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Control of a chaotic economic system

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Abstract: A chaotic system describing the FDI in China is investigated. Two-dimensional map of the system has been controlled by the use of time delay feedback method. The factors affecting FDI are also discussed in the paper.

Keywords: Economic system, Chaos, Control

1 Introduction

Foreign direct investment (FDI) is defined as the flow of capital from a foreign country to a host country to establish production or service facilities and to conduct business activities. It is widely believed that the advantages that FDI brings to the standard of living and prospects for economic growth of the host nation largely outweigh its disadvantages. Since the adoption of the “open-door” policy in late 1978, China has experienced rapid expansion in FDI inflows. Attracting FDI has been a strategic economic policy adopted by China to upgrade technology and boost economic growth. It has absorbed US$535 billion in 2003 and is now the largest recipient of FDI in the world. There are many papers discussing the effects of FDI on China’s development. In this work, we investigate how to control the chaotic behavior of the FDI in China.

It is generally accepted that economy belongs to extremely complex systems [1] and that many economical systems can exhibit a phenomenon of deterministic chaos. Chaos theory is a blanketing theory that covers all aspects of science. It shows up everywhere in world today: mathematics, physics, chemistry, biology, communications, computer and even music. The range of potential applications of chaos theory in economics is very broad: from forecasting movements in foreign exchange rates and stock markets, to understanding international business cycles. As more and more knowledge is gained about the nature of chaos, recent interests are now directed to controlling a chaotic system. Both chaos utilization and elimination are important depending on the specific applications. Chaos control is an effective method for both chaos utilization and elimination. Since the pioneering work of Ott *et al.* [2], much progress has been achieved in this field. Controlling chaos has become more and more interesting in academic research and practical applications. Chaos control algorithms can be broadly classified into two categories: feedback control and non-feedback control. Feedback methods [2-5] control chaos by stabilizing a desired unstable periodic orbit which is embedded in a chaotic attractor. Time-delayed feedback method is one of the feedback control methods. Controlling of some economical processes seems to be one of the most important and challenging tasks facing the economists and politicians responsible for economical policy. It is of interesting to examine this method in a economic context and to explore its implications to economic system.

2. The model system

The model to be investigated in present paper was proposed by Yanchuk and his co-workers [6]. Before making our studies, we at first give some descriptions about this model.

In the process of constructing the model, it is assumed that the consumption C_t at time t to be the following function of income

$$C_t = \alpha_0 + (\alpha_1 - \alpha_2 Y_{t-1})Y_t \qquad (1)$$

where α_0, α_1 and α_2 are constant parameters. In the absence of the regional trade the income is formed mostly by consumption and FDI:

$$Y_t = C_t + \mathrm{FDI}_t \qquad (2)$$

In order to take into account the adoption of FDI and its influence on the development of the region, the standard

Keynesian adjustment algorithm is used:

$$\text{FDI}_t = \gamma_1' (Y_{t-1} - Y_{t-2}) + \gamma_0 \text{FDI}_{t-1} \qquad (3)$$

In order to prevent FDI to be negative, Eq. (3) is equipped with the "floor":

$$\text{FDI}_t = \max\{ \gamma_1' (Y_{t-1} - Y_{t-2}) + \gamma_0 \text{FDI}_{t-1}, 0 \} \qquad (4)$$

The set of Eqs. (1), (2) and (4) constitutes the closed dynamical model for the description of the fluctuations of consumption level, income, and foreign direct investments (FDI). With an appropriate rescaling procedure $y = Y(\alpha_2/\alpha_1)$, $c = C(\alpha_2/\alpha_1^2)$, and $x =$ FDI (α_2/α_1^2), these equations can be written as follows:

$$c_t = \alpha + (1 - y_{t-1}) y_t \qquad (5)$$

$$\alpha_t y_t = c_t + x_t \qquad (6)$$

$$x_t = \max\{ \gamma_0 x_{t-1} + \gamma_1 (y_{t-1} - y_{t-2}), 0\} \qquad (7)$$

where $\alpha = \alpha_0 \alpha_2 / \alpha_1^2$ and $\gamma_1 = \gamma_1' / \alpha_1$ are new parameters. Writing x_t in terms of y_{t-1} and y_{t-2} by combing Eq.(5) and Eq.(6), and then substituting it into Eq.(7), a two dimensional piecewise-smooth map($R_+^2 \to R_+^2$) is obtained:

$$F : \begin{pmatrix} y \\ z \end{pmatrix} \to \begin{pmatrix} \frac{1}{\beta + y}[\alpha + \max\{-\alpha\gamma_0 + \gamma_0(\beta + z)y + \gamma_1(y - z), 0\}] \\ y \end{pmatrix} \qquad (8)$$

When the parameters are selected as follows: $\alpha = 0.9 \times 10^{-5}$, $\beta = 0.01$, $\gamma_0 = 0.7$ and $\gamma_1 = 0.03$, the system (8) exhibits chaotic behavior. The corresponding chaotic attractor is illustrated in Fig.1.

