r \geq 0$$

Again, formula (3-2) is a restriction condition because the productivity of all DMU (formula (1)) becomes 100% or more. And the objective function (3-1) represents the minimization of the virtual outputs of DMU<sub>k</sub>, setting that the virtual inputs of DMU<sub>k</sub> is equal to 1 (formula (3-3)). Therefore, the optimal solution of (v<sub>i</sub>, u<sub>r</sub>) represents the inconvenient weight for DMU<sub>k</sub>. Especially, the inverse

number of optimal objective function value indicates the “inefficiency score” in the manner that  $\theta_k^{IE} = 1(100\%)$  means the state of inefficiency, while  $\theta_k^{IE} < 1(100\%)$  means the state of efficiency.

#### 2.4. Problems in Traditional Approaches

As shown in Figure 1,  $DMU_B$  and  $DMU_F$  are evaluated as both states of “efficiency ( $\theta_k^E = 1$ )” and “inefficiency ( $\theta_k^{IE} = 1$ )”. This result clearly shows mathematical difference in two approaches. For the example,  $DMU_B$  has the best productivity for the Output 2 / input, while it has worst productivity for the Output 1 / input. In efficiency approach, the weight of  $DMU_B$  is evaluated by the aspect of the strong points. Therefore, the weight of Output 2 / input becomes a positive value and the weight of Output 1 / input becomes zero. On the other hand, in inefficiency approach, the weight of  $DMU_B$  is evaluated by the aspect of the weak points. Therefore, the weight of Output 2 / input becomes zero and the weight of Output 1 / input becomes a positive value. This difference of the weight estimation causes the mathematical problems as follow:

(a) No robustness of evaluation value

Both approaches may produce zero weights for most inputs and outputs. The zero weight indicates that the corresponding inputs or outputs are not used for the evaluation value. Moreover, if the specific inputs or output items are removed from the analysis, the evaluation value may change greatly [17]. This type of DEA problem is usually tackled by multiplier restriction approaches [15] and cone ratio approaches [16]. Such multiplier restrictions usually reduce the number of zero weight, and these approaches often produce an infeasible solution. The development of DEA model which has robustness of the evaluation value is required.

(b) Lack of unification between DEA-efficiency and DEA-inefficiency model

Fundamentally, efficient DMU can not be inefficient while inefficient DMU can not be efficient. However, the evaluation value may be not consistent like the  $DMU_B$  and  $DMU_F$  in the Figure 1 where they are in the both states of “efficiency” and “inefficiency”. Thus, it is not easy for a decision maker to understand the difference between

evaluation values. The basis of the evaluation value which has unification between DEA-efficiency model and DEA-inefficiency model is required.

### 3. Multi-Viewpoint DEA Model

Let us propose a new decision support technique referred to as Multi-Viewpoint DEA model. The proposed model is a re-formulation of the DEA-efficiency and DEA-inefficiency model into one mathematical formulation.

#### 3.1. DEA-efficiency model based on GP technique

This paper applies the following formula (4) which added the variable ( $d_j^+, d_j^-$ ) to formula (2-2):

$$-\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} + d_j^+ - d_j^- = 0 \quad (j=1, 2, \dots, n) \quad (4)$$

Here  $d_j^+$  indicates the slack variables, and  $d_j^-$  indicates the artificial variables. Therefore, the objective function (2-1) can be replaced by mathematically using several big M as follows:

$$\sum_{r=1}^s u_r y_{rk} - M \sum_{j=1}^n d_j^- \quad (5)$$

From the formula (4) and formula (2-3), the objective function (5) can be rewritten as follows:

$$\begin{aligned} \sum_{r=1}^s u_r y_{rk} - M \sum_{j=1}^n d_j^- &= (\sum_{i=1}^m v_i x_{ik} - d_k^+ + d_k^-) - M \sum_{j=1}^n d_j^- \\ &= 1 - d_k^+ + d_k^- - M \sum_{j=1}^n d_j^- \\ &= 1 - d_k^+ + (1-M)d_k^- - M \sum_{j=1, j \neq k}^n d_j^- \end{aligned} \quad (6)$$

