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The Application of Wavelet Transform in Stock Market

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Abstract

In this study, the wavelet transform is briefly described and is also compared with the popular Fourier transformation (FT), as well as a specific presentation of the continuous wavelet transform and the discrete wavelet transform. Different application of wavelet analysis in stock market is introduced, including decomposition of stock series with multi-resolution analysis, denoising of stock price series, characterizing abrupt changes in the stock prices, and detecting the self-similarity of stock series. In addition, Examples are given with the wavelet analysis on some real estate stock price data in China mainland.

Keywords: Wavelet, Stock market

1 Introduction to wavelet transforms

Mathematical transformations are applied to time series to obtain further information that is not available in the original time-domain series, among which the Fourier transforms are probably the most popular since the 19th century. However, the FT gives only the frequency information of the series, but not the time information simultaneously. That means FT is suitable for stationary series whose frequency does not change in time, but not for non-stationary series. Wavelet transform is of the types that can give the time-frequency representation of series, as well as short time Fourier transform (STFT).

It is a long story to present the advantage and disadvantage of STFT. The wavelet transform is a relatively new concept (about 20 years old). It was developed to overcome the resolution problem of the STFT. Since in 1988 Mallat and Meyer presented the basic frame of multi-resolution analysis (MRA), a lot of wavelet functions have been developed, three common examples are shown in Figure 1.

(a) morlet   (b) db3   (c) db4

Figure 1. Several mother wavelet

In mathematics, the analyzing wavelet is defined as follows:

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}} \psi(\frac{t-b}{a}) \qquad (1)$$

For a series function $f \in L^2(R)$, the continuous wavelet transform (CWT) is defined

$$\begin{aligned} Wf(a,b) &= \langle f, \psi_{a,b} \rangle \\ &= |a|^{-\frac{1}{2}} \int f(t) \psi(\frac{t-b}{a}) dt \end{aligned} \qquad (2)$$

As seen in the above equations, $\psi(t)$ is the mother wavelet (as seen in Figure.1). The term “wavelet” means a small wave which refers that the function is oscillatory like a wave and is compactly supported. $\{\psi_{a,b}\}$ is called the analyzing wavelet, in which “a” is interpreted as a reciprocal of frequency, or scale as called, and “b” represents time or space. As a decrease the oscillations of proceed series become more intense and show high-frequency behavior; similarly, it becomes drawn out and shows low-frequency behavior as a increase, i.e. the change of parameters "a" decides the WT window expanding or shrinking; and "b" is related to the location of the window. The coefficients of wavelet transform of a series function f is symbolized "*Wf*", as seen in Equation 2.

In CWT, the parameters a, b, and t are all continuous variables, so the CWT of a series yields a lot of information, as seen in the following Figure 1. However, it is obviously that there is quite a few redundant data in the CWT. Thus the discrete wavelet transform (DWT) is advanced which can be regarded as subsamples that retain certain key features of CWT.

As we pick a (in Equation 1) to be the form of a_0^{-j} (here a_0 is a constant, $a_0 > 1$), and generally $a_0 = 2$ (which is called Dyadic Wavelet), the dyadic wavelet and the DWT are defined as follows:

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k) \quad j,k \in Z \tag{3}$$

$$\begin{aligned} Wf(j,k) &= \langle f, \psi_{j,k} \rangle \\ &= 2^{\frac{j}{2}} \int f(t)\psi(2^j t - k)dt \end{aligned} \tag{4}$$

As seen in all the above equations, the scale a and time b in CWT are discrete by integer parameter j and k in DWT. Recall that the CWT is a correlation between a wavelet at different scales and the series with the scale (or the frequency) being used as a measure of similarity. The CWT coefficients are computed by changing the scale of the analysis window, shifting the window in time, multiplying by the series, and integrating over all times. In the discrete case, filters of different cutoff frequencies are used to analyze the series at different scales. The DWT employs two sets of functions, called scaling functions and wavelet functions, which are associated with low-pass and high-pass filters, respectively. The series is passed through a series of high-pass filters to analyze the high frequencies, and it is passed through a series of low-pass filters to analyze the low frequencies, which is how the decomposition of the series into different frequency bands is simply obtained.

