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Description	

Performance Analysis of RS-Coded M -ary FSK for Frequency-Hopping Spread Spectrum Mobile Radios

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Abstract—Decoding performance of Reed–Solomon (RS) coded M -ary FSK with noncoherent detection in a frequency-hopping (FH) spread spectrum (SS) mobile radio channel is theoretically analyzed. Exact formulas and an approximate one for evaluating word error rates (WER's) of error correction and error-and-erasure correction schemes upon decoding the RS codes are derived. It is shown that with K symbol erasure and C symbol error correction, RS coded M -ary FSK achieves the equivalent diversity order of $(K + 1)(C + 1)$.

I. INTRODUCTION

IN land mobile communications, the received signal suffers from fast fading due to multipath propagation of the transmitted signal. Long-term burst errors are caused by deep fades in the received signal strength, thus error randomization schemes are required when applying forward error correction (FEC). Interleaving can randomize errors caused by fading and enhance the effectiveness of coding. However, the major drawback of using interleaving is that an unreasonably long delay time is usually required to perfectly randomize errors. Frequency hopping (FH) can solve this problem because the error randomization function is incorporated within frequency hopping itself if the hopping carriers are sufficiently separated in frequency so that each received symbol suffers from independent fading.

Recently, it has been recognized that spread spectrum (SS) techniques have the possibility to reduce fading effects and increase the user capacity of commercial mobile communication systems. Several papers have estimated the increase in user capacity of cellular mobile radio systems upon employing SS techniques [1]–[3]. Gilhousen *et al.* showed that, assuming perfect transmitter power control, a direct sequence code division multiple access (DS/CDMA) scheme can greatly increase user capacity compared to the capacity of existing systems [3]. However, under fading the DS/CDMA transmission may still require a delay time for interleaving [4].

Much work has been done for the performance analysis of coded FH-SS systems in various signaling environments. Pursley and Stark analyzed the bit error rate (BER) performance of Reed–Solomon (RS) coded M -ary orthogonal signaling in a partial band interference environment [5]. Stark also investigated coding for the FH-SS communications with partial band interference [6], [7], and derived the channel capacity and cutoff rate. Geraniotis and Pursley analyzed

the BER performance of binary FSK with slow frequency hopping (SFH) in fading channels [8]. However, no analysis has been presented for the performance evaluation of coded M -ary FSK in Rayleigh fading channels encountered in mobile communication systems.

This paper applies RS¹ coded M -ary FSK with slow frequency hopping to mobile radio communications. Decoding performance of RS-coded M -ary FSK is analyzed in a frequency hopping Rayleigh fading channel. Section II describes the system model under investigation. Section III describes the word error rates (WER's) of error correction and error-and-erasure correction schemes upon decoding of the RS codes. In fading, the primary objective in good code design is to maximize the equivalent diversity order of the code [17]. It is shown that with K symbol erasure and C symbol error correction, RS coded M -ary FSK achieves a diversity order equivalent to $(K + 1)(C + 1)$. Therefore, the most powerful decoding performance is achieved by maximizing the $(K + 1)(C + 1)$ value. In Section IV, numerical calculations are given to support the theoretical analysis.

II. SYSTEM MODEL

The block diagram of the system under investigation is shown in Fig. 1. The input bit stream with a bit rate of f_b b/s is segmented into q -bit symbols, and the resulting symbol sequence with a symbol rate of $f_s = f_b/q$ symbols/s is fed to the RS code encoder. The RS (N, K_b) code defined over the Galois field $GF(2^q)$ is used for error correction, where N is the code length and K_b is the information length of the code. The coded symbol, to be transmitted with a transmission symbol rate of $f_s \times N/K_b$, is assigned to one of the $M (= 2^q)$ -ary FSK signals, where orthogonality of the M frequencies is assumed.

The frequency hopping M -ary FSK signal is output from a frequency hopper where the hopping local signal is multiplied to the M -ary FSK signal. A frequency synthesizer is used to generate the hopping frequencies, which are determined by the output from the PN sequence generator. A frequency hop is assumed to be made in each symbol interval. The hopping carriers are separated in frequency so that the received dehopped M -ary FSK signal suffers from independent Rayleigh fading symbol-by-symbol [9]. For each symbol, fading is assumed to be nonfrequency selective.