3. Controlling chaos

Delayed feedback control method proposed by Pyragas [5] is a simple and convenient method of controlling chaos in nonlinear dynamical systems. This method is based on a feedback of the difference between the current state and the delayed state. The delay time is set to correspond to the period of the desired unstable periodic orbit (UPO) so that the feedback term vanishes after the UPO is stabilized. Pyraga's method is a continuous method, which continuously applies control signals to the system by a feedback function, $F(t,\tau) = K[z(t-\tau) - z(t)]$. Here z is a time dependent measurement of the system state, τ the delay time, and K the feedback strength. The advantage of this method is that it does not require full information about the UPO. In this work, we use time delayed feedback method to control the chaos of FDI system (8).

In our studies, the control force F is assumed to be as follows:

$$F = K\ (y_n - y_{n-m}) \quad (9)$$

where m is the delay time, K the feedback strength. In such a case, the equation describing the evolution of y in Eq. (8) reads

$$y_{n+1} = \frac{1}{\beta + y_n}\left[\alpha + \max\{-\alpha\gamma_0 + \gamma_0(\beta + z_n)y_n + \gamma_1(y_n - z_n),0\}\right] + K\ (y_n - y_{n-m}) \quad (10)$$

Results of simulations of such a control are given in Fig.2. One can see that when m=2, the system is stabilized at the period-one orbit with K being within 0.121-- 0.225. If K is increased to 0.227, the system exhibits chaotic behavior. When K reached 0.241, the control of the period-two orbit was switched on. After K overruns 0.316, the system came back to the chaotic behavior. By adjusting the feedback strength K, the system can be controlled on the period-1, period-2, period-4 orbits, etc. There is a time interval between the periodic orbit and the chaotic state. In other words, chaotic states and periodic states appear by turns.

4. Conclusions

In this paper we have shown that one can control the deterministic chaos in a simple economic model describing foreign direct investment (FDI). The control consists of adding to the system variable a time-delayed feedback. One may ask whether present control method is possible for practical application in real economical systems. Our opinion is positive. It is reasonable to think the control force (the time-delayed control term) as the intervention force of the government. In other words, we can take government's intervention on the FDI as the above control term figuratively for the practical application. It is well known that there are many factors affecting FDI, such as intellectual property rights protection [7], labor market [8], opening policy [9], foreign exchange rate [10,11], relative wages and income convergence[12], GDP in the host country[13], bureaucratic corruption and environmental policy [14], economic stability and the political climate [15], and so on. If we want to attract more FDI and make the best use of it, we must take into account above factors.

In summary, the general conclusion of our simulations is that the Chinese authorities can control the FDI chaos by exerting an additional force or intervention on the system and play a positive role in attracting more inward FDI. However, it should be indicated here that the control force (or feedback strength) can not be too big if we want to obtain appropriate stabilized periodic orbits. For example, in the case of delay time m =2 if we take K = 0.5, the phase portrait and the time series are all in negative region, which is shown in Fig.3. It is clear that this control process under this condition has no economically significance because the system variables and their changes are all in negative area.