In a summary, the DWT is an important practical tool for series/ time series/or images analysis for the following basic reasons [1]:

(1) The DWT re-expresses a time series in terms of coefficients that are associated with a particular time and a particular dyadic scale $2^{j/2}$. These coefficients are fully equivalent to the original series in that we can perfectly reconstruct it from the DWT coefficients.

(2) The DWT allows us to partition the energy in a time series into pieces that are associated with different scales and times.

(3) The DWT can be computed using an algorithm that is faster than the celebrated fast Fourier transform algorithm.

2 The application of wavelet analysis on stock market

2.1 Decomposition and reconstruction of stock price series

The DWT procedure decomposes a signal with a multi-resolution analysis, and the coefficients obtained in every scale includes two parts of the low frequencies and high frequencies corresponding to the approximate part signal and detail signal respectively. With the scale increase, the approximate signal becomes prominent which reveals the developing tendency of the signal, and the detail part becomes less and less. So, with a multi-resolution analysis, the DWT of a stock signal allow a prediction of the developing tendency of the signal.

As seen in Figure 2, a) is the price series of a real estate stock in China mainland, b), c) and d) are the low frequencies DWT coefficients in scale 1, 3 and 5 respectively. Comparing with the original signal, the approximate part signal appears less coarse, which is helpful to determine a long term developing tendency of the signal.

Figure.3 illustrates the reconstruction of the signal with the low frequency part coefficients in different scales. In Figure.3, the upper graphic is the return series of the real estate stock, the others are reconstruction signals in scale 1 to 5 from the up direction. It is usually quite difficult to detect the developing tendency from the upper original signal. As seen from the plot, it is easy to determine the sharp point in the lower scale and to distinguish the long term tendency in a higher scale (such as a4 and a5 in Figure.3).