Using GP (Goal Programming) technique, the DEA-efficiency-model (formula (2)) can be replaced by the following Linear Programming:

$$\begin{aligned}
\text{Max} \quad & 1 - d_k^+ + (1-M)d_k^- - M \sum_{j=1, j \neq k}^n d_j^- \\
\text{s.t.} \quad & -\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} + d_j^+ - d_j^- = 0 \\
& (j = 1, 2, \dots, n) \\
& \sum_{i=1}^m v_i x_{ik} = 1 \\
& v_i \geq 0, u_r \geq 0, d_j^+, d_j^- \geq 0
\end{aligned} \tag{7}$$

The efficiency score ( $\theta_k^E$ ) of DMU<sub>k</sub> as follows:

$$\theta_k^E = 1 - d_k^{+*} \left( \frac{\sum_{r=1}^s u_r^* y_{rj}}{\sum_{i=1}^m v_i^* x_{ik} (=1)} \right) \tag{8}$$

Where superscript “\*” indicate the optimal solution of formula (7).

### 3.2. DEA-inefficiency model based on GP technique

Let us apply the formula (4) which added the variable ( $d_j^+, d_j^-$ ) to formula (3-2). This paper notes that  $d_j^+$  indicates the artificial variables and  $d_j^-$  indicates the slack variables in DEA-inefficiency model. Using GP technique, the DEA-inefficiency-model (formula (3)) can be replaced by the following Linear Programming:

$$\begin{aligned}
\text{Min} \quad & 1 + (M-1)d_k^+ + d_k^- + M \sum_{j=1, j \neq k}^n d_j^+ \\
\text{s.t.} \quad & -\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} + d_j^+ - d_j^- = 0 \\
& (j = 1, 2, \dots, n) \\
& \sum_{i=1}^m v_i x_{ik} = 1 \\
& v_i \geq 0, u_r \geq 0, d_j^+, d_j^- \geq 0
\end{aligned} \tag{9}$$

The inefficiency score ( $\theta_k^{IE}$ ) of DMU<sub>k</sub> as follows:

$$\theta_k^{IE} = \frac{1}{1 + d_k^{+*}} \tag{10}$$

Where superscript “\*” indicate the optimal solution of formula (9).

### 3.3. Mathematical integration of the efficiency and inefficiency model

In order to integrate two DEA models into one formula mathematically, this paper introduces slack variables. As seen in formula (7) and (9), it is understood that the both models have the same restriction conditions. Then, this paper applies the following formula (11) which added any constant ( $\alpha, \beta$ ) to the objective function of formula (7) and (9).

$$\begin{aligned}
& \alpha \{1 - d_k^+ + (1-M)d_k^- - M \sum_{j=1, j \neq k}^n d_j^-\} \\
& - \beta \{1 + (M-1)d_k^+ + d_k^- + M \sum_{j=1, j \neq k}^n d_j^+\} \\
= & (\alpha - \beta) - \{\alpha - \beta(1-M)\}d_k^+ + \{\alpha(1-M) - \beta\}d_k^- \\
& - (\alpha M \sum_{j=1, j \neq k}^n d_j^+ + \beta M \sum_{j=1, j \neq k}^n d_j^-)
\end{aligned} \tag{11}$$

When formula (11) is divided by several big M mathematically, it can be developed as follows:

$$\begin{aligned}
& -(\beta d_k^+ + \alpha d_k^-) - (\beta \sum_{j=1, j \neq k}^n d_j^+ + \alpha \sum_{j=1, j \neq k}^n d_j^-) \\
= & -\beta \sum_{j=1}^n d_j^+ - \alpha \sum_{j=1}^n d_j^-
\end{aligned} \tag{12}$$

Where these constants can be estimated  $\alpha + \beta = 1$ , because the constants ( $\alpha, \beta$ ) indicate relative ratios of the DEA-efficient and the DEA-inefficiency model. Then the proposed model is formulated as the following Linear Programming:

$$\begin{aligned}
\text{Max} \quad & -(1-\alpha) \sum_{j=1}^n d_j^+ - \alpha \sum_{j=1}^n d_j^- \\
\text{s.t.} \quad & -\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} + d_j^+ - d_j^- = 0 \\
& (j = 1, 2, \dots, n) \\
& \sum_{i=1}^m v_i x_{ik} = 1 \\
& v_i \geq 0, u_r \geq 0, d_j^+, d_j^- \geq 0
\end{aligned} \tag{13}$$

Where  $x_{ij}$ :  $i^{\text{th}}$  input value of  $j^{\text{th}}$  DMU,  
 $y_{rj}$ :  $r^{\text{th}}$  output value of  $j^{\text{th}}$  DMU,  
 $v_i, u_r$ : input and output weight,  
 $d_j^+, d_j^-$ : slack variables.