The receiver has an identical PN generator, which controls the frequency synthesizer output for dehopping. The receiver

¹The major conclusion of this paper is applicable not only to RS codes but also to codes whose error and erasure correction capabilities are the same as those of the RS codes, given the minimum distance of the code.

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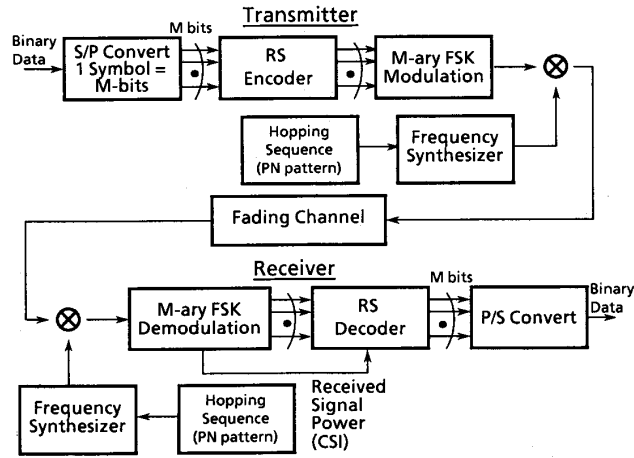


Fig. 1. System model.

input signal is multiplied by the dehopped signal to obtain the received *M*-ary FSK signal. We assume perfect synchronization of the PN generators in the transmitter and receiver. Noncoherent detection is used for the reception of the *M*-ary FSK signal.

The detector output symbol sequence and samples of the envelope detector output for each received symbol are then fed to the RS decoder. Unfortunately, constructing a soft decision decoder for an RS code is impractical except for some restricted classes of codes. Therefore, we restrict our investigation to error correction and error-and-erasure correction (decisions based on the comparison between the *a posteriori* probabilities for the code words as in Chase's algorithms [10] are not used because of decoder complexity). In error-and-erasure correction, several symbols having the lowest signal envelope samples are regarded as the erasures. The envelope samples for each received symbol are used to determine the erasures in the received word.

III. ANALYSIS

For an RS(*N*, *K*_b) code defined over the Galois field GF(2^{*q*}), there are several approximate formulas [11], [12, pp. 429–433] for evaluating the BER after decoding of the RS code from the WER. Hence, we use WER as the performance measure instead of BER.

A. Error Correction

The symbol error probability *p*_s(γ) for *M*-ary FSK with noncoherent detection is given by [12, pp. 295–298]

$$p_s(\gamma) = \sum_{i=1}^{M-1} \frac{(-1)^{i+1}}{1+i} \binom{M-1}{i} \exp\left\{-\frac{i\gamma}{1+i}\right\} \quad (1)$$

where γ is the received signal energy per symbol-to-jamming power spectrum density ratio (*E*_s/*N*_J). Here, we have assumed that the composite signal from multiple interference sources can be regarded as a complex Gaussian process with

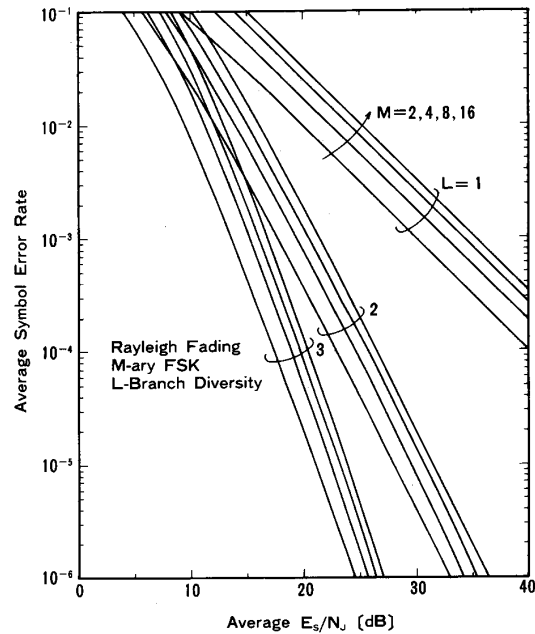


Fig. 2. Average symbol error rate.

a flat frequency spectrum [1]; its single sided power spectrum density is *N*_J.