2.2 Characterization of self-similarity of stock series

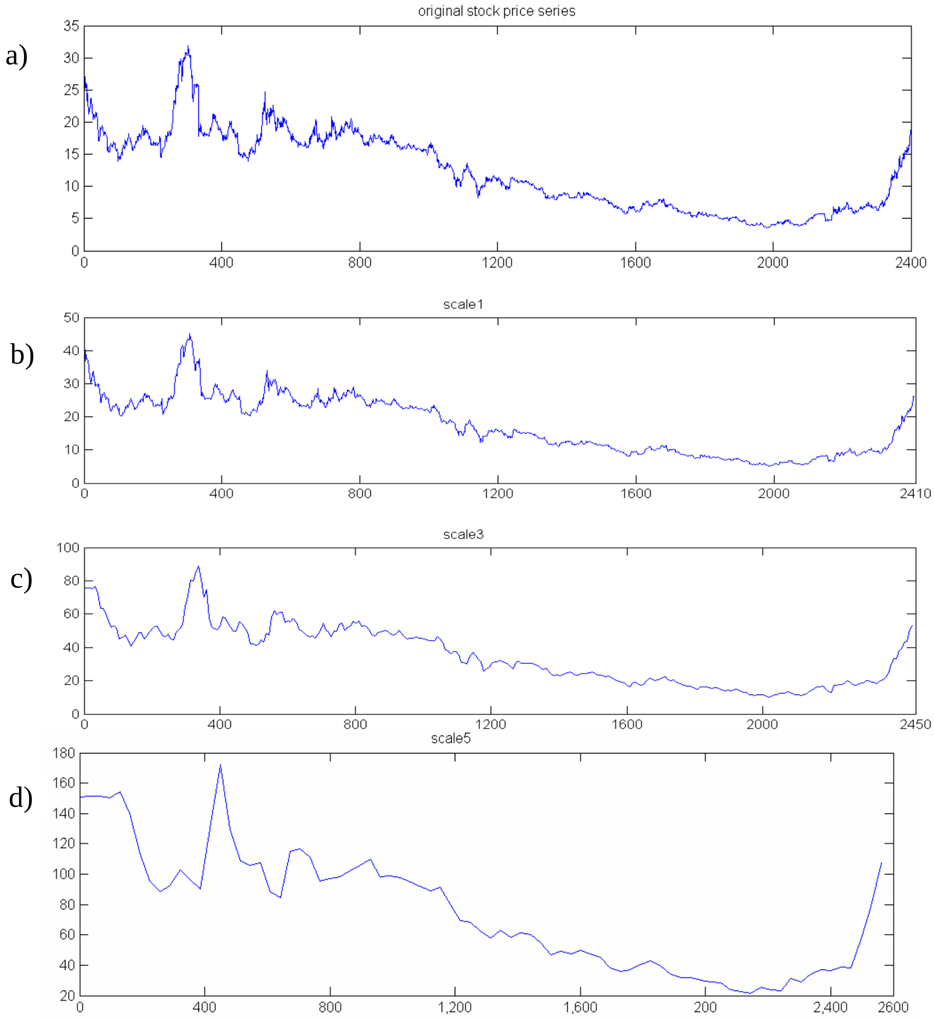

Figure 2. The low frequencies of signal with a DWT in different scales

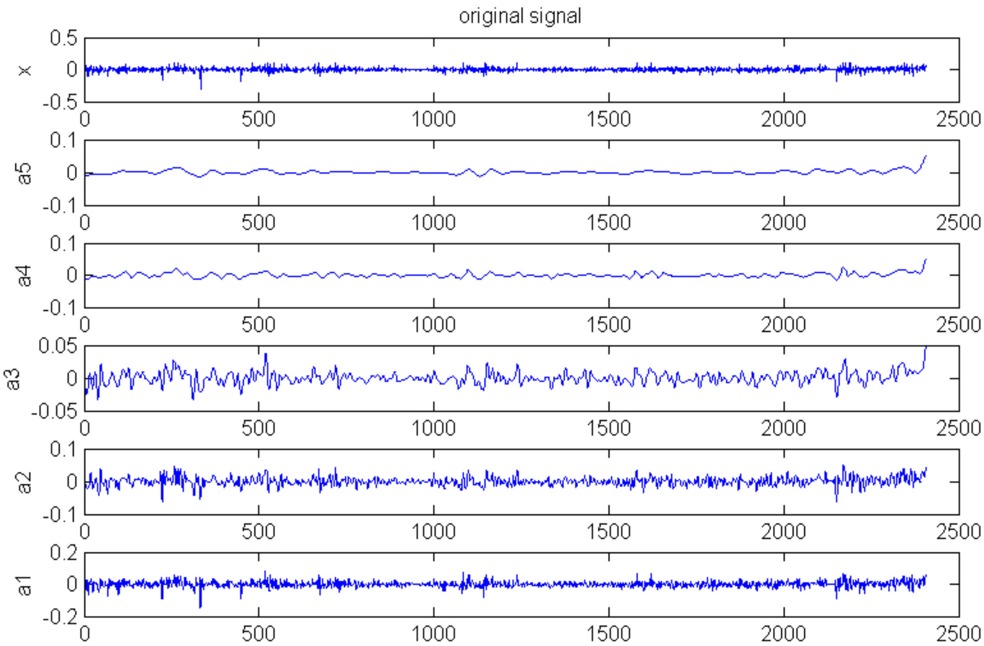

Figure 3. The reconstruction with the low frequency coefficients in different scale

Many things in the nature have the property of self-similarity, such as snow-flake, map leaf, cloud, coastline, etc. In Mandelbrot's definition, self-similarity means that each piece is similar to the whole - not necessarily identical to the whole. His classic example of self-similarity is the coastline of Britain. From a satellite view, it looks similarly jagged at any altitude. The concept of self-similarity doesn't require that the coastline looked exactly the same from each altitude, only that it was similar in its texture, irregularity, or coarseness.