The formula (13) includes the viewpoint's parameter  $\alpha$ , and allows us to analyze the performance of DMU by changing the parameter  $\alpha$  between the strong points (especially, if  $\alpha = 1$  then the optimal solutions is the same with one of DEA-efficiency model) and weak points (if  $\alpha = 0$  then the optimal solutions is the same with one of DEA-inefficiency model).

And if  $\alpha = \alpha'$  then this paper defines the evaluation value ( $\theta_k^{MVP, \alpha'}$ ) of DMU<sub>k</sub> as follows:

$$\begin{aligned} \theta_k^{MVP, \alpha'} &= \alpha' \theta_k^E - (1 - \alpha') \theta_k^{IE} \\ &= \alpha' (1 - d_k^+) - (1 - \alpha') \left( \frac{1}{1 + d_k^-} \right) \end{aligned} \quad (14)$$

Where superscript “\*” indicate the optimal solution of formula (13).

The first term of formula (14) indicates the evaluation value by the aspect of the strong points and the second term indicates it by the aspect of the weak points. Therefore, the evaluation value ( $\theta_k^{MVP, \alpha'}$ ) is measured on the range between -1 (-100%: inefficiency) and 1 (100%: efficiency).

## 4. Case Study

### 4.1. A data set

A data set used in this paper is demonstrated in Table 1. There are twelve DMU whose performance is measured by two inputs and three outputs.

Table 1. A data set

No.	DMU	Input		Output		
		Input1	Input2	Output1	Output2	Output3
1	A	10	8	23	32	21
2	B	26	10	37	47	32
3	C	40	15	80	148	68
4	D	35	28	76	104	60
5	E	30	21	23	40	20
6	F	33	10	38	89	41
7	G	37	12	78	175	65
8	H	50	22	68	200	77
9	I	31	15	48	86	33
10	J	12	10	16	35	16
11	K	20	12	64	74	23
12	L	45	26	72	58	35

Table 2. Evaluation value ( $\theta^{MVP, 1}$ ) and optimal solutions ( $v^*$ ,  $u^*$ ,  $d^{+*}$ ,  $d^{-*}$ )

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.100	0	0.017	0.006	0.019	0	0	1.055	0	0.383	0	0.389	0	1.968	0	1.293	0
2	B	0.033	0.015	0	0	0.021	0	0	0.316	0	0.080	0	0.288	0	0.872	0	0.352	0
3	C	0.021	0.010	0	0	0.014	0	0	0.206	0	0.052	0	0.188	0	0.569	0	0.230	0
4	D	0.029	0	0.005	0.002	0.006	0	0	0.301	0	0.109	0	0.111	0	0.562	0	0.369	0
5	E	0.033	0	0.006	0.002	0.006	0	0	0.352	0	0.128	0	0.130	0	0.656	0	0.431	0
6	F	0	0.100	0	0	0.018	0.412	0	0.409	0	0.245	0	1.692	0	1.731	0	0.243	0
7	G	0.024	0.008	0.006	0	0.008	0	0	0.232	0	0.062	0	0.133	0	0.601	0	0.327	0
8	H	0.020	0	0	0.002	0.007	0	0	0.218	0	0.082	0	0.109	0	0.394	0	0.227	0
9	I	0.032	0	0.005	0.002	0.006	0	0	0.340	0	0.123	0	0.125	0	0.635	0	0.417	0
10	J	0.083	0	0	0.007	0.030	0	0	0.909	0	0.340	0	0.452	0	1.643	0	0.947	0
11	K	0.042	0.014	0.011	0	0.013	0	0	0.396	0	0.105	0	0.228	0	1.027	0	0.558	0
12	L	0.019	0.006	0.005	0	0.006	0	0	0.177	0	0.047	0	0.102	0	0.459	0	0.250	0