The average symbol error probability *P*_S can be calculated by averaging *p*_s(γ) over the pdf of the received *E*_s/*N*_J γ . If *L* branch maximum-selection diversity is used for *M*-ary FSK signal reception, *P*_S is, in the Rayleigh fading channel, given by [13]

$$P_S = \sum_{i=1}^{M-1} \frac{(-1)^{i+1}}{1+i} \binom{M-1}{i} \cdot \prod_{j=1}^L \frac{j}{j + \Gamma i / (1+i)} \quad (2)$$

where Γ is the average *E*_s/*N*_J. Fig. 2 shows the average symbol error probability *P*_S as a function of Γ with the diversity order *L* as a parameter. It is found from this figure

TABLE I
VALUES OF THE CONSTANT $k(M, L)$

L	M				
	2	4	8	16	32
1	1	1.8333	2.5929	3.3182	4.0272
2	1	2.0972	3.3551	4.8069	6.4716
3	1	2.3043	4.0226	6.2551	9.0985
4	1	2.4660	4.5931	7.6064	11.761
5	1	2.5912	5.0716	8.8275	14.346
6	1	2.6879	5.4667	9.9037	16.770
7	1	2.7623	5.7889	10.833	18.984
8	1	2.8192	6.0489	11.622	20.959
9	1	2.8628	6.2568	12.283	22.690
10	1	2.8960	6.4219	12.829	24.181

that, for a large value of Γ , P_S reduces inversely proportional to Γ^L for any value of M . This implies that when $\Gamma \rightarrow \infty$, ratio of (P_S for M -ary FSK with L -branch diversity)/(P_S for binary FSK with L -branch diversity) becomes constant as

$$k(M, L) = \lim_{\Gamma \rightarrow \infty} \frac{P_S \text{ for } M\text{-ary FSK with } L \text{ branch diversity}}{P_S \text{ for binary FSK with } L \text{ branch diversity}} \\ = 2 \sum_{i=1}^{M-1} \frac{(-1)^{i+1}}{1+i} \binom{M-1}{i} \cdot \left(\frac{i+1}{2i}\right)^L. \quad (3)$$

The values of $k(M, L)$'s are listed in Table I. Since P_S for $M = 2$ (binary FSK) can be approximated by [14]

$$P_S \approx 2^{L-1} L! \Gamma^{-L} \quad (4)$$

we can rewrite P_S with any values of L and M using the constant $k(M, L)$ as

$$P_S \approx 2^{L-1} L! k(M, L) \Gamma^{-L} \quad (5)$$

for a large value of Γ .

If only error correction is applied, since it is assumed that each received symbol suffers from independent fading, WER is given by

$$\text{WER} = \sum_{i=C+1}^N \binom{N}{i} \cdot P_S^i \cdot (1 - P_S)^{N-i} \quad (6)$$

where C is the error correction capability of hard decision decoding. For a small value of P_S , WER can be approximated as

$$\text{WER} \approx \binom{N}{C+1} \cdot P_S^{C+1} \\ \approx \binom{N}{C+1} \cdot \{k(M, L)/\Gamma\}^{C+1}. \quad (7)$$

It is found from (7) that the WER reduces inversely proportional to Γ^{C+1} , and its corresponding BER reduction is the same as that with $C+1$ branch diversity. Therefore, RS coded M -ary FSK with C symbol error correction has an equivalent diversity order of $C+1$. However, the binomial coefficient (7) rapidly increases with the code length N . This feature can be a disadvantage when using long size codes. On the other

hand, shorter code lengths lead to decreased error correction capability. Therefore, the WER improvement obtained from the RS code M -ary FSK with error correction alone is not large enough.

B. Error-and-Erasure Correction

If error-and-erasure correction is applied, the equivalent diversity order can be increased further. K symbols that have the lowest signal envelope samples are regarded as the erasures. If the number of error symbols arising in the remainder symbols is, for the code whose minimum distance is an odd number $2d+1$, less than or equal to $\lfloor (2d-K)/2 \rfloor$, the received block can be decoded correctly by the error-and-erasure correction algorithm [15]. Therefore, the error correction capability C becomes

$$C \leq \lfloor (2d-K)/2 \rfloor. \quad (8)$$

For the code whose minimum distance is an even number $2d$, the capability C becomes

$$C \leq \lfloor (2d-K-1)/2 \rfloor. \quad (9)$$

Since random error is assumed, the WER for error-and-erasure correction is given by

$$\text{WER} = \sum_{i=C+1}^{N-K} \binom{N-K}{i} \cdot P_{S1}^i \cdot (1 - P_{S1})^{N-K-i}, \quad (10)$$

where P_{S1} is the average error probability of the $N-K$ remainder symbols. P_{S1} can be calculated by averaging $p_S(\gamma)$ given by (1) over the probability density function (pdf) $p_c(\gamma)$ of the received E_s/N_J for the $N-K$ remainder symbols.

