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Description	

TRELLIS DECODING OF LINEAR BLOCK CODES IN DIGITAL MOBILE RADIO

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ABSTRACT

Word error rate (WER) performance with soft decision decoding of linear block codes using a trellis is investigated for noncoherent FSK in a Rayleigh fading channel. The received signal envelopes corresponding to each bit of the code are sampled and used for estimating reliability of the bits. Hamming (7, 4) code is used for WER performance evaluation via computer simulation and laboratory experiments. The simulation results show that required SNR to obtain a WER of 10^{-3} *for trellis decoding is about 5dB lower than that for minimum distance decoding with 1-bit error correction when (maximum Doppler frequency)×(bit interleaving degree) / (bit rate)* $\geqq 0.2$. *The effect of n-bit A/D-conversion in signal envelope sampling is also investigated. A simplified trellis decoding algorithm, in which the hard decision output of a bit with the envelope sample greater than the threshold value is accepted as correct, is presented for complexity reduction.*

1. INTRODUCTION

In land mobile radio, the digital signal transmission performance is severely degraded due to multipath Rayleigh fading (1). Error control coding, in which random error correcting block codes such as the BCH codes have been widely used, is an effective technique for reducing the effect of fading (2). It is well known that soft decision decoding can achieve better performance than conventionally-used minimum distance decoding (3)-(5). This is because soft decision decoding uses channel measurement information (CMI) for estimating reliability of received bits, while minimum distance decoding uses only the algebraic redundancy of the codes. However, most soft decision algorithms are complex, and are applicable only to restricted classes of codes.

It is widely recognized that the Viterbi algorithm for convolutional codes performs maximum-likelihood decoding with reduced search complexity. Wolf (5) has shown that the Viterbi algorithm can be applied to decoding of linear block codes using a trellis. Since the trellis algorithm can also perform maximum-likelihood decoding for the block codes, word error rate (WER) performance is improved over minimum distance decoding.

This paper deals with the trellis decoding of linear block codes in digital mobile radio. In Section 2, a method for applying trellis decoding in a Rayleigh fading channel is presented. The received signal envelopes corresponding to each bit of the code are sampled and used as the CMI. The metric values corresponding to each node of the trellis are derived from the CMI.

The WER performances of trellis decoding and minimum distance decoding with 1-bit error correction for Hamming (7, 4) code are investigated through computer simulations in Section 3. Simplified algorithms to reduce the complexity of full-search trellis decoding are also presented. The trade-off between the algorithm complexity and efficiency is demonstrated.

In Section 4, the algorithm is applied to 16kb/s signal transmission using GMSK modulation and frequency detection. The experimental results show that a WER performance improvement roughly equivalent to the computer simulation results can be obtained in an experimentally-simulated Rayleigh fading channel.

2. TRELLIS DECODING USING RECEIVED SIGNAL ENVELOPE

Consider an (N, K) linear block code. Let the

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parity check matrix be denoted as

$$H=(\mathbf{h}_1\ \mathbf{h}_2 \cdots\cdots \mathbf{h}_N), \qquad [1]$$

where $\mathbf{h}_i$ is the i-th column vector of H. For a code word $\mathbf{X}_j=(X_{j1}, \cdots, X_{jN})$, in which X_{ji} is an element of a Grois Field GF (q), the node $\mathbf{S}_k$ is defined as

$$\mathbf{S}_k = \sum_{i=1}^{k} X_{ji}\mathbf{h}_i, \qquad [2]$$

where $k=1 \sim N$ and $\mathbf{S}_0=(0\ 0 \cdots\cdots 0)^t$. The series $\mathbf{S}_1$, $\mathbf{S}_2, \cdots, \mathbf{S}_N$ is the locus of the code word $\mathbf{X}_j$. The set of all loci described by the series for all code words constitutes a trellis. A trellis of Hamming (7, 4) code is shown in Fig. 1.

The metric value of the k-th node for the code word $\mathbf{X}_j$ is given by

$$L_k(\mathbf{X}_j) = \sum_{i=1}^{k} \log f_i, \qquad [3]$$

where $k=1 \sim N$, and f_i is the likelihood ratio of the hard decision result of the $\mathbf{X}_j$'s i-th bit. The f_i is given by

$$f_i = \begin{cases} p_i/(1-p_i) \cdots\cdots Y_i \neq X_{ji} \\ (1-p_i)/p_i \cdots\cdots Y_i = X_{ji}, \end{cases} \qquad [4]$$

where p_i is the error probability of the i-th bit, and Y_i is the decision result of the bit. The sampled value of the received signal envelope is used as the CMI to estimate the bit error probability.