No.	DMU	Slack Variable											
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-
1	A	0	0	1.075	0	1.098	0	0.395	0	0	0	2.236	0
2	B	0	0	0.321	0	0.535	0	0.203	0	0.344	0	1.117	0
3	C	0	0	0.209	0	0.349	0	0.132	0	0.225	0	0.729	0
4	D	0	0	0.307	0	0.314	0	0.113	0	0	0	0.639	0
5	E	0	0	0.358	0	0.366	0	0.132	0	0	0	0.745	0
6	F	0	0	0.778	0	0.891	0	0.705	0	0.775	0	1.954	0
7	G	0	0	0.369	0	0.317	0	0.148	0	0	0	0.580	0
8	H	0	0	0.135	0	0.249	0	0.071	0	0.119	0	0.559	0
9	I	0	0	0.347	0	0.354	0	0.127	0	0	0	0.721	0
10	J	0	0	0.563	0	1.037	0	0.295	0	0.496	0	2.330	0
11	K	0	0	0.631	0	0.541	0	0.253	0	0	0	0.990	0
12	L	0	0	0.282	0	0.242	0	0.113	0	0	0	0.443	0

Evaluation Value		
$1 \times \theta^E$	$0 \times \theta^{IE}$	$\theta^{MVP, 1}$
1	0	1
0.684	0	0.684
0.948	0	0.948
0.889	0	0.889
0.344	0	0.344
0.757	0	0.757
1	0	1
0.865	0	0.865
0.646	0	0.646
0.705	0	0.705
1	0	1
0.557	0	0.557

#### 4.2. Multi-Viewpoint DEA's result

This paper calculates five patterns ( $\alpha = 1, 0.75, 0.5, 0.25, 0$ ). Table 2 shows the evaluation value ( $\theta^{MVP,1}$ ) and optimal solutions ( $v^*, u^*, d^{+*}, d^{-*}$ ) when the viewpoint parameter  $\alpha = 1$ . And Table 3 shows the evaluation value ( $\theta^{MVP,0.75}$ ) and optimal solutions ( $v^*, u^*, d^{+*}, d^{-*}$ ) when the viewpoint parameter  $\alpha = 0.75$ . The other pattern's results are put on Table A-1, A-2, and A-3 in Appendix.

##### Finding 1:

As shown in Table 1, when the viewpoint parameter  $\alpha = 1$ , the optimal solutions of formula (2) and formula (13) were the same about inefficiency DMU ( $\theta^E < 1$ ). However in this case study, the optimal solutions of formula (2) was different from it of formula (13) about efficiency DMU ( $\theta^E = 1$ ). The same result applied to inefficiency model (formula (9) and formula (13) with parameter  $\alpha = 0$  in Appendix Table A-3). The cause of this result is the multiple solutions problem [3]. Thus, the

proposed model is considered to integration DEA model of the efficiency and inefficiency model.

##### Finding 2:

Comparing the results of Table 1 to ones of Table 2, the combination of the optimal solution was change. Especially, focusing the ranking of evaluation value about DMU<sub>2</sub> and DMU<sub>6</sub>, we can find that the order of DMU<sub>2</sub> and DMU<sub>6</sub> was reversed. As shown in Table 1, because DMU<sub>2</sub> has the multiplicity of strong points such as output 2 and output3, it is understood that DMU<sub>2</sub> has high rank roughly. In contrast, because DMU<sub>6</sub> has the only one strong point such as output 2, it is understood that DMU<sub>6</sub> has low rank roughly. Thus, proposed model can allow us to know a change in strong points and weak points for each DMU. The important point of this result is that proposed model can allow us to know the robustness for each DMU's evaluation value.

Table 3. Evaluation value ( $\theta^{MVP,0.75}$ ) and optimal solutions ( $v^*, u^*, d^{+*}, d^{-*}$ )

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.069	0.039	0.019	0	0.026	0	0	0.622	0	0	0	0.444	0	1.911	0	0.849	0
2	B	0.038	0	0.007	0	0.013	0	0.060	0.308	0	0.057	0	0	0	0.722	0	0.451	0
3	C	0.025	0	0.005	0	0.009	0	0.039	0.200	0	0.037	0	0	0	0.469	0	0.293	0
4	D	0.020	0.011	0.006	0	0.008	0	0	0.178	0	0	0	0.127	0	0.546	0	0.243	0
5	E	0.031	0.003	0.006	0	0.012	0	0.054	0.237	0	0	0	0	0	0.622	0	0.343	0
6	F	0.030	0	0.006	0	0.010	0	0.048	0.242	0	0.045	0	0	0	0.569	0	0.355	0
7	G	0.027	0	0.005	0	0.009	0	0.042	0.216	0	0.040	0	0	0	0.507	0	0.317	0
8	H	0.020	0	0.004	0	0.007	0	0.031	0.160	0	0.030	0	0	0	0.375	0	0.234	0
9	I	0.031	0.003	0.006	0	0.012	0	0.053	0.234	0	0	0	0	0	0.613	0	0.338	0
10	J	0.057	0.032	0.016	0	0.022	0	0	0.512	0	0	0	0.365	0	1.572	0	0.699	0
11	K	0.047	0.004	0.009	0	0.018	0	0.082	0.359	0	0	0	0	0	0.941	0	0.519	0
12	L	0.021	0.002	0.004	0	0.008	0	0.036	0.160	0	0	0	0	0	0.419	0	0.231	0

