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Multi-criteria Decision Making with Fuzzy Targets

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Abstract

This paper discusses the issue of how to solve multi-criteria decision making (MCDM) problems from the target perspective. In particular, instead of making use of the traditional normalization method, it tries to calculate the probability of meeting some pre-designed targets for every criterion, where target values can be of different types: such as fuzzy number, fuzzy interval numbers and so on, and then uses weighted aggregation method to calculate the overall value for each alternative, at last selects the alternative(s) maximizing these overall values. The method is illustrated by the same application example taken from literature to compare with previous methods.

Keywords: Multi-criteria, Decision Making, Target-oriented Preferences, Targets Type

1 Instruction

Multi-criteria decision making (MCDM) refers to the problems of selecting among alternatives associated with multi-criteria. It is a problem with extensive theoretical and practical backgrounds, and it has received a great deal of attention from researchers in many disciplines for more than three decades [1; 2; 3; 4; 5]. In general, basically it involves the following three phases:

- Collecting the information about criteria values and criteria weights;
- Obtaining an overall value by weighted aggregation of the criteria values across all the criteria for each alternative;
- Ranking the overall values to get the best alternative(s).

So far, there are numerous methods proposed in the MCDM literatures. In this paper, we aim at applying the target-based approach to solving MCDM problems. Practically, thinking about targets is very natural in many situations [6], therefore it is quite interesting to think of MCDM problems from the target-based point of view. For example, when someone wants to buy a car, based on his/her personal hobbies and financial capacity, he/she may firstly establish a targeted car which should have some desirable properties in terms of criteria for example as colors, features, functions and so on.

In general, target-based approach has been used for decision making under uncertainty (DMUU) with a single criterion. Many methods and models have been proposed to solve the DMUU problems [6; 7; 8; 9; 10; 11]. Due to the mathematical and structural relation between DMUU and the MCDM models established in [12], we can apply the target-based decision model to MCDM problems. Bordely and Kirkwood [13] used performance targets to valuate the multi-criteria performance analysis, where the target-oriented decision maker has (DM in short) only two different utility levels, and these two utility levels can be set to one (if the target is achieved) or zero (if the target is not achieved). However, target achievements can be of different levels (i.e., from 0 to 1). As it is much easier and intuitively natural to define the fuzzy target, fuzzy target values can be of different types, such as fuzzy numbers, fuzzy interval numbers and so on.

The main focus of this paper is to solve MCDM problems with fuzzy targets. Essentially, instead of using the traditional MCDM method, it tries to calculate the probabilities of meeting some predefined targets for every criterion, where the targets can be of different types, and then the weighted aggregation method is used to calculate the overall value for each alternative, at last selects the maximal overall values according to the optimization principle.

The organization of this paper is as follows. In section 2 we present a general decision matrix. In section 3 we introduce the target-based decision making under uncertainty and in section 4 a multi-criteria decision making model with fuzzy targets has been proposed. The method is illustrated by the same application example taken from literature to compare with previous methods in section 5. This paper is concluded in the last section.

2 Preliminaries

In this section, we will enunciate the relationships between multi-criteria decision making (MCDM) and decision making under uncertainty (DMUU) based on [17]. In general, for any kind of decision problem, the DM has to choose one alternative out of a set of m mutually exclusive alternatives $A_i (i = 1,2,\ldots,m)$.

In the case of a decision problem with multi-criteria, the quality of the different alternatives depends on the n criteria $C_j (j = 1,2,\ldots,n)$. In general, given any criterion of the alternative, every criterion contribution to different alternatives depends on k states of nature $S_l (l = 1,2,\ldots,k)$, which cannot be influenced by the DM and may lead, for each alternative, to possibly different and more or less favorable contributions. Thereby, a criterion $C_j (j = 1,2,\ldots,n)$ is a real-valued function defined on the set of alternatives $A_i (i = 1,2,\ldots,m)$ parameterized by the set of states of nature $S = \{S_1, S_2,\ldots, S_k\}$.

