Soft Decision Decoding Of Block Codes Using Received Signal Envelopes In Fading Channels

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Abstract

The word error rate (WER) performance of non-coherent FSK with soft decision decoding of block codes using Chase's second algorithm is investigated in a Rayleigh fading channel. The received signal envelope is sampled and used as channel measurement information. The theoretical upper and lower bounds of the WER are derived assuming independent Rayleigh envelope samples in a received block. The effects of bit interleaving on the WER performance when fading envelope variation is slow compared with the bit rate are investigated through computer simulations. The theoretical analysis was supported by laboratory experiments.

1. Introduction

Multipath fading [1] severely degrades the digital signal transmission performance in mobile radios. Forward error correction (FEC) [2] is an effective technique for combatting fading. Random error correcting block code, such as the BCH codes, with minimum distance decoding has been widely utilized because of its simplicity in implementation. However, performance improvement by FEC with minimum distance decoding is unsatisfactory for high quality digital mobile radio. The signal transmission performance is further improved by soft decision decoding using channel measurement information (CMI), which has recently attracted much attention because its performance is better than that of minimum distance decoding [3]-[5]. However, most soft decision algorithms are complex, and are applicable only to restricted classes of codes.

One approach to reducing decoding complexity is to generate a small set of candidate code words and to select the most likely code word as the output of the decoder. Chase [7] proposed three simplified algorithms based on this idea and investigated the word error rate (WER) performance in a Rayleigh fading environment through computer simulations. It has been shown that the second algorithm results in WER performance only slightly inferior to that of maximum likelihood decoding, while decoding complexity is greatly reduced. Experimental evaluations of bit error rate (BER) performance of a coherent GMSK system with soft decision decoding using the second algorithm were presented by Stjernvall et al. [6] and Ekemark et al. [7]. Great improvement in BER performance was obtained. However, there has been no theoretical analysis of improvement with soft decision decoding using the second algorithm.

This paper theoretically investigates the WER performance of a noncoherent FSK system with soft decision decoding using Chase's second algorithm in a Rayleigh fading environment. The received signal envelope is sampled and used as CMI to select unreliable bits and weaken the contribution of these bits to decoding. The decoding algorithm is described in Section 2. In Section 3, assuming statistically independent signal envelope samples, the upper and lower bound estimates of the WER is theoretically analyzed. In Section 4, the effects of bit interleaving on the WER performance when fading envelope variation is slow compared with the bit rate are investigated through computer simulations. Laboratory experimental results are presented in Section 5.

2. Algorithm

In Rayleigh fading, the signal-to-noise power ratio (SNR) varies over a wide range and the bit error probability of each bit in the received block is widely distributed. The bits having a low SNR are unreliable. The WER can be improved by weakening the contribution of these unreliable bits to decoding. This can be achieved by erasure correction. Erasure bits are selected by finding K bits, from the received block, having the lowest envelope samples. The resulting word with N-K remainder bits is decoded into a code word after minimum distance decoding with C-bit error correction is carried out 2^K times, where N is the word length.

Chase's second algorithm, using received signal envelope samples, consists of the following three steps:

Step 1 : Find the K bits having the lowest signal envelope samples from the received word $Y = (y_1, \cdots, y_N)$.

Step 2 : These K bits are assumed to have been erased. Applying 2^K patterns to the erasure bits, C-bit error correction is performed for each of the 2^K patterns by minimum distance decoding, where $0 \leqq C \leqq d$, and $2d+1$ is the minimum distance of the code. Then, the set Ω of the candidate code words is obtained.

Step 3 : Using the bit error probability, with the SNR's derived from the signal envelope samlpes, calculate the *a posteriori* probability that Y is received when each candidate code word is assumed to have been transmitted. Then, the code word with the maximum *a posteriori* probability is selected as the output of the decoder.