No.	DMU	Slack Variable											
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-
1	A	0	0.205	0.956	0	0.922	0	0.483	0	0	0	1.797	0
2	B	0	0.004	0.410	0	0.407	0	0.134	0	0	0	0.745	0
3	C	0	0.002	0.266	0	0.264	0	0.087	0	0	0	0.484	0
4	D	0	0.059	0.273	0	0.263	0	0.138	0	0	0	0.513	0
5	E	0	0.055	0.298	0	0.330	0	0.116	0	0	0	0.631	0
6	F	0	0.003	0.323	0	0.321	0	0.105	0	0	0	0.587	0
7	G	0	0.003	0.288	0	0.286	0	0.094	0	0	0	0.523	0
8	H	0	0.002	0.213	0	0.212	0	0.069	0	0	0	0.387	0
9	I	0	0.054	0.294	0	0.325	0	0.115	0	0	0	0.623	0
10	J	0	0.169	0.786	0	0.758	0	0.397	0	0	0	1.478	0
11	K	0	0.083	0.450	0	0.499	0	0.176	0	0	0	0.955	0
12	L	0	0.037	0.201	0	0.222	0	0.078	0	0	0	0.425	0

Evaluation Value		
$0.75 \times \theta^E$	$0.25 \times \theta^E$	$\theta^{MVP,0.75}$
0.750	0.250	0.5
0.519	0.250	0.269
0.722	0.250	0.472
0.655	0.250	0.405
0.284	0.250	0.034
0.484	0.250	0.234
0.750	0.249	0.501
0.590	0.250	0.340
0.506	0.250	0.256
0.452	0.250	0.202
0.750	0.250	0.5
0.431	0.250	0.181

## 5. Conclusion

This paper has proposed a new decision support technique referred to as Multi-Viewpoint DEA model which integrated the DEA-efficiency model and the DEA-inefficiency model into one mathematical formulation. The proposed model allows us to analyze the performance of DMU by changing the viewpoint's parameter  $\alpha$  between the strong points (especially, if  $\alpha=1$  then it becomes DEA-efficiency model) and weak points (if  $\alpha=0$  then it becomes DEA-inefficiency model). A case study has shown that the proposed model has two desirable features: (1) robustness of the evaluation value, and (2) unification between DEA-efficiency model and DEA-inefficiency model.

For the future study, we will apply the proposed method to practical problems which include volumes of data. We will also analytically compare our method to the traditional approaches [15, 16] and explore how to set the viewpoint's parameter.