MCDM problems, in other words, are characterized by the fact that, for each criterion $C_j (j = 1,2,\ldots,n)$ of the alternative $A_i (i = 1,2,\ldots,m)$, the outcome of every criterion C_j is characterized by a k-dimensional vector a_{ij} of criteria values with $a_{ij} = (a_{ij}(S_1), a_{ij}(S_2),\ldots, a_{ij}(S_k))$. This vector denotes the criteria value if the DM chooses alternative A_i. Then we can get the general decision matrix $D = A \times C \times S$.

According to the decision matrix, usually in the case of MCDM only one state of nature of each criterion is considered, i.e., with $k = 1$, then decision matrix reduces to a $m \times n$ matrix; in the case of DMUU only one criterion is considered, i.e., with $n = 1$, then the decision matrix reduces to a $m \times k$ decision matrix. In this paper, in the context the MCDM problems, only one state of nature is considered.

3 Target-oriented Decision Making Under Uncertainty

In this section, we will introduce the target-based method for DMUU. The DMUU problems can be effectively described by the decision matrix shown in Table 1.

Table 1. DMUU Problems

Alternatives	State of Nature			
	S_1	S_2	$\ldots$	S_k
A_1	c_{11}	c_{12}	$\ldots$	c_{1k}
A_2	c_{21}	c_{22}	$\ldots$	c_{2k}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	c_{m1}	c_{m2}	$\ldots$	c_{mk}

In Table 1, the set $A_i (i = 1,2,\ldots,m)$ represents the alternatives available to a DM, one of which must be selected; $S_l (l = 1,2,\cdots,k)$ denotes the possible values or states associated with the state of nature S. Each element c_{il} of the matrix is the payoff the DM receives if the alternative A_i is selected and state S_l occurs. Most often, it is assumed that there exist a probability distribution P_s over $S = \{S_1, S_2, \cdots, S_k\}$, such that $P_l = P_S(S = S_l)$, where $\sum_{l=1}^{k} P_l = 1, \forall P_l > 0$. A bounded domain of the payoff variable can be restricted such that $D = [c_{\min}, c_{\max}]$, i.e., $c_{\min} \le c_{il} \le c_{\max}$.

As is well known, the most common method to valuate alternatives A_i is to use the expected utility defined as:

$$v(A_i) = \sum_{l=1}^{k} U(c_{il}) P_S(S_l) \tag{1}$$

where U is a utility function defined over D.

On the other hand, each alternative A_i can be formally considered as a random payoff having the probability distribution P_i defined, with an abuse of notation as follows:

$$P_i(A_i = c) = P_S(\{S_l : c_{il} = c\}) \tag{2}$$

Then, the target-based model [6; 8; 13] suggests using the following value function:

$$v(A_i) == \mathrm{P}(A_i \succeq T) = \sum_{l=1}^{k} P_S(S_l) \mathrm{P}(c_{il} \succeq T) \tag{3}$$

where $\mathrm{P}(c_{il} \succeq T)$ is a formal notation indicting the probability of meeting the target of value c_{il}, or equivalently, the utility $U(c_{il}) = \mathrm{P}(c_{il} \succeq T)$ in the utility-based language.

Thus for a target-oriented DM it is not necessary to assess a utility function; instead, it is necessary to determine the probability function of meeting the targets. In many situations, due to the lack of information or inability of DM to assess a probabilistic uncertain target, but based on his feeling or experience, he may be able to assess some fuzzy target instead. This motivated the authors in [11] to consider using fuzzy targets in the target-based decision model for DMUU.

A direct way to define $\mathrm{P}(c_{il} \succeq T)$ is to use Yager's method [15] for converting the possibility distribution into an associated probability distribution. Then $\mathrm{P}(c_{il} \succeq T)$ can be as follows according to [11]:

$$\mathrm{P}(c_{il} \succeq T) = \frac{\int_{c_{\min}}^{c_{il}} \mu_T(t)dt}{\int_{c_{\min}}^{c_{\max}} \mu_T(t)dt}, \tag{4}$$

Then the fuzzy target-based model for DMUU can be defined as follows [11]:

$$v(A_i) = \frac{\sum_{l=1}^{k} \left[\int_{c_{\min}}^{c_{il}} \mu_T(t)dt \right] P_l}{\int_{c_{\min}}^{c_{\max}} \mu_T(t)dt}, \tag{5}$$

where $\mu_T(t)$ denotes the membership function of the target T.