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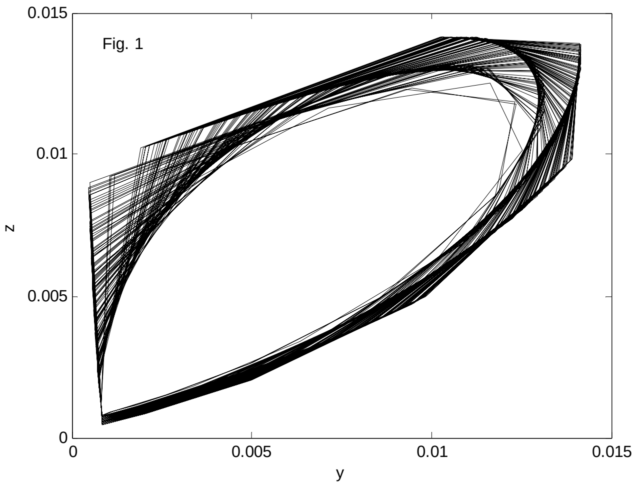
Figures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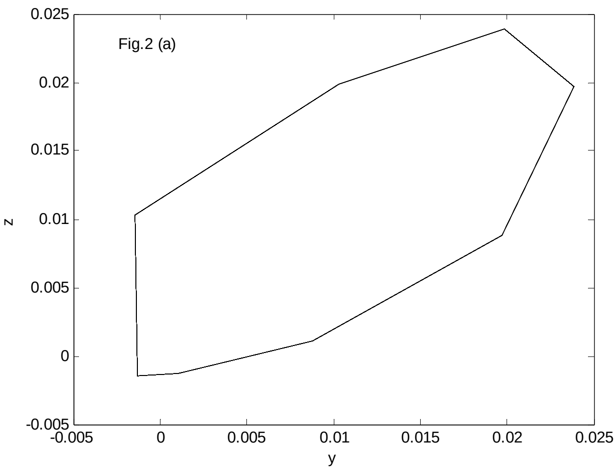
0.005
0
0.005
0.01
0.015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y
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0.005
0
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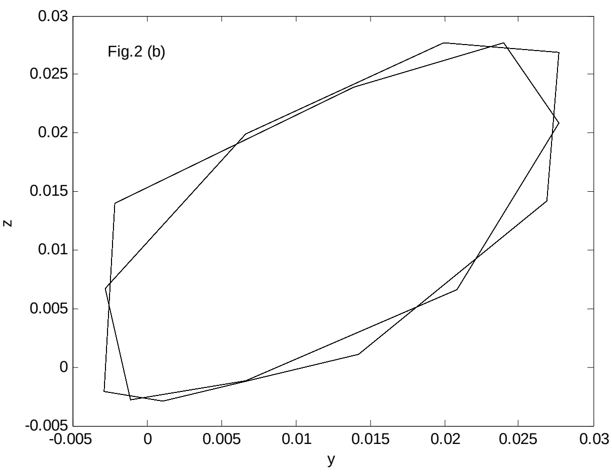
0
0.005
0.01
0.015
0.02
0.025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z
y
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0.025
0.02
0.015
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0
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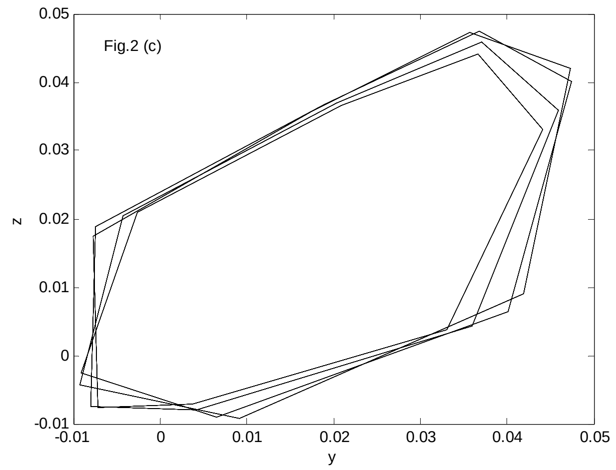
0
0.005
0.01
0.015
0.02
0.025
0.03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z
y
0.05
0.04
0.03
0.02
0.01
0
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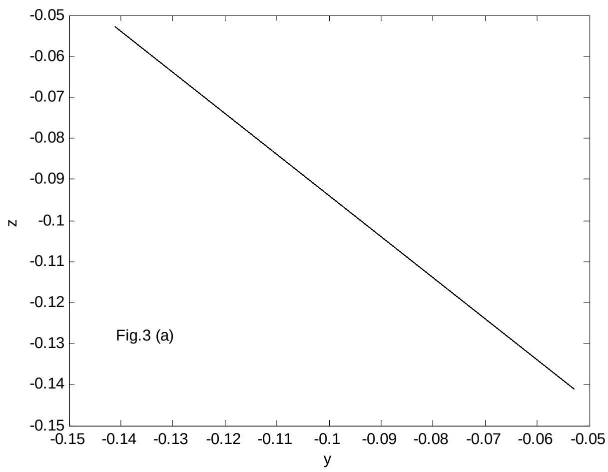
-0.01
0
0.01
0.02
0.03
0.04
0.05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z
y
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-0.09
-0.1
-0.11
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-0.13
-0.14
-0.15
-0.15
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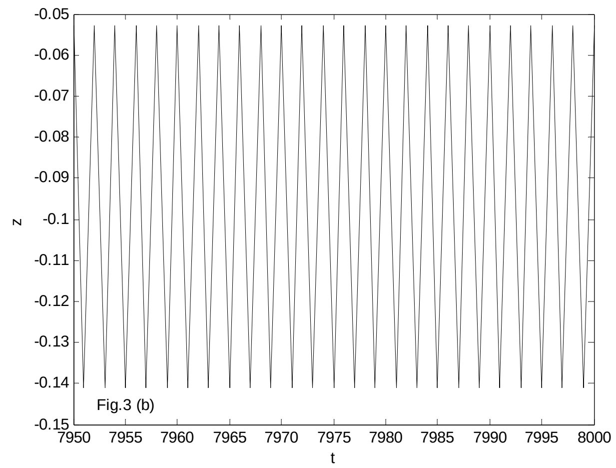
Fig.3 (b)
z
t
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-0.1
-0.11
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-0.13
-0.14
-0.15
7950
7955
7960
7965
7970
7975
7980
7985
7990
7995
8000

Captions

Fig.1. The chaotic attractor of the FDI system (8). The parameters are: $\alpha = 0.9 \times 10^{-5}$, $\beta = 0.01$, $\gamma_0 = 0.7$ and $\gamma_1 = 0.03$.

Fig.2. Results of control simulation for the case of delay time *m* = 2, when the controller (9) is added to the system (8). The feedback strength *K* in order are: (a) 0.2 (period-1); (b) 0.25 (period-2); (c) 0.437 (period-4). The other parameters are the same as Fig.1.

Fig.3. (a) Phase portrait and (b) time series for the case of delay time *m* = 2 and feedback strength *K* = 0.5, when the controller (9) is added to the system (8). The other parameters are the same as Fig.1.