## 6. References

- [1] A.Charnes, W.W.Cooper, and E.Rhodes, "Measuring the efficiency of decision making units", *European Journal of Operational Research*, 1978, Vol.2, pp429-444.
- [2] T. Sueyoshi and S. Aoki, "A use of a nonparametric statistic for DEA frontier shift: the Kruskal and Wallis rank test", *OMEGA: The International Journal of Management Science*, Vol.29, No.1, 2001, pp1-18.
- [3] T.Sueyoshi, K.Onishi, and Y.Kinase, "A Bench Mark Approach for Baseball Evaluation", *European Journal of Operational Research*, Vol.115, 1999, pp.429-428.
- [4] T. Sueyoshi, Y. Kinase and S. Aoki, "DEA Duality on Returns to Scale in Production and Cost Analysis", *Proceedings of the Sixth Asia Pacific Management Conference 2000*, 2000, pp1-7.
- [5] W. W. Cooper, L. M. Seiford, K. Tone, *Data Envelopment Analysis: A comprehensive text with models, applications, references and DEA-Solver software*, Kluwer Academic Publishers, 2000.
- [6] R. Coombs, P. Sabiotti and V. Walsh, *Economics and Technological Change*, Macmillan, 1987.
- [7] Y. Yamada, T. Matui and M. Sugiyama, "An inefficiency measurement method for management systems", *Journal of Operations Research Society of Japan*, vol. 37, 1994, pp. 158-168 (In Japanese).
- [8] Y. Yamada, T. Sueyoshi, M. Sugiyama, T. Nukina and T. Makino "The DEA Method for Japanese Management: The Evaluation of Local Governmental Investments to the Japanese Economy", *Journal of the Operations Research Society of Japan*, Vol.38, No.4, 1995, pp.381-396.
- [9] S. Aoki, K. Mishima, H. Tsuji: Two-Staged DEA model with Malmquist Index for Brand Value Estimation, The 8th World Multiconference on Systemics, Cybernetics and Informatics, Vol. 10, pp.1-6, 2004.
- [10] R. D. Banker, A. Charnes, W. W. Cooper, "Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis", *Management Science*, Vol.30, 1984, pp.1078-1092.
- [11] R. D. Banker, and R. M. Thrall, "Estimation of Returns to Scale Using Data Envelopment Analysis", *European Journal of Operational Research*, Vol.62, 1992, pp.74-82.
- [12] H. Nakayama, M. Arakawa, Y. B. Yun, *Data Envelopment Analysis in Multicriteria Decision Making*, M. Ehrgott and X. Gandibleux (eds.) *Multiple Criteria Optimization: State of the Art Annotated Bibliographic Surveys*, Kluwer Academic Publishers, 2002.
- [13] E. W. N. Bernroider, V. Stix , "The Evaluation of ERP Systems Using Data Envelopment Analysis", *Information Technology and Organizations*, Idea Group Pub, 2003, pp.283-286.
- [14] Y. Zhou, Y. Chen, "DEA-based Performance Predictive Design of Complex Dynamic System Business Process Improvement", *Proceeding of Systems, Man and Cybernetics, 2003. IEEE International Conference*, 2003, pp.3008-3013.
- [15] R. G. Thompson, L. N. Langemeier, C. T. Lee, and R. M. Thrall, "The Role of Multiplier Bounds in Efficiency Analysis with Application to Kansas Farming", *Journal of Econometrics*, Vol.46, 1990, pp.93-108.
- [16] W. W. Cooper, W. Quanling and G. Yu, "Using Displaced Cone Representation in DEA models for Nondominated Solutions in Multiobjective Programming", *Systems Science and Mathematical Sciences*, Vol.10, 1997, pp.41-49.
- [17] S. Aoki, Y. Naito, and H. Tsuji, "DEA-based Indicator for performance Improvement", *Proceeding of The Third International Conference on Active Media Technology*, 2005, pp.327-330.



## Appendix

Table A-1. Evaluation value ( $\theta^{MVP,0.50}$ ) and optimal solutions ( $v^*, u^*, d^{+*}, d^{-*}$ )

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.038	0.078	0.030	0	0.020	0	0.116	0	0	0	1.104	0	0	1.670	0	0.053	0
2	B	0.038	0.0	0.004	0	0.021	0	0.160	0.162	0	0	0.249	0	0.255	0.631	0	0.237	0
3	C	0.025	0.0	0.003	0	0.014	0	0.104	0.105	0	0	0.162	0	0.166	0.410	0	0.154	0
4	D	0.011	0.022	0.009	0	0.006	0	0.033	0	0	0	0.315	0	0	0.477	0	0.015	0
5	E	0.021	0.017	0.005	0	0.015	0	0.062	0.085	0	0	0.257	0	0	0.601	0	0.096	0
6	F	0.030	0	0.004	0	0.017	0	0.126	0.127	0	0	0.196	0	0.201	0.497	0	0.187	0
7	G	0.027	0	0.003	0	0.015	0	0.112	0.114	0	0	0.175	0	0.179	0.443	0	0.167	0
8	H	0.020	0	0.002	0	0.011	0	0.083	0.084	0	0	0.129	0	0.133	0.328	0	0.123	0
9	I	0.032	0	0.004	0	0.018	0	0.134	0.136	0	0	0.209	0	0.214	0.529	0	0.199	0
10	J	0.031	0.063	0.025	0	0.016	0	0.094	0	0	0	0.897	0	0	1.357	0	0.043	0
11	K	0.033	0.028	0.007	0	0.023	0	0.099	0.134	0	0	0.407	0	0	0.952	0	0.152	0
12	L	0.015	0.012	0.003	0	0.010	0	0.044	0.060	0	0	0.183	0	0	0.428	0	0.069	0