For example, the bit error probability p_i for non-coherent FSK is estimated by

$$p_i = \frac{1}{2} \exp(-\gamma_i/2), \qquad [5]$$

where $\gamma_i=R_i^2/2$ is the signal-to-noise power ratio (SNR) with R_i the envelope sample of the i-th bit and N_0 the average in-band noise power.

The Viterbi algorithm can be applied to the trellis. When the k-th node of the code word $\mathbf{X}_i$ is the same as that of $\mathbf{X}_j$, the metric values of these code words, $L_k(\mathbf{X}_i)$ and $L_k(\mathbf{X}_j)$, are compared, and the code word with smaller one is discarded. In all the nodes which have two entering branches, the code word of greater metric value is selected. In the n-th node, the most likely of the code words is selected and delivered as the output of decoding.

3. COMPUTER SIMULATION RESULTS

In this section, the computer simulation results of the WER performance using trellis decoding and minimum distance decoding are shown for non-coherent FSK. Hamming (7, 4) code, whose minimum distance is 3, is used for the computer simulations.

WER performance

A time-varying Rayleigh envelope is generated based on a model in which a mobile station moves with constant speed and many multipath waves with identical amplitude and uniformly-distributed phase come from all directions. In the simulation, a bit interleaving technique with degree M_i was employed : M_i code words are written as rows of an $N \times M_i$ bit array in a memory and the bits transmitted by reading the columns sequentially. The envelope is sampled with a normalized sampling period $f_D T$, where f_D is the maximum Doppler frequency given by *(vehicle speed) / (carrier wave length)*, and $T=M_i T_b$ with bit rate T_b^{-1}.

WER simulation results for trellis decoding and minimum distance decoding with 1-bit error correction versus average SNR are shown in Fig. 2 for $f_D T=1.0$, where the envelope variations are considered statistically independent. Required SNR to obtain a WER of 10^{-3} for trellis decoding is about 5 dB lower than that for minimum distance decoding.

When the degree of bit interleaving is not sufficiently large, the signal envelope samples in the received block are statistically correlated. As predicted from the above, envelope correlation degrades the WER performance of trellis decoding. The average SNR required for a WER of 10^{-2} versus $f_D T$ is plotted in Fig. 3. When $f_D T \geqq 0.2$, the required SNR is almost the same as that when $f_D T=1$. Thus, $M_i T_b \geqq 0.2/f_D$ is necessary to obtain the full advantage of using trellis decoding.

For implementation of the trellis decoder, A/D-conversion is necessary in sampling received signal envelopes. Therefor, the value of a sampled envelope is quantized in the A/D converter resolution width. The effect of quantization is also investigated through computer simulation. The signal envelope generated by computer is logarithmically compressed, as is done in a real receiver, and sampled. The sampled value is limitted in range of 0~24 dB so that no overflow occurs in 8-bit floating point calculation of (1/2) exp $(-\gamma/2)$, and quantized in 2^n steps corresponding to n-bit A/D-conversion. The WER's for trellis decoding of Hamming (7, 4) code versus the resolution n of the n-bit A/D-converter are plotted in Fig. 4 for an average SNR of 13 dB with $f_D T$ as a parameter. When $n \geqq 3$, no degradation in WER performance occurs for $f_D T=1$ or $f_D T=0.1$.

Complexity Reduction in Decoding

The trellis for the Golay (24, 12) code, for example, contains $2^{12}=4096$ nodes. Therefor, some complexity reduction is necessary for the practical realization of the trellis decoder. K.R.Matis et al. (6) presented a reduced-complexity algorithm of trellis decoding, in which the hard decision output of a bit was accepted as correct when the bit could be regarded reliable. In this section, a simplified algorithm of trellis decoding with reduced complexity based on the same idea as given by K.R.Matis et al., is investigated.

From Eq. [5], it is seen that any bit with a high SNR is likely to be reliable. Therefor, the hard decision result of the i-th bit with SNR of $\gamma_i > \gamma_{th}$, where $\gamma_{th}=R_{th}^2/2No$ is the threshold SNR with R_{th} the signal envelope threshold, might be accepted as correct for $1 \leqq i \leqq K$. In terms of the trellis, this means that for a certain bit with signal envelope $R > R_{th}$, we extend the trellis along only that branch corresponding to the hard decision result. However, too small value of R_{th} causes degradation of WER performance.