Since wavelet transform is a representation of time-frequency of series in multi-resolution, it is often used in analysis of self-similarity of series or time series. The following figures are examples of analysis of self-similarity with a wavelet transform. In Figure.4, one is Koch curve which is obviously self-similar for it is a synthetic series built recursively, and the other is the grey figure of wavelet transforms coefficients in different scales. From the lower grey figure, it can be seen that it is similar in grey shape at any scale, and also has very clear profile as looks along the column of figure. The first one in Figure 5 is the real estate stock return series, and the second grey figure is its wavelet decomposition coefficients in different scale. It also can be concluded that the stock does own self-similar character.

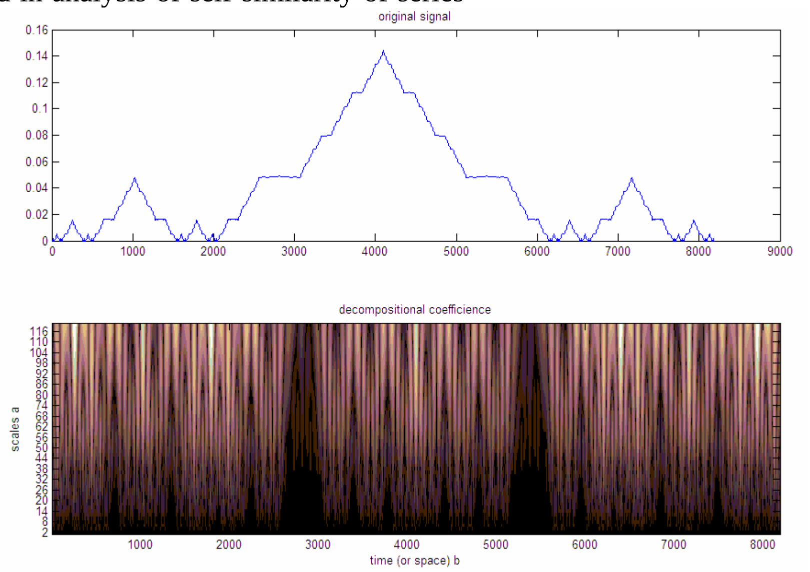

Figure4. Koch curve and its decomposition coefficients

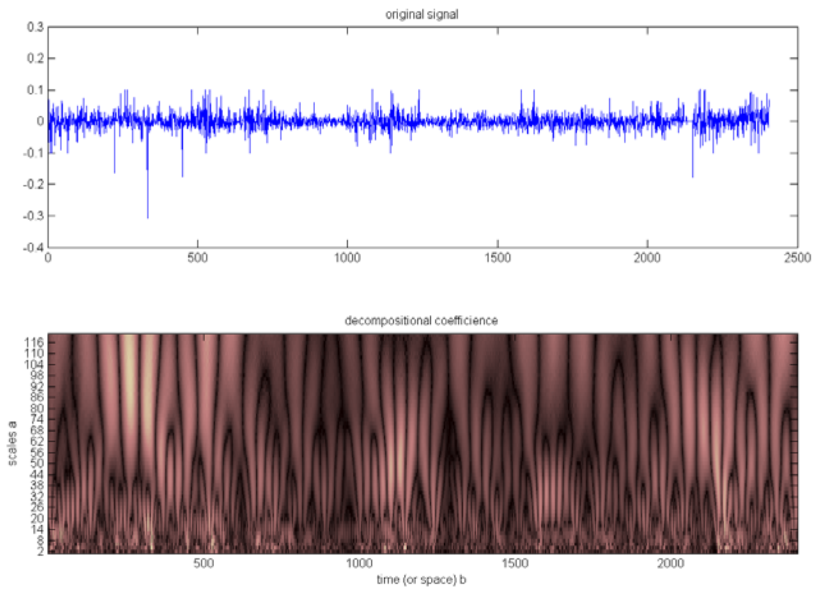

Figure 5. The real estate stock return series and its decomposition coefficients

2.3 Denoising of stock price series based on wavelet-filter

Sometimes market participants can be divided into two parts: those making rational choices based on some adopted strategies and those making effectively random decisions (regarded as noise). Denoising price series will be helpful to understand the impact of rational strategies on stock market. A key problem is how to define a rigorous noise in performing denosing process.

Compared with Fourier denoising method, the wavelet denoising can retain the peak and chop parts for its superior time-frequency localization property. In many studies, wavelet-based filters are widely used in multiscale denoising of stock price series; see e.g. [2, 3, 4, 5].