No.	DMU	Slack Variable												Evaluation Value			
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-	$0.50 \times \theta^*$	$0.50 \times \theta^{**}$	$\theta^{MVP,0.50}$	
1	A	0	1.330	0	0	0.226	0	0.427	0	0	0.706	0.845	0	0.500	0.448	0.052	
2	B	0	0.292	0	0	0.284	0	0.054	0	0	0	0.673	0	0.419	0.500	-0.081	
3	C	0	0.190	0	0	0.185	0	0.035	0	0	0	0.438	0	0.500	0.430	0.070	
4	D	0	0.380	0	0	0.064	0	0.122	0	0	0.202	0.241	0	0.500	0.500	0.000	
5	E	0	0.320	0	0	0.212	0	0.120	0	0	0	0.560	0	0.199	0.500	-0.301	
6	F	0	0.230	0	0	0.224	0	0.042	0	0	0	0.531	0	0.407	0.500	-0.093	
7	G	0	0.205	0	0	0.200	0	0.038	0	0	0	0.473	0	0.500	0.415	0.085	
8	H	0	0.152	0	0	0.148	0	0.028	0	0	0	0.350	0	0.500	0.500	0.000	
9	I	0	0.245	0	0	0.238	0	0.045	0	0	0	0.565	0	0.381	0.500	-0.119	
10	J	0	1.080	0	0	0.183	0	0.347	0	0	0.573	0.687	0	0.327	0.500	-0.173	
11	K	0	0.506	0	0	0.336	0	0.190	0	0	0	0.886	0	0.500	0.500	0.000	
12	L	0	0.228	0	0	0.151	0	0.086	0	0	0	0.399	0	0.301	0.500	-0.199	

Table A-2. Evaluation value ( $\theta^{MVP,0.25}$ ) and optimal solutions ( $v^*, u^*, d^{+*}, d^{-*}$ )

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.036	0.080	0.051	0	0.001	0	0.195	0	0.187	0	1.515	0	0.436	1.565	0	0	0
2	B	0.038	0	0.020	0	0.008	0	0.246	0	0	0	0.614	0	0.663	0.531	0	0.178	0
3	C	0.025	0	0.013	0	0.005	0	0.160	0	0	0	0.399	0	0.431	0.345	0	0.116	0
4	D	0.010	0.023	0.014	0	0.000	0	0.056	0	0.053	0	0.433	0	0.125	0.447	0	0	0
5	E	0.013	0.029	0.018	0	0.001	0	0.071	0	0.068	0	0.549	0	0.158	0.567	0	0	0
6	F	0.030	0	0.016	0	0.006	0	0.194	0	0	0	0.484	0	0.522	0.419	0	0.140	0
7	G	0.027	0	0.014	0	0.006	0	0.173	0	0	0	0.432	0	0.466	0.373	0	0.125	0
8	H	0.020	0	0.011	0	0.004	0	0.128	0	0	0	0.319	0	0.345	0.276	0	0.093	0
9	I	0.032	0	0.017	0	0.007	0	0.206	0	0	0	0.515	0	0.556	0.446	0	0.149	0
10	J	0.029	0.065	0.041	0	0.001	0	0.159	0	0.152	0	1.229	0	0.354	1.270	0	0	0
11	K	0.050	0	0.026	0	0.010	0	0.319	0	0	0	0.799	0	0.861	0.691	0	0.231	0
12	L	0.022	0	0.012	0	0.005	0	0.142	0	0	0	0.355	0	0.383	0.307	0	0.103	0