A quantity which affects trellis decoder complexity is the number Nc of nodes having two entering branches. The smaller Nc becomes, the more the complexity is reduced, and vice versa. Therefor, the trellis decoder with reduced complexity must have a value of Nc smaller than that of the decoder with full search algorithm.

Since the acceptance of the hard decision output of a bit as correct depends on the sampled value of the fading signal envelope, N_c becomes a random variable. When the received signal envelope samples are statistcally independent, the average value of Nc, $<Nc>$, is given by

$$<Nc> = \sum_{i=1}^{Nco} \sum_{j=1}^{k} i \cdot s_{ij} \cdot P(\gamma_{th})^j (1-P(\gamma_{th}))^{k-j}, \qquad [6]$$

where

$$P(\gamma_{th}) = \int_0^{\gamma_{th}} p(\gamma)\, d\gamma . \qquad [7]$$

Here, $p(\gamma)$ is the probability density function of γ ; Nco, the number of nodes with two entering branches when no complexity reduction is applied ; and s_{ij}, the number of cases in which the trellis has i nodes having two entering branches when any of the j envelope samples corresponding to j bits of the K information bits becomes lower than the threshold.

The WER of the simplified trellis decoder is investigated through computer simulation. The WER and the $<Nc>$ versus γ_{th} are shown in Fig. 5 for an average SNR of 13 dB and $f_D T=1.0$. WER/WERo and $<Nc>/Nco$ are plotted, where $WERo=7.2 \times 10^{-3}$ is the word error rate without complexity reduction for average SNR=13dB and $f_D T=1.0$, and $Nco=11$. The theoretical $<Nc>$ given by Eq.[6] is also shown. The simulation results for $<Nc>$ agree well with the theoretical value. The value of the $<Nc>$ is reduced to a quarter of the value for full search algorithm with no degradation in WER for $f_D T=1.0$ when γ_{th} =10.5 dB.

4. LABORATORY EXPERIMENT RESULTS

The trellis decoding algorithm was applied to 16 kb/s signal transmission using a GMSK modulation and frequency detection system. Laboratory experiments were conducted for the Hamming (7, 4) code. A 16 kb/s bit stream of the coded data was interleaved, differentially-encoded, and fed to the IF stage GMSK-modulator. The differential encoding was necessary to apply a 1/2-bit offset decision rule (7) in receiver so that no error propagation would be caused. The Gaussian-shaped low pass filter with a 3 dB bandwidth of 8 kHz was used for pre-modulation band limitation. The fading GMSK signal was generated by a Rayleigh fading simulator operating on a 90 MHz band. The maximum Doppler frequency f_D of the fading simulator was set at 120 Hz, corresponding to a typical vehicle speed of 64 km/h for the RF frequency of a 2 GHz band. The bit interleaving degree M_i was set at 64 (i. e., $f_DT=0.48$).

A limiter-discriminator type receiver and a 1/2-bit offset decision were used*. An approximately Gaussian-shaped ceramic filter with a center frequency of 455 kHz and a 3-dB bandwidth of 6 kHz was adopted for the pre-detection bandpass filter. The frequency discriminator output was lowpass-filtered by a Gaussian-shaped filter with 3-dB bandwidth of 8 kHz for post-detection noise reduction. The filter output was fed to a 1/2-bit offset decision circuit, and the data stream was regenerated.

The logarithmically compressed IF signal was envelope-detected. The envelope detector output was lowpass-filtered by a 4-pole Butterworth filter with a 3-dB bandwidth of 1 kHz. The filter output was sampled and value-limited in range of 20 dB using an n-bit A/D converter with resolution n of 0~7. A large amount of (n+1)-bit-data segments consisting of regenerated data and the envelope sample were stored for later processing. The trellis decoding algorithm was performed using the stored data.

The experimental WER's versus the A/D-converter resolution n is shown in Fig. 6 for $M_i=64$, when the channel bit error rate was 1.4×10^{-2}. The WER performance improvement plateaus when $n\geqq3$. This agrees with the computer simulation result. The experimental WER's versus the channel BER's are shown in Fig. 7 for trellis decoding and minimum distance decoding with a bit error correction for $M_i=64$ and $n=3$. The computer simulation results of WER for $f_DT=1.0$ are also plotted in the figure. A little degradation from the simulation results for $f_DT=1.0$ is observed in the experimental WER's. Further study will clarify the reason for this degradation.