Supposing the series f_n with length n is denoised by noise signal e_n , then the signal observed including noise is $X_n = f_n + e_n$. So the objective is to optimize the approximation of signal f_n, that is, preserving the local characters with sharp, but not blurring brink while denoising.

Basic strategy is :

For the linear character of wavelet transform, it follows:

$$WX = Wf + We$$

So denoised signal can be obtained by a converse transform to the residual effective wavelet transform coefficients Wf after defining and removing the ones We controlled by noise signal.

The filtering procedure discussed is based on the universal threshold wavelet filtering [6,7], because it is most effective and simple among the several denoising approaches of wavelet transform. Its theoretical supporting is that the amplitude of wavelet coefficients of effective series may be larger than those including noise. Then the specific filtering process includes three parts: first, keep the whole coefficients in large scale after a multiscale wavelet decomposition of the series including noise; secondly, for the left small scale coefficients, by setting a threshold keep the part which is larger than it or perform a shrinkage process, and take the coefficients which is lower than the threshold as zero; finally, reconstruct the processed coefficients by a reverse wavelet transform, and effective series without noise is obtained. The universal method of choosing the threshold is VisuShrink, which gives an overall threshold $\sigma\sqrt{2\log n}$ for Gaussian-type noise. However, a further study is still needed on the type of noise signals, which is basis of the threshold representation.

The following figures are two examples of denoising results. Figure.6 is cited from literature [8], and Figure.7 illustrates the denoising of a real estate stock return series with the universal threshold wavelet filtering method.