4 Multi-criteria Decision Making with Fuzzy Targets

In this section, we aim at solving MCDM problems from the target perspective. A general MCDM problem can be effectively described as in Table 2.

In this decision matrix, the set $A_i (i = 1,2,\ldots,m)$ represents the alternatives available to a DM, one of which must be selected; $C_j (l = 1,2,\cdots,n)$ denotes criteria. Each element a_{ij} of the matrix is the payoff the DM receives if the alternative A_i is selected with respect to criterion j, where $i = 1,2,\cdots,m; j = 1,2,\cdots n$. $w = (w_1, w_2, \cdots, w_n)^T$ denotes the vector with criteria weights (or weights thereafter), where $\sum_{j=1}^{n} w_j = 1, w_j \geq 0, \forall j$. Here a bounded domain of the variable $D_j = [a_j^{\min}, a_j^{\max}]$ is defined, where $i = 1,2,\cdots,m$, $j = 1,2,\cdots n$, $a_j^{\max} = \max\{a_{1j}, \cdots, a_{mj}\}$ and $a_j^{\min} = \min\{a_{1j}, \cdots, a_{mj}\}$.

Table 2. MCDM Problems

Alternatives	The criteria of alternatives		
	$C_1 : w_1$	$\ldots$	$C_n : w_n$
A_1	a_{11}	$\ldots$	a_{1n}
A_2	a_{21}	$\ldots$	a_{2n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	a_{m1}	$\ldots$	a_{mn}

In MCDM, we assume that each criterion is defined either as benefit criterion (*i.e., the larger the criterion value, the greater the preference*) or cost criterion (*i.e., the smaller the criterion value, the greater the preference*). Usually the criteria values need to be normalized, one common normalization method [14] is as follows:

$$b_{ij} = \begin{cases} \dfrac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}}, & \text{for a benefit criterion} \\ \dfrac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}}, & \text{for a cost criterion} \end{cases} \tag{6}$$

where $i = 1,2,\cdots,m$ and $j = 1,2,\cdots n$.

Now let's focus on the target-based model for MCDM problems. Motivated from [12], by the structural relation between DDMU and MCDM models, we can apply the target-based model mentioned in Section 3 to MCDM in a similar way. Assume that DM assesses for each criterion C_j a target T_j having membership function $\mu_{T_j} : D_j \rightarrow [0,1]$, where $T = (T_1, T_2, \cdots, T_n)$ represents the target set.

As the benefit criteria in MCDM is similar with the payoff variable in MDUU, according to [7; 11; 12], firstly, we consider a simple case, a random target T_j which has a uniform distribution on D_j with the probability density function P_{T_j} defined by

$$P_{T_j}(a) = \begin{cases} \frac{1}{a_j^{\max} - a_j^{\min}}, a \in [a_j^{\min}, a_j^{\max}] \\ 0, otherwise \end{cases} \quad (7)$$

Then under the assumption that the random target T_j is stochastically independent of any alternative A_i, for a benefit criterion we have

$$v_j(A_i) = \mathrm{P}(a_{ij} \succeq T_j) = \int_{-\infty}^{a} P_{T_j}(t)dt \quad (8a)$$

In the situation of a cost criterion, we can define the cumulative distribution function for $\mathrm{P}(a_{ij} \preceq T_j)$ as follows:

$$\begin{aligned} v_j(A_i) &= \mathrm{P}(a_{ij} \preceq T_j) \\ &= 1 - \mathrm{P}(a_{ij} \succeq T_j) \quad (8b) \\ &= \int_{a}^{+\infty} P_{T_j}(t)dt \end{aligned}$$

According to (7), (8a), and (8b), we can get the probability of A_i meeting the target T_j at the criterion C_j as follows:

$$v_j(A_i) = \begin{cases} \frac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}}, \text{for a benefit criterion} \\ \frac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}}, \text{for a cost criterion} \end{cases} \quad (9)$$

From (6) and (9), we can easily see that there is no way to tell if the DM selects an alternative by traditional method or by maximizing the probability of meeting the uncertainty target, in other words, the target-based decision model with the decision function $v_j(A_i)$ is equal to the traditional normalization function.

