

Title	平面における伸縮を許したリンケージの裏返し判定問題
Author(s)	藤本, 洋一
Citation	
Issue Date	2008-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/4308
Rights	
Description	Supervisor: 上原隆平准教授, 情報科学研究科, 修士

Inverting Linkages with Stretchable Links

Youichi Fujimoto (0610076)

School of Information Science,
Japan Advanced Institute of Science and Technology

February 7, 2008

Keywords: algorithm, inversion, linear time, linkage, reconfiguration.

A *linkage* is a collection of fixed-length line segments joined at their endpoints to form a graph. While we can ignore the length of edges in general graph embedding, a linkage must be embedded in order to keep the lengths of edges. A linkage is also considered as a physical model which consists of straight rigid bars and joints permitting arbitrary rotation. Linkages have a number of practical applications such as smooth graph deformation, analysis of physical structures, motion planning in robotics, molecular modeling, and so on.

A *configuration* of a linkage is a specification of the location of all the link endpoints. Given an initial configuration and a final configuration, the *reconfiguration* problem asks whether the linkage can be continuously reconfigured from the initial configuration to the final one, keeping all links rigid. The reconfiguration problem is related to graph theory, computational geometry, computational complexity and topology because it has properties of graphs and geometry.

Linkages can be distinguished according to their graph structure, the dimension and intersection conditions. The graph structures may be a general graph, a path, a tree, or a cycle. The second parameter is the dimension of the space in which the linkage is embedded: 2, 3 or higher dimensions. The last parameter classifies how the linkage may intersect itself. It is known that reconfiguration problems in 2 and 3 dimensions are especially difficult. For example, Alt et al. showed that the reconfiguration problem of trees

without self-intersection in 2 dimension is PSPACE-complete. Furthermore, in 3 dimension, they also proved that the reconfiguration problem is PSPACE-complete even for paths without self-intersection. As results of linkages with self-intersection, Hopcroft et al. showed linkage reconfiguration problem is PSPACE-hard in 2 dimensional plane. As results of linkages without self-intersection, Connery et al. showed that paths and trees can be reconfigured onto a line and that cycles can be reconfigured to convex configurations. Canterella et al. showed that those motions can be calculated by energy driven motions and their steps are proportional to polynomials of the number of vertices and calculated in polynomial time.

Inversion of a linkage is a special reconfiguration from a given configuration to its mirror-image configuration. We say that a linkage is *invertible* if there exists the inversion of a given configuration. Of course, there exists noninvertible linkages. For example, linkages including a triangle are not invertible since any triangle cannot continuously change its interior angles without changing the length of edges. Lenhart and Whitesides proved the necessary and sufficient condition of inverting cycles. The inversion can be executed in $O(n)$ steps for invertible cycles.

In this thesis, we consider inversion of linkages with expansion and contraction of links permitting self-intersection on 2-dimensional plane. In molecular modeling, the distances between molecules are not strictly fixed. Thus, it is practical to consider linkages with expansion and contraction of links. We introduce a notion *stretch ratio* to denote how each link can be stretched. When a stretch ratio is α ($\alpha \geq 1$), each link with length l can independently stretch from l/α to αl . Note that $\alpha = 1$ means ordinary linkages. We remark that any linkage can be inverted with appropriate stretch ratio. We say that the *optimal stretch ratio* of a linkage is the minimum stretch ratio which enables inverting it. Introducing stretch ratio, we can extend decision problems to optimization problems which are more applicational problems of minimizing α .

We obtained that the optimal stretch ratios of cycles are at most $\sqrt{2}$. We also showed that the optimal stretch ratio of any cycle can be computed in linear time.

We extended these results to Outer-Planar graphs. Outer-Planar graphs

are graphs which have a plane drawing with all vertices face to the outer-plane. We proposed an algorithm which computes optimal stretch ratio of an Outer-Planar graph and showed its computation time is linear to the number of vertices. The key fact is that the dual graphs of Outer-Planar graphs excepting Outer-Plane are trees.

For general planar graph, we proved that there is no upper bound of optimal stretch ratio by constructing wheels W_n which cannot be inverted within a given stretch ratio. We also proposed an algorithm which can invert wheels W_n ($n = 7, n \geq 9$) with the optimal stretch ratio. For wheel W_4 , we showed non-trivial upper bound and lower bound of optimal stretch ratio.