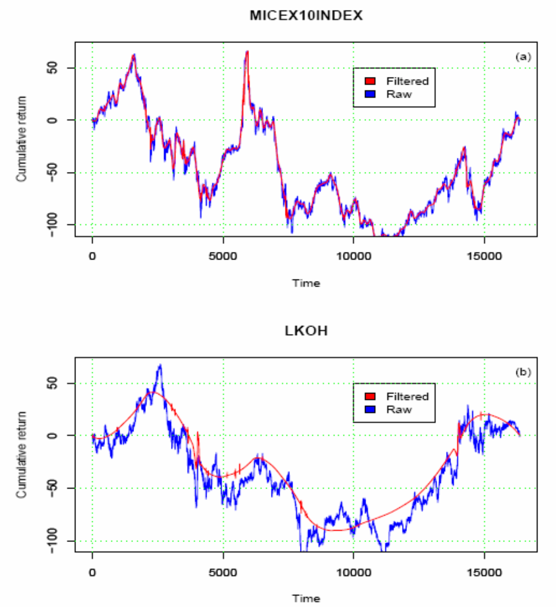

Figure.6 Raw versus wavelet-filtered price dynamics

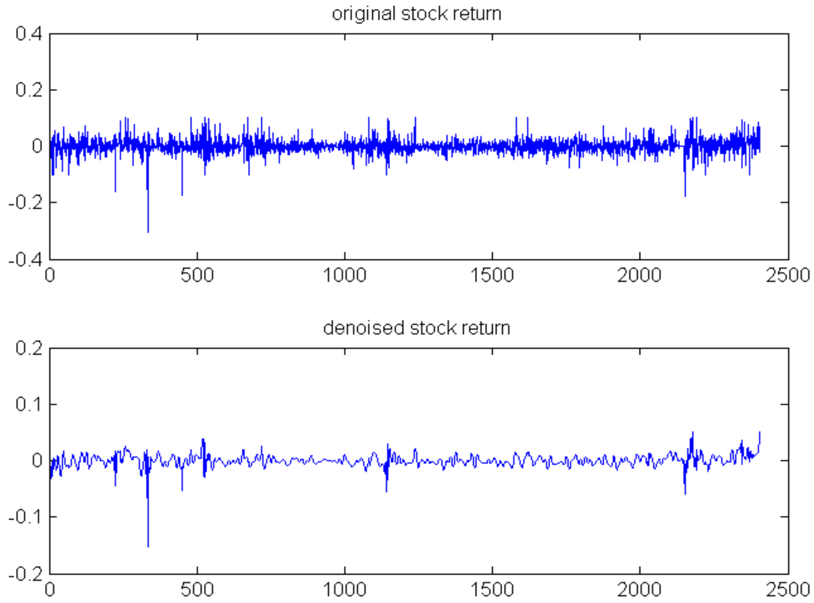

Figure 7. Denoising of a real estate stock return series with wavelet filtering

2.4 Characterizing abrupt changes in the stock prices with a wavelet analysis

Abrupt changes in the stock prices, either upwards or downwards, are usually preceded by an oscillatory behavior with frequencies that tend to increase as the moment of transition becomes closer.

Stock Markets have experienced rapid growth during recent years in many countries and efforts have been directed into studying their dynamic behavior. One group of research attempts to adapt physical and mathematical modeling techniques with a view on characterizing trends in the stock market prices (as in Refs. [9,10,11]).

A successful characterization of the dynamics of the stock prices, particularly of sudden large drops, can have a profound impact on risk management. Some of the many quantitative concepts and tools that are used to assist the decision making process include: moving average modeling, intra-day maximum and minimum price estimates, volume analysis, linear regression, correlation, support and resistance, moment sensor, cycle study, relative force index and others.

One very interesting approach considers that the stock prices have a dynamic behavior that resembles, very closely, some physical phenomena [11-13]. In particular, Johansen [11] showed that the behavior of the prices in the stock market, just prior to abrupt changes, present a profile very similar to those observed in the earthquake phenomenon. In many studies, it is frequently mentioned about the onset of small amplitude and high frequency fluctuations in the buy–sell actions, as an abrupt change (crash) becomes imminent.

Mathematical models have been presented to describe abrupt changes in the stock market; however, it is difficult to obtain the numerical values for the model parameters. In the wavelet transform, the function must satisfy the condition: , so it is required to remove the linear trend from stock price data. It is also available to remove the cyclic term with very low frequencies for the study of abrupt changes. After the first pre-processing of the stock price data, the wavelet decomposition technique is applied. The resulting coefficients are then presented on a grey graph. Then, it is possible to visualize graphically the dynamic behavior of data modeled by Johansen (as in Ref. [11]), so that some simple rules based on thresholds can be heuristically established to detect the eventual occurrence of short-time abrupt changes in the stock prices.

The proposed method (as in [9]) was applied to a number of actual data from the São Paulo Stock Exchange (BOVESPA) and draw downs in the IBOVESPA and other minor abrupt changes were shown to be successfully detectable. Moreover, the proposed method was also shown to be of use in the analysis of intraday behavior of the market.

2.5 Approximation and forecasting of stock series

Forecasting of financial time series is often difficult and complex due to the interaction of many variables involved, but it is full of challenge for its practical prospect. It is well known that linear models are inadequate for financial time series as in practice almost all economic processes are nonlinear to some extent.