5. CONCLUSION

WER performance with trellis decoding of linear block codes was investigated for noncoherent FSK in a Rayleigh fading channel. The Hamming (7, 4) code was used for the WER performance evaluation via computer simulation and laboratory experiments. In the simulation, the bit interleaving technique was employed. The simulation results show that required SNR to obtain a WER of 10-3 for trellis decoding is about 5dB lower than that for minimum distance decoding with 1-bit error correction when *(maximum Doppler frequency)*×*(bit interleaving degree)* / *(bit rate)* $\geqq0.2$. The effect of n-bit A/D-conversion in the signal envelope sampling was also investigated. When $n\geqq3$, no degradation in WER performance occured for *(maximum Doppler frequency)*×*(bit interleaving degree)* / *(bit rate)*=0.1 or 1.0. A simplified trellis decoding algorithm, in which the hard decision result of the bit with the envelope sample greater than the threshold value was accepted as correct, was presented. The trade-off between the complexity and the efficiency of the algorithm was demonstrated. The complexity could be reduced to a quarter of the value for the full search algorithm with little degradation in WER.

The trellis decoding algorithm was applied to 16 kb/s signal transmission using GMSK modulation and frequency detection system. A little degradation from the simulation results was observed in experimental WER's for *(bit interleaving degree)*=64, when *(maximum Doppler frequency)*=120 Hz. Further study will clarify the reason for this degradation, and will resolve the problem of random phase variation in the received carrier due to fading. Such study will confirm effectiveness of trellis decoding in real digital mobile radio.

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* Limiter-discriminator detection is widely used for FSK signal reception. The BER for FSK with limiter-discriminator can be approximated by $(1/2)\exp(-a\gamma)$, in which parameter a is experimentally obtained. Hence, this algorithm can also be applied.

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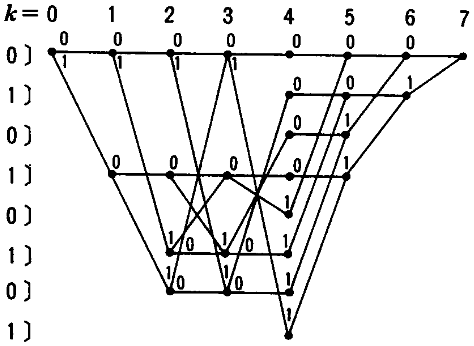

Fig. 1 Trellis forHamming (7, 4) Code

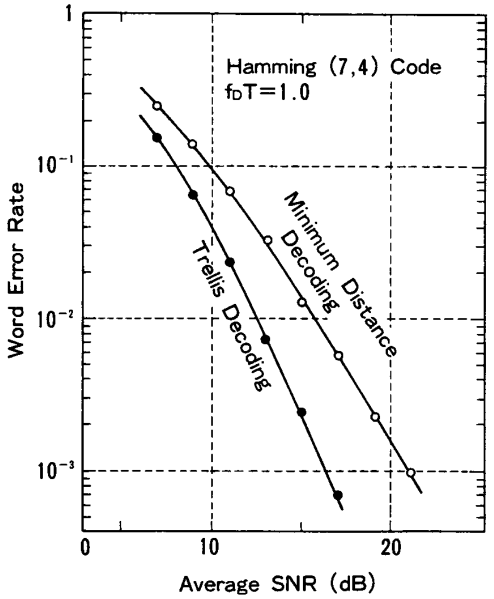

Fig. 2 Simulation Results of WER

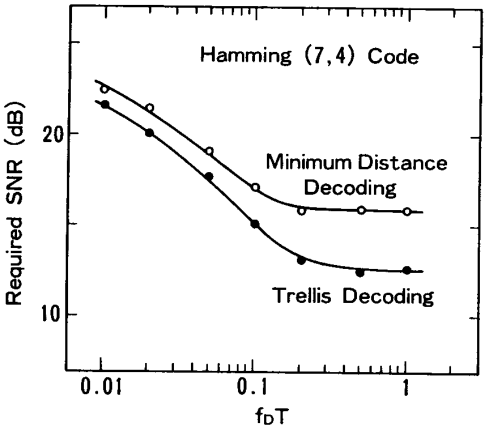

Fig. 3 Required SNR for Obtaining WER$=10^{-2}$ vs. Normalized Doppler Frequency f_DT