The pdf $p_c(\gamma)$ is given by [16]

$$p_c(\gamma) = \frac{N}{N-K} \cdot p(\gamma) \sum_{i=1}^{N-K} \binom{N-1}{K+i-1} \cdot P(\gamma)^{K+i-1} \\ \cdot \{1 - P(\gamma)\}^{N-K-i} \quad (11)$$

with $p(\gamma) = (1/\Gamma) \exp(-\gamma/\Gamma)$ and $P(\gamma) = 1 - \exp(-\gamma/\Gamma)$ being the pdf and the cumulative distribution function of γ in Rayleigh fading, respectively. The resulting calculation for

P_{S1} is

$$P_{S1} = \frac{N}{N-K} \sum_{j=1}^{M-1} \sum_{i=1}^{N-K} \sum_{r=0}^{K+i-1} \binom{M-1}{j} \binom{N-1}{K+i-1} \cdot \binom{K+i-1}{r} \frac{(-1)^{j+r+1}}{(1+j)(N-K-i+r+1)+j\Gamma}. \quad (12)$$

We observe that the pdf $p_c(\gamma)$ can be rewritten as

$$p_c(\gamma) = \frac{N}{N-K} \sum_{i=1}^{N-K} \binom{N-1}{K+i-1} \frac{q_{K+i}(\gamma)}{K+i} \quad (13)$$

where $q_{K+i}(\gamma)$ is the pdf of the received E_b/N_J for $K+i$ branch maximum-selection diversity given by

$$q_{K+i}(\gamma) = (K+i)p(\gamma)P(\gamma)^{K+i-1}. \quad (14)$$

Thus, for a large Γ , the term in (14) which dominates the average symbol error probability P_{S1} of the remainders is the pdf $q_{K+1}(\gamma)$ for $K+1$ branch diversity.

Since the average symbol error probability P_S of the $K+1$ branch diversity can be approximated by

$$P_S \approx 2^K (K+1)! k(M, K+1) \Gamma^{-(K+1)} \quad (15)$$

for a large value of Γ , P_{S1} can be approximated by

$$P_{S1} \approx \frac{2^K N k(M, K+1) (K+1)!}{(N-K)(K+1)} \binom{N-1}{K} \Gamma^{-(K+1)}. \quad (16)$$

Substituting (16) into (10) and taking the dominant term of $i = C+1$ yields

$$\text{WER} \approx \binom{N-K}{C+1} \left\{ \frac{2^K N k(M, K+1) (K+1)!}{(N-K)(K+1)} \cdot \binom{N-1}{K} \right\}^{C+1} \Gamma^{-(K+1)(C+1)}. \quad (17)$$

Consequently, RS coded M -ary FSK with K symbol erasure and C symbol error correction is found to have an equivalent diversity order of $(K+1)(C+1)$.

Therefore, the most powerful decoding performance is achieved when the value of $(K+1)(C+1)$ is maximized. The value K is doubled when the value of C is decreased by one. Consider the RS(7,3) code defined over GF(2^3) for example whose minimum distance is five. Possible values of K and C are $(K, C) = (0, 2), (2, 1)$ and $(4, 0)$. If two-error correction $(K, C) = (0, 2)$ is applied, the equivalent diversity order becomes 3. If one-error and two-erasure correction $(K, C) = (2, 1)$ is applied, the equivalent diversity order becomes six. If four-erasure correction $(K, C) = (4, 0)$ is applied, the equivalent diversity order becomes five. Therefore, the most powerful strategy is $(K, C) = (2, 1)$. For another example, consider the RS(8,3) code defined over GF(2^3) whose minimum distance is six. Possible values of K and C are $(K, C) = (1, 2), (3, 1)$ and $(5, 0)$. The maximum equivalent diversity order of eight is achieved with $(K, C) = (3, 1)$.