Now let us turn to the problem of MCDM using fuzzy targets. Based on the target model for DMUU [11], we use Yager's method [15] to define the probability of A_i meeting the target T_j at the benefit criterion C_j as follows:

$$v_j(A_i) = \mathrm{P}(a_{ij} \succeq T_j) = \frac{\int_{a_j^{\min}}^{a_{ij}} \mu_{T_j}(t)dt}{\int_{a_j^{\min}}^{a_j^{\max}} \mu_{T_j}(t)dt},$$

For benefit criteria (10a)

Similarly we can get the probability of A_i meeting the target T_j at the cost C_j as follows:

$$v_j(A_i) = \mathrm{P}(a_{ij} \preceq T_j) = \frac{\int_{a_{ij}}^{a_j^{\max}} \mu_{T_j}(t)dt}{\int_{a_j^{\min}}^{a_j^{\max}} \mu_{T_j}(t)dt},$$

For cost criteria (10b)

Then the problems here are how to define the target membership function $\mu_{T_j}(t)$ for target T_j. Bordley nd Kirkwood [13] used the crisp performance targets to valuate the multi-criteria performance analysis where he target-oriented DM has only two different utility levels, and these two utility levels can be set to one (if the target is achieved) or zero (if the target is not achieved). However, target achievements can be of different levels (i.e., from 0 to 1). Furthermore target values can be of different types, such as fuzzy numbers, fuzzy interval numbers and so on. In the following, we will discuss two special cases: fuzzy numbers and fuzzy interval numbers.

4.1 Fuzzy numbers

In the target-based model with a target T_j with a fuzzy number a_j^0, for example, when a consumer wants to buy a house with "size is 100 m^2", then we can view this as "about 100 m^2". Unlike [13], the target-oriented DM can have more than 2 different utility levels. We can define the following membership function for $\mu_{T_j}(t)$.

$$\mu_{T_j}(t) = \begin{cases} \frac{t - a_j^{\min}}{a_j^0 - a_j^{\min}}, t \in [a_j^{\min}, a_j^0] \\ \frac{a_j^{\max} - t}{a_j^{\max} - a_j^0}, t \in [a_j^0, a_j^{\max}] \end{cases} \quad (11)$$

Then according to (10a) and (7), we can get the probability of A_i meeting the fuzzy target T_j at the benefit criterion C_j as follows:

$$\mathrm{P}(a_{ij} \succeq T_j) = \begin{cases} \frac{(a_{ij} - a_j^{\min})^2}{(a_j^{\max} - a_j^{\min})(a_j^0 - a_j^{\min})}, a_{ij} \in [a_j^{\min}, a_j^0] \\ \frac{a_j^0 - a_j^{\min}}{(a_j^{\max} - a_j^{\min})} + \frac{(a_{ij} - a_j^0)(1 + \frac{a_j^{\max} - a_j^0}{a_j^{\max} - a_j^{\min}})}{(a_j^{\max} - a_j^{\min})}, a_{ij} \in [a_j^0, a_j^{\max}] \end{cases}$$

(12a)

If the criterion C_j is a cost criterion, then we can get the probability of A_i meeting the crisp target T_j at the cost criterion C_j according to (6b) and (7) as follows:

$$P(a_{ij} \preceq T_j) = \begin{cases} \dfrac{a_j^{\max} - a_j^0}{(a_j^{\max} - a_j^{\min})} + \dfrac{(a_{ij} - a_j^0)((a_{ij} + a_j^0) - 2a_j^{\min})}{(a_j^0 - a_j^{\min})(a_j^{\max} - a_j^{\min})}, a_{ij} \in [a_j^{\min}, a_j^0] \\ \dfrac{(a_j^{\max} - a_{ij})^2}{(a_j^{\max} - a_j^{\min})(a_j^{\max} - a_j^0)}, a_{ij} \in [a_j^0, a_j^{\max}] \end{cases} \tag{12b}$$

4.2 Fuzzy Interval Values

In practice, based on the DM's feeling/experience, the DM may also assess his/her target with interval numbers. For example, usually, when a consumer wants to buy a car, he/she may define the price target as a interval number, such as from p_1 to p_2.