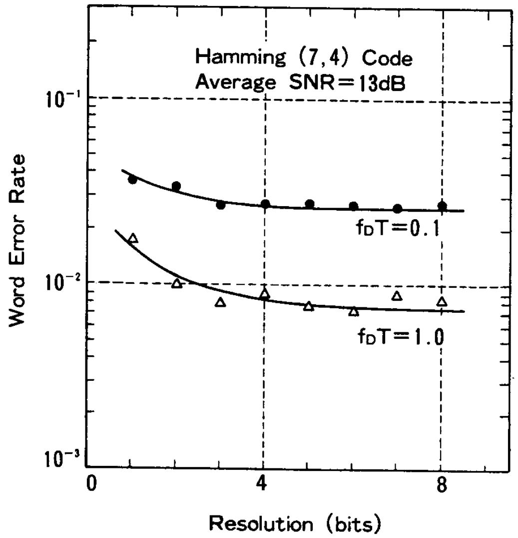

Fig. 4 Simulated WER vs. Resolution of n-bit A/D-Conversion

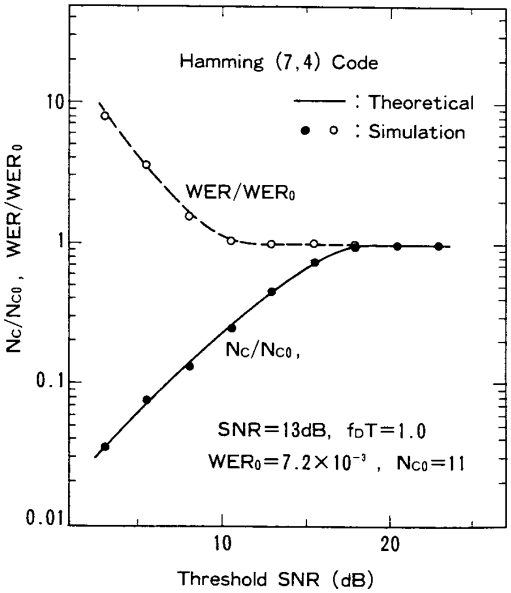

Fig. 5 WER and $<Nc>$ vs. Threshold SNR

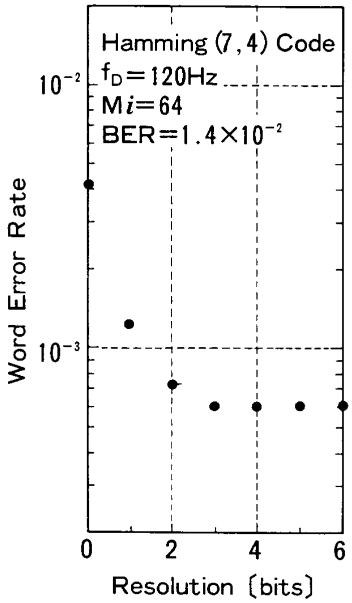

Fig. 6 Experimental WER vs. Resolution of n-bit A/D-Conversion

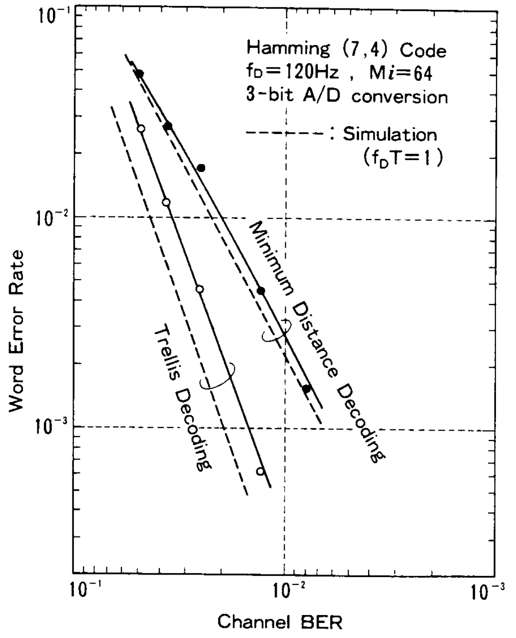

Fig. 7. WER vs. Channel Bit Error Rate