A Code System and Optimization for Rectangle Packing in 2D Torus Space

Shibata Takayuki (610042)

School of Information Science,
Japan Advanced Institute of Science and Technology

February 7, 2008

Keywords: 2D Torus, VLSI layout, Sequence-pair, Module placement.

Rectangle packing problem can be defined as a problem of packing rectangles without mutually overlapping in a certain coordinate space, and relevant optimization problems have various applications such as VLSI layout design, and task assignment on a multiprocessor system. Unfortunately it is known that area minimization in rectangle packing belongs to the NP-hard class.

This research focuses on the rectangle packing in a two-dimensional (2D) torus space. 2D torus space is a space where each of two axes has a recurrence structure; T=$\{(x, y)|x \in [0, L_x), y \in [0, L_y)\}$, $(a, b) + (c, d) = ((a + c)_{\mathrm{mod}L_x}, (b + d)_{\mathrm{mod}L_y})$. Rectangle packing in a 2D torus space can be thought as it is literally meant or alternatively, it can be thought as a 2D repeated packing of a set of rectangles such that each rectangle is repeatedly placed at $(x + kL_x, y + \ell L_y)$ with x- and y-directional periods L_x and L_y which are common for all rectangles. Such a rectangle packing has applications in VLSI layout especially for FPGA layout and multiprocessor layout, and task assignment on a multiprocessor system having torus network structure.

Sequence-pair is an approach to express individual packing solution in a 2D plane. Sequence-pair has the form of the ordered pair of permutations of rectangle names. This code does not have the information on the coordinate, instead it has the information on the relative position between

rectangles. Hence, the code system is incorporated with a decoding algorithm to compute coordinates of rectangles from a Sequence-pair code. To optimize the placement (that is, to find the best code), a stochastic search method such as Simulated Annealing (SA) and Genetic Algorithm is applied to the solution space defined by the set of all Sequence-pair codes. One feature of this method is the adaptability to a variety of placement problems by selecting proper objective function. Moreover, it can find a near optimum solution by the expense of computation time. The concept of Sequence-pair has been extended to Sequence-triple and Sequence- quintuple for three dimensional cube packing, Sequence-triple and Modified Sequence-pair for one-directional repeated rectangle placement, etc. For the rectangle packing in a 2D torus space, Sequence-pair and it's variants proposed so far do not work well. Sequence-pair represents relative position between rectangles in pair-wise fashion, and the difficulty in applying Sequence-pair to 2D torus space arises from the infinity (or the largeness) of rectangle-pairs for which Sequence-pair and it 's variants attempt to specify relative position. To solve this difficulty, we have introduced another type of relative position between rectangles, and we have limited rectangle-pairs for which Sequence-pair type of relative position is assigned. To do this, first we define a rectangular base area, which is just a rectangle obtained by cutting 2D torus space by two rectangular straight lines (cut lines), and partition rectangles into interior rectangles which are completely included in the base area without being separated by cut lines and the other rectangles which are split by a cut line. It is interesting that, if the torus space is cut-opened such that the left-bottom corner of an rectangle is the cross point of two cut lines, every rectangle is split at most once by either a horizontal cut line or a vertical cut line. As a result, rectangles are partitioned into three types, interior rectangles, rectangles split by horizontal cut line, and rectangles split by a vertical cut line. Next we define an extended base packing, which consists of one set of interior rectangles, two sets of rectangles split by a vertical cut line (one set of them locates on the left boundary of the rectangular base area, and one other set locates on the right boundary), and two sets of rectangles split by a horizontal cut line (one set on the bottom boundary, and the other on the top boundary). Finally, we design a code system as a Sequence-pair for this extended base

packing, and name it Enlarged Sequence-pair (Π_+, Π_-). Being different from the original Sequence-pair, each sequence Π_+, Π_- includes each rectangle once for interior rectangles and twice for rectangles split by a cut line. The size of the solution space defined by this Enlarged Sequence-pair is $\mathcal{O}(((2n)!)^2)$.

In this research, we show that there always exists a corresponding code for an arbitrary rectangle packing in 2D torus space. We also derive a condition for a pair of sequences of rectangle names to be a valid code, and an algorithm to compute positions of rectangles from a given Enlarged Sequence-pair code. Next we implemented SA search of the solution space defined by our Enlarged Sequence-pair, and achieved experiments on minimum area rectangles packing in 2D torus space. The ability of our Enlarged Sequence-pair to express vast variety of rectangle packing has been verified through experiments. On the other hard, we found that some of move operations designed for our Enlarged Sequence-pair result in a large change of packing, which prevents SA from converging to a good packing solution. Effective move operations proper to our Enlarged Sequence-pair, and a better code system in computational complexity and the size of solution space remain as future problems.