We can define an fuzzy interval value target for criterion C_j as $T_j = [a_j^L, a_j^U]$. The interval value target can be viewed as at least a_j^L and at most a_j^U. Then we can define (13) as the membership function $\mu_{T_j}(t)$ of the target T_j:

$$\mu_{T_j}(t) = \begin{cases} \dfrac{t - a_j^{\min}}{a_j^L - a_j^{\min}}, t \in [a_j^{\min}, a_j^L] \\ 1, t \in [a_j^L, a_j^U] \\ \dfrac{a_j^{\max} - t}{a_j^{\max} - a_j^U}, t \in [a_j^U, a_j^{\max}] \end{cases} \tag{13}$$

Then according to (10a), (10b), and (13), we can get the probability of A_i meeting the target T_j at the criterion C_j for different types of criteria as follows:

$$P(a_{ij} \succeq T_j) = \begin{cases} \dfrac{(a_{ij} - a_j^{\min})^2}{(a_j^L - a_j^{\min})((a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L))}, a_{ij} \in [a_j^{\min}, a_j^L] \\ \dfrac{2a_{ij} - (a_j^{\min} + a_j^L)}{(a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L)}, a_{ij} \in [a_j^L, a_j^U] \\ \dfrac{2a_j^U - (a_j^{\min} + a_j^L)}{(a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L)} + \dfrac{[2a_j^{\max} - (a_{ij} + a_j^U)] \times (a_{ij} - a_j^U)}{[(a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L)](a_j^{\max} - a_j^U)}, a_{ij} \in [a_j^U, a_j^{\max}] \end{cases}$$

For benefit criteria (14a)

$$P(a_{ij} \preceq T_j) = \begin{cases} \dfrac{(a_j^{\max} + a_j^U) - 2a_j^L}{(a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L)} + \dfrac{[(a_{ij} + a_j^L) - 2a_j^{\min}] \times (a_j^L - a_{ij})}{[(a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L)](a_j^L - a_j^{\min})}, a_{ij} \in [a_j^{\min}, a_j^L]; \\ \dfrac{(a_j^{\max} + a_j^U) - 2a_{ij}}{(a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L)}, a_{ij} \in [a_j^L, a_j^U]; \\ \dfrac{(a_j^{\max} - a_{ij})^2}{(a_j^{\max} - a_j^U) \times ((a_j^{\max} - a_j^{\min}) + (a_j^U - a_j^L))}, a_{ij} \in [a_j^U, a_j^{\max}]. \end{cases}$$

For cost criteria (14b)

It should be noted that, there are two special cases: at least a_j^0 (at most $a_j^{\max}$) and at most a_j^0 (at least $a_j^{\min}$). Similarly we firstly define the membership function $\mu_{T_j}(t)$ of target T_j for a criterion as follows:

$$\mu_{T_j}(t) = \begin{cases} \dfrac{t - a_j^{\min}}{a_j^L - a_j^{\min}}, t \in [a_j^{\min}, a_j^0] \\ 1, t \in [a_j^0, a_j^{\max}] \end{cases}$$

, at least a_j^0 (15)

And then according to (10a), (10b) and (15), we can get the probability of A_i meeting the target T_j at the criterion C_j as follows:

$$P(a_{ij} \succeq T_j) = \begin{cases} \dfrac{(a_{ij} - a_j^{\min})^2}{(2a_j^{\max} - (a_j^0 + a_j^{\min}))(a_j^0 - a_j^{\min})}, a_{ij} \in [a_j^{\min}, a_j^0] \\ \dfrac{2a_{ij} - (a_j^0 + a_j^{\min})}{2a_j^{\max} - (a_j^0 + a_j^{\min})}, a_{ij} \in [a_j^0, a_j^{\max}] \end{cases}$$

For a benefit criterion at least a_j^0 (16a)

$$P(a_{ij} \preceq T_j) = \begin{cases} \dfrac{2(a_j^{\max} - a_j^0) + (a_j^0 - a_{ij})(1 + \dfrac{a_{ij} - a_j^{\min}}{a_j^0 - a_j^{\min}})}{2a_j^{\max} - (a_j^0 + a_j^{\min})}, a_{ij} \in [a_j^{\min}, a_j^0]; \\ \dfrac{2(a_j^{\max} - a_{ij})}{2a_j^{\max} - (a_j^0 + a_j^{\min})}, a_{ij} \in [a_j^0, a_j^{\max}] \end{cases}$$