The geometric sketch for decoding with $2d+1=7$ is shown in Fig. 1. When $C \leqq \lfloor (2d-K)/2 \rfloor$, the set Ω contains a single code word (see Fig. 1 (a)), where $\lfloor X \rfloor$ denotes the greatest integer less than or equal to X. Thus, Step 3 is not needed. When $\lfloor (2d-K)/2 \rfloor < C \leqq (2d-K)$, Ω contains several code words, and Step 3 is required to select the most likely among the candidates (see Fig. 1 (b)).

The above algorithm is applied to a non-coherent FSK system. The bit error probability is given by

$$p_E(\gamma) = \frac{1}{2} exp(-\gamma/2) \qquad \cdots(1)$$

where $\gamma = R^2/2N_0$ is the SNR, with R and N_0 being the received signal envelope and average noise power, respectively. Therefore, Step 3 is almost equivalent to finding a code word which minimizes the sum of SNR's associated with the bit positions where the received word and candidate code word have different symbols. Thus, the algorithm can be summarized as follows :

Find the code word X_j
such that

$$\sum_{i=1}^{N} \gamma_i (y_i \oplus x_{ji}) \rightarrow Min \quad \text{subject to } X_j \in \Omega, \qquad \cdots(2)$$

where γ_i is the SNR associated with the i-th bit, y_i, of the received word, $X_j = (x_{j1}, \cdots, x_{jN})$ is the j-th code word of Ω, and $\oplus$ denotes the modulo two sum.

3. Bound Estimation

A. Expression for lower bound

The WER P_w is represented as

$$P_w = P_w' + P_w'', \qquad \cdots(3)$$

where P_w' is the probability that $C+1$ or more errors occur in the remainder bits, and P_w'' is the probability that the number of errors in the remainder bits is C or less, but the code word selected from the candidates is incorrect. When $C \leqq \lfloor (2d-K)/2 \rfloor$, obviously $P_w''=0$ and the WER is given exactly by P_w'. Otherwise, $P_w''>0$. Therefore, the lower bound of the WER is represented by P_w' and is given by

$$lower\ bound\ of\ WER = P_w'$$

$$= 1 - \sum_{i=0}^{C} {}_{N-K}C_i \cdot P_{b1}{}^i \cdot (1-P_{b1})^{N-K-i}, \qquad \cdots(4)$$

where P_{b1} is the average bit error probability of the N-K remainder bits and ${}_AC_B$ is the binominal coefficient. Equation (4) gives the exact WER when $C \leqq \lfloor (2d-K)/2 \rfloor$. The P_{b1} value can be calculated by averaging $p_E(\gamma)$ with the probability density function (pdf) $p_c(\gamma)$ of the SNR associated with the N-K remainder bits. The $p_c(\gamma)$ is given by

$$p_c(\gamma) = \frac{N}{N-K} \cdot p(\gamma)$$

$$\cdot \sum_{i=1}^{N-K} {}_{N-1}C_{K+i-1} \cdot P(\gamma)^{K+i-1} \cdot \{1-P(\gamma)\}^{N-K-i}, \cdots(5)$$

where $p(\gamma)$ is the pdf and $P(\gamma)$ is the cumulative distribution function of γ for any N bits in the received block. In the Rayleigh fading environment, $p(\gamma) = 1/\Gamma \cdot exp(-\gamma/\Gamma)$ and $P(\gamma) = 1 - exp(-\gamma/\Gamma)$, where Γ is the average SNR. Figure 2 shows $p_c(\gamma)$ for $N=23$ with K as a parameter. It is shown that as the number K of the erasure bits increases, the SNR's of the N-K remainder bits increase. It can be anticipated from Fig.2 that the bit error probability P_{b1} of N-K remainder bits is reduced as K increases. The P_{b1} value is given by

$$P_{b1} = \frac{N}{N-K} \sum_{i=1}^{N-K} \cdot \sum_{r=0}^{K+i-1} (-1)^r \cdot \frac{{}_{N-1}C_{K+i-1} \cdot {}_{K+i-1}C_r}{2(N-K-i+r+1)+\Gamma}. \qquad \cdots(6)$$

The P_w's are calculated using the P_{b1} values and are plotted in Fig. 3, versus K, with C as a parameter for $\Gamma = 13$ dB. The greater K becomes, the smaller the P_w' obtained.