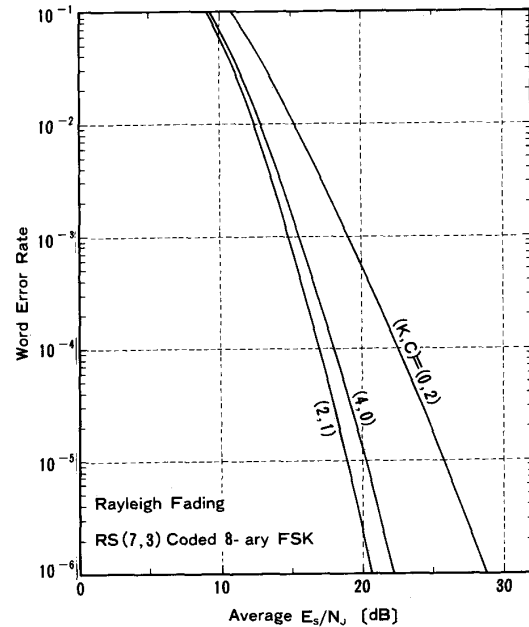


Fig. 3. Word error rate.

IV. NUMERICAL CALCULATIONS

Numerical calculation results for the WER performances of the RS(7,3) coded 8-ary FSK with $(K, C) = (0, 2), (2, 1)$ and $(4, 0)$ are presented in this section. The WER's calculated using (10) and (12) versus average E_s/N_J Γ are shown in Fig. 3. A reduction in WER that is inversely proportional to $\Gamma^{(K+1)(C+1)}$ is observed for each combination of (K, C) . Four-erasure correction requires an average E_s/N_J of 15.6 dB for a WER of 10^{-3} , which is 3.4 dB lower than that with two-error correction. Two-erasure and one-error correction requires an average E_s/N_J of 14.7 dB.

The values of WER at average $E_s/N_J = 30$ dB and 40 dB were calculated using the approximate formula of (17). The exact and approximate values are listed in Table II for $(K, C) = (0, 2), (2, 1)$ and $(4, 0)$. The approximate formula given by (17) is found to be accurate enough to estimate the WER when the average $E_s/N_J \geq 30$ dB. For each pair of (K, C) , the WER value for the average E_s/N_J of 40 dB is $10^{-(K+1)(C+1)}$ times that for the average E_s/N_J of 30 dB, and this confirms the equivalent diversity order of $(K+1)(C+1)$.

V. CONCLUSION

Decoding performance of Reed-Solomon coded M -ary FSK with slow frequency hopping has been analyzed in a Rayleigh fading channel. Exact and an approximate formulas for evaluating word error rates of error correction and error-and-erasure correction schemes upon decoding of RS codes were derived. It has been shown that with K symbol erasure and C symbol error correction, the word error rate of the RS coded M -ary FSK reduces inversely in proportion to the average E_s/N_J to the power of $(K+1)(C+1)$ for a

TABLE II
EXACT AND APPROXIMATED WER'S OF RS(7,3) CODE WITH ERROR CORRECTION AND ERROR-AND-ERASURE
CORRECTION SCHEMES AT AVERAGE $E_s/N_J = 30$ AND 40 dB

Capability		Average $E_s/N_J = 30$ dB		Average $E_s/N_J = 40$ dB	
		Exact WER	Approximate WER	Exact WER	Approximate WER
Errors	Erasures				
2	0	6.0×10^{-7}	6.1×10^{-7}	6.1×10^{-10}	6.1×10^{-10}
1	2	4.3×10^{-12}	4.6×10^{-12}	4.5×10^{-18}	4.6×10^{-18}
0	4	1.9×10^{-10}	2.0×10^{-10}	2.0×10^{-15}	2.1×10^{-15}

large value of E_s/N_J . This is equivalent to a $(K+1)(C+1)$ -order diversity. Therefore, the most powerful decoding performance is achieved when the value of $(K+1)(C+1)$ is a maximum, given the minimum distance of the code. For a small value of the average E_s/N_J , say, E_s/N_J values for $10^{-1} \sim 10^{-2}$ word error rate, codes having the maximum equivalent diversity order cannot always achieve the best performance. This condition may be encountered in zone fringe area, and other code design criteria may apply to this area. This problem, together with the optimum hopping pattern assignment problem for cellular system with frequency hopping, is left for further study.

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