For a cost criterion at least a_j^0 (16b)

Now let us turn to at most a_j^0. Firstly, we can define the membership function of target T_j for a criterion as follows:

$$\mu_{T_j}(t) = \begin{cases} 1, t \in [a_j^{\min}, a_j^0] \\ \dfrac{a_j^{\max} - t}{a_j^{\max} - a_j^0}, t \in [a_j^0, a_j^{\max}] \end{cases} \text{ at most } a_j^0 \quad (17)$$

And then according to (10a), (10b) and (17), we can easily get the cumulative probability function as follows:

$$P(a_{ij} \succeq T_j) = \begin{cases} \dfrac{2(a_{ij} - a_j^{\min})}{(a_j^0 + a_j^{\max}) - 2a_j^{\min}}, a_{ij} \in [a_j^{\min}, a_j^0] \\ \dfrac{2(a_j^{\max} - a_j^0)(a_j^0 - a_j^{\min}) + (2a_j^{\max} - (a_{ij} + a_j^0))(a_{ij} - a_j^0)}{((a_j^0 + a_j^{\max}) - 2a_j^{\min})(a_j^{\max} - a_j^0)}, a_{ij} \in [a_j^0, a_j^{\max}] \end{cases}$$

For a benefit criterion at most a_j^0 (18a)

$$P(a_{ij} \preceq T_j) = \begin{cases} \dfrac{2(a_{ij} - a_j^{\min})}{(a_j^0 + a_j^{\max}) - 2a_j^{\min}}, a_{ij} \in [a_j^{\min}, a_j^0] \\ \dfrac{(a_j^{\max} - a_{ij})^2}{((a_j^0 + a_j^{\max}) - 2a_j^{\min})(a_j^{\max} - a_j^0)}, a_{ij} \in [a_j^0, a_j^{\max}] \end{cases}$$

For a cost criterion at most a_j^0 (18b)

4.3 Decision Procedure

After getting the probability of meeting the target for each alternative A_i with respect to criterion C_j, we can aggregate the probabilities using different methods. Bordley and Kirkwood [13] had formulated the target-oriented multi-criteria decision making, in this paper; we use the *additive target method*. Then we can aggregate the probabilities to get the following function:

$$v(A_i) = \sum_{j=1}^{n} w_j \times v_j(A_i) \quad (19)$$

At last we maximize the probability of A_i meeting the target T by the following function:

$$V^* = \arg\max_{A_i}\{v(A_i)\} \quad (20)$$

Thus the algorithm for MCDM with targets has the following steps:

- Collecting alternatives information and weight information.
- Setting up targets for every criterion.

  For a target-oriented DM, targets can be different targets, fuzzy targets, fuzzy interval targets (both common interval and special interval).
- Calculating the probability of target achievements of every criterion.

Fuzzy targets If the target T_j for criterion C_j is of fuzzy number, then calculate the probability of A_i meeting the target T_j according to (12a) for benefit criteria, and according to (12b) for cost criteria.

Fuzzy Interval targets If the target for criterion C_j is fuzzy interval type, then calculate the probability of A_i meeting the target according to (14a) ~ (18b) depending on the criterion type and target value.

- Aggregating target probability for every alternative according to (19).
- Selecting the maximal probability the target has been achieved according to (20).

5 A Numeric Experiment

5.1 An Illustrative Example

Example: Consider the following decision matrix with alternatives and eight criteria, where C_1, C_2, and C_3 are ***cost criteria*** and $C_4 \sim C_8$ ***benefit criteria***. The example was ever examined by [14], and effectively described as in Table 3.

Now let us solve this decision problem based on the decision procedure. The weights information and criteria value for each alternative have been collected. The target assigned by DM with respect each criterion can be described as $T = (T_1, T_2, \cdots, T_n)$. With the targets, we can calculate the probability of A_i meeting the target T_j at the criterion C_j. And then by weighted aggregating method, we can get the overall value for each alternative A_i, see Table 4. It is clear that $A_3 \succ A_2 \succ A_1$, and A_3 is the best choice for $T = (T_1, T_2, \cdots, T_n)$.