During the last three decades, various nonlinear approaches have been developed for time series prediction. Of the nonlinear methods, neural networks have become very popular. Many different types of neural networks such as MLP and RBF have been proven to be universal function approximators, which make neural networks attractive for time series modeling, and for financial time-series forecasting in particular. However, an important prerequisite for the successful application of some modern advanced modeling techniques such as neural networks, however, is a certain uniformity of the data [14].

In financial time series, such an assumption of stationarity has to be discarded. Generally speaking, there may exist in different kinds of nonstationarities. To overcome the problems of monolithic global models, another efficient way is to design a hybrid scheme incorporating multiresolution decomposition techniques such as the wavelet transform, which can produce a good local representation of the signal in both the time and the frequency domain.

Recently some financial forecasting strategies have been discussed that used wavelet transforms to preprocess the data(as Ref. [15], [16]). In some literatures(as Ref. [17, 18]), a neuro-wavelet hybrid system had been developed that incorporates multiscale wavelet analysis into a set of neural networks for a multistage time series prediction. Article [19] uses wavelet packet theory to decompose price series into several subseries whose rule is relatively easy to learn by NN, so the task of forecasting stock price is decomposed into forecasting in the decomposed series using NN.

On the other side, many other approaches except neural network are employed to the prediction of financial time series, such as genetic algorithm, grey system model, fractal theory, etc. It was testified that the stock series owns a self-similar behavior, as mentioned in many literature, so a fractal theory may be effective in approximation and prediction of stock series. A linear fractional interpolation is introduced as following:

For a given observe data $(x_i, y_i), i = 0,1,2,...,n$, in an iterated function system (IFS) $\left\{R^2; w_i, i = 1,2,...n\right\}$, every function w_i is determined by an affine transformation

$$\begin{pmatrix} x \\ y \end{pmatrix} = w_i \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}, \quad i = 1,2,...,n \quad (5)$$

where a_i, c_i, d_i, e_i, f_i are real parameters, and meets the requirements that

$$w_i \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_{i-1} \\ y_{i-1} \end{pmatrix}; \quad w_i \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \quad (6)$$

Then all the parameters except d_i can be obtained by the above equations as follows:

$$a_i = \frac{x_i - x_{i-1}}{x_n - x_0}, \qquad c_i = \frac{y_i - y_{i-1}}{x_n - x_0} - d_i \frac{y_n - y_0}{x_n - x_0}$$

$$e_i = \frac{x_n x_{i-1} - x_0 x_i}{x_n - x_0}, \quad (7)$$

$$f_i = \frac{x_n y_{i-1} - x_0 y_i}{x_n - x_0} - d_i \frac{x_n y_0 - x_0 y_n}{x_n - x_0}$$

where d_i is a free parameter undetermined, as called “vertical scale factor”.

With Eq. (5) and (7), $\left\{w_i\right\}$ forms an iterated function system, whose attractor is the figure of fractal iterated function of the observed data series.

Some literatures have reported quite a good approximation of stock data with linear fractal theory after a multiple iteration of IFS (see e.g. [20-22]). The key problem in constructing the IFS is the choice of the free parameter d_i, which is usually defined as the ration of given data in vertical. To predict the stock data y_{i+1}, d_i is assumed to be the average value $\sum_{i=1}^{n} d_i / n$, which results in the difference between the predicted and observed data.

3 Conclusion

(1) The wavelet transform is described by comparing the specific presentation of the continuous wavelet transform with the discrete wavelet transform.
(2) Then the applications of wavelet analysis in stock price series are summarized in several parts, including decomposition of stock series with multiresolution analysis, denoising of stock price series, characterizing abrupt changes in the stock prices, and detecting the self-similarity of stock series. Examples are given with the wavelet analysis using some real estate stock price data in China mainland.
(3) As for the forecasting of financial signal, neuro-wavelet hybrid system is simply introduced. Since it has been well accepted that stock price series own a self similar property, the fractal interpolation method could be employed to approximate and predict it, which is also presented.

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