No.	DMU	Slack Variable												Evaluation Value			
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-	$0.25 \times \theta^*$	$0.75 \times \theta^{**}$	$\theta^{MVP,0.25}$	
1	A	0	1.757	0	0	0.165	0	0.627	0.399	0	0	1.596	0	0.250	0.627	-0.377	
2	B	0	0.666	0	0.058	0	0.038	0.012	0	0	0.705	0	0	0.250	0.750	-0.500	
3	C	0	0.433	0	0.038	0	0.025	0.008	0	0	0.458	0	0	0.250	0.536	-0.286	
4	D	0	0.502	0	0	0.047	0.114	0	0	0	0.456	0	0	0.250	0.667	-0.417	
5	E	0	0.637	0	0	0.060	0.144	0	0	0	0.578	0	0	0.108	0.750	-0.642	
6	F	0	0.524	0	0.046	0	0.030	0.009	0	0	0.556	0	0	0.215	0.750	-0.535	
7	G	0	0.468	0	0.041	0	0.027	0.008	0	0	0.496	0	0	0.250	0.511	-0.261	
8	H	0	0.346	0	0.030	0	0.020	0.006	0	0	0.367	0	0	0.250	0.728	-0.478	
9	I	0	0.558	0	0.049	0	0.032	0.010	0	0	0.591	0	0	0.250	0.727	-0.477	
10	J	0	1.426	0	0	0.134	0.323	0	0	0	1.295	0	0	0.169	0.750	-0.581	
11	K	0	0.865	0	0.076	0	0.049	0.016	0	0	0.917	0	0	0.250	0.391	-0.141	
12	L	0	0.385	0	0.034	0	0.022	0.007	0	0	0.407	0	0	0.250	0.750	-0.500	

Table A-3. Evaluation value ( $\theta^{MVP,0}$ ) and optimal solutions ( $v^*, u^*, d^{+*}, d^{-*}$ )

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0	0.125	0	0.066	0	0	1.100	0	1.834	0	7.838	0	3.325	0	0	0	4.591
2	B	0.038	0	0.001	0.028	0	0	0.545	0	0.371	0	2.723	0	1.678	0	0	0	1.278
3	C	0.025	0	0	0	0.038	0	0.538	0	0.550	0	1.550	0	1.375	0	0	0	0.713
4	D	0	0.036	0	0.019	0	0	0.314	0	0.524	0	2.239	0	0.950	0	0	0	1.312
5	E	0.033	0	0	0.021	0.008	0	0.508	0	0.379	0	2.317	0	1.900	0	0	0	1.096
6	F	0.030	0	0.040	0	0	0	0.606	0	0.675	0	1.950	0	1.943	0	0	0	0.502
7	G	0.027	0	0	0	0.041	0	0.581	0	0.595	0	1.676	0	1.486	0	0	0	0.770
8	H	0.020	0	0.026	0	0	0	0.400	0	0.445	0	1.287	0	1.283	0	0	0	0.331
9	I	0.052	0	0	0	0.048	0	0.694	0	0.710	0	2.000	0	1.774	0	0	0	0.919
10	J	0	0.100	0.091	0	0	0	1.300	0	2.378	0	5.804	0	4.139	0	0	0	2.470
11	K	0.050	0	0	0	0.075	0	1.075	0	1.100	0	3.100	0	2.750	0	0	0	1.425
12	L	0.022	0	0	0.014	0.006	0	0.339	0	0.253	0	1.544	0	1.000	0	0	0	0.731

No.	DMU	Slack Variable												Evaluation Value			
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-	$0 \times \theta^*$	$1 \times \theta^{**}$	$\theta^{MVP,0}$	
1	A	0	9.984	0	10.375	0	3.769	0	1.047	0	3.356	0	0.556	0	0.476	-0.476	
2	B	0	3.591	0	3.776	0	1.286	0	0.542	0	1.397	0	0	0	0.730	-0.730	
3	C	0	1.513	0	1.638	0	0.463	0	0.300	0	0.363	0	0.188	0	0.392	-0.392	
4	D	0	2.853	0	2.964	0	1.077	0	0.299	0	0.959	0	0.159	0	0.513	-0.513	
5	E	0	2.954	0	3.142	0	1.033	0	0.463	0	1.067	0	0	0	1.000	-1.000	
6	F	0	1.962	0	1.173	0	0.958	0	0.269	0	1.924	0	1.482	0	0.666	-0.666	
7	G	0	1.635	0	1.770	0	0.500	0	0.324	0	0.392	0	0.203	0	0.379	-0.379	
8	H	0	1.295	0	0.774	0	0.632	0	0.177	0	1.270	0	0.978	0	0.564	-0.564	
9	I	0	1.952	0	2.113	0	0.597	0	0.387	0	0.468	0	0.242	0	0.626	-0.626	
10	J	0	5.922	0	4.009	0	2.883	0	0.461	0	4.643	0	3.974	0	0.685	-0.685	
11	K	0	3.025	0	3.275	0	0.925	0	0.600	0	0.725	0	0.375	0	0.580	-0.580	
12	L	0	1.969	0	2.094	0	0.689	0	0.308	0	0.711	0	0	0	1.000	-1.000	