5.2 A Comparative Study

Here, as a comparative analysis, we will briefly comparer our method with other two methods: traditional MCDM methods and Bordley's method by using the example showed in Table 3. To compare our proposed method with Bordley's method, here we assume that the DM assigned fuzzy value target for each criterion. The targets are $T = (T_1, T_2, \cdots, T_n)$ **= (19000, 4.5, 350, 110, 350, 0.85, 45, 1.4)**.

- **Traditional MCDM Method**

According to (6), we can get the normalized value b_{ij} for a_{ij}, and then by using aggregated method $v(A_i) = \sum_{j=1}^{n} w_j \times b_{ij}$ we get the overall value. The result and ranking order is as follows:

A_1	A_2	A_3	Ranking Order
0.357	0.532	0.622	$A_3 \succ A_2 \succ A_1$

- **Bordley's Method**

In [13], Bordley and C. Kirkwood used the crisp performance targets to valuate the multi-criteria performance analysis. We use the targets defined before. In their model, they defined the probability $P(a_{ij} \succeq T_j)$ of A_i meeting the target T_j at the criterion C_j as follows:

$$P(a_{ij} \succeq T_j) = \begin{cases} 1, T_j \geq a_{ij} \\ 0, T_j < a_{ij} \end{cases} \quad \text{for benefit criteria}$$

$$P(a_{ij} \preceq T_j) = \begin{cases} 1, T_j \leq a_{ij} \\ 0, T_j > a_{ij} \end{cases} \quad \text{for cost criteria}$$

And then according to the pre-defined targets, by using Bordely's method, the overall value for each alternative and ranking order is as follows:

A_1	A_2	A_3	Ranking Order
0.327	0.165	0.622	$A_3 \succ A_1 \succ A_2$

- **Our Proposed Method**

By using our method proposed above, the overall value for each alternative and ranking order is as follows:

A_1	A_2	A_3	Ranking Order
0.349	0.459	0.622	$A_3 \succ A_2 \succ A_1$

Based on the overall value for each alternative and ranking order of different method, it is clear that A_3 is the best choice. It is possible that with the change of target value, the ranking order of our method will change.

Table 3. MCDM: Example

Criteria: weight	Alternatives		
	A_1	A_2	A_3
C_1 : 0.2126	18400	19600	29360
C_2 : 0.0713	3	4	6
C_3 : 0.0417	100	120	540
C_4 : 0.1605	80	100	120
C_5 : 0.0524	300	400	150
C_6 : 0.1115	0.6	0.8	1.0
C_7 : 0.15	40	40	50
C_8 : 0.20	1.2	1.3	1.5

Table 4. Multi-criteria Decision Making with Targets

Criteria: weight	Alternatives			Target	Target achievements		
	A_1	A_2	A_3		A_1	A_2	A_3
C_1 : 0.2126	18400	19600	29360	19000	1.0	0.8389	0.0
C_2 : 0.0713	3	4	6	[3,5]	1.0	0.4444	0.0
C_3 : 0.0417	100	120	540	at least 300	1.0	0.9706	0.0
C_4 : 0.1605	80	100	120	[90,110]	0.0	0.6667	1.0
C_5 : 0.0524	300	400	150	at most 200	0.8333	1.0	0.0
C_6 : 0.1115	0.6	0.8	1.0	0.7	0.0	0.6667	1.0
C_7 : 0.15	40	40	50	at least 40	0.0	0.0	1.0
C_8 : 0.20	1.2	1.3	1.5	1.2	0.0	0.5556	1.0
				Overall Value	**0.3693**	**0.5954**	***0.6220***

6 Conclusion

In this paper, we have proposed a new method to solve multi-criteria decision making problems with fuzzy targets. In particular, instead of using the traditional MCDM problems, we calculate the probability for each criterion of meeting specified target, and then calculate the overall value using weighted aggregation method. Unlike [13], the target achievement can range from 0 to 1. And the target value type can be of different types, such as fuzzy numbers and fuzzy interval numbers. As illustrated by the example taken from [14], the proposed method has been compared with two other methods.

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