B. Expression for upper bound

Since P_w'' depends on the algebraic structure of the code, an exact derivation of P_w'' is difficult. However, it is possible to derive the upper bound of P_w''. The upper bound of the WER is then expressed as P_w' plus the upper bound of P_w''. To derive the upper bound of P_w'', it is sufficient to consider only the neighboring code words. The received word is decoded either into the transmitted code word (correct decoding) or into neighboring code words (incorrect decoding). In the soft decision decoding described in Section 2, the decoder selects a code word, as output, which minimizes the sum of SNR's associated with the bit positions where the received word and candidate code word have different symbols. To derive the expression of the upper bound of P_w'', we assume that m errors in the K erasure bits and n errors in the N-K remainder bits are produced in the received word. The Hamming distance between the received word and the transmitted code word is $m+n$. Thus, the Hamming distance e between the received word and the neighboring code words satisfies the following triangle inequality [8, p.16]:

$$e+(m+n) \geqq 2d+1. \qquad \cdots(7)$$

We assume that the e bits consist of s bits in erasures and t bits in the remainders. Let the sum of the SNR's associated with the $m+n$ error bits be denoted by γ_e and that associated with the e bits by γ_c. The transmitted code word is correctly selected if $\gamma_e<\gamma_c$. It is obvious that the greater the probability of γ_c being large, the smaller the probability of erroneous decoding. Therefore, the upper bound of the probability that the received word containing m errors in the erasure bits and n errors in the remainder bits will be erroneously decoded is equal to the probability of $\gamma_e \geqq \gamma_c$ when e, s and t are determined so that γ_c is smallest. This condition is satisfied for a candidate code word with $e=2d+1$-m-n and $s=K$-m. Accordingly, the e bits exist in the N-$(m+n)$ bits which are assumed to be correctly received for the transmitted code word.

Thus, the upper bound of P_w'' is obtained by

$$\text{upper bound of } P_w'' = \sum_{m=0}^{K} \sum_{n=2d+1-K-C}^{C} {}_K C_m \cdot {}_{N-K}C_n \cdot P_w^{(mn)}, \qquad \cdots(8)$$

where $P_w^{(mn)}$ is the probability that $\gamma_e \geqq \gamma_c$ when m errors are produced in the erasures and n errors in the remainders. This upper bound is obviously independent of the algebraic structure of the code. Thus, the upper bound of the WER is given by

$$\text{upper bound of WER} = 1-\sum_{i=0}^{C} {}_{N-K}C_i \cdot P_{b1}{}^{i} \cdot (1-P_{b1})^{N-K-i} + \sum_{m=0}^{K} \sum_{n=2d+1-K-C}^{C} {}_K C_m \cdot {}_{N-K}C_n \cdot P_w^{(mn)}. \qquad \cdots(9)$$

The equation for $P_w^{(mn)}$ is $P_w^{(mn)}=P_0^{(mn)} \cdot Prob(\gamma_e \geqq \gamma_c)$, where $P_0^{(mn)}$ is the probability that m errors are produced in the erasures and n errors in the remainders. Letting the average bit error probability of the erasure bits be denoted by P_{b2}, the $P_0^{(mn)}$ is obtained from

$$P_0^{(mn)} = {}_K C_m \cdot {}_{N-K}C_n \cdot P_{b2}^{m} \cdot (1-P_{b2})^{K-m} \cdot P_{b1}^{n} \cdot (1-P_{b1})^{N-K-n}. \qquad \cdots(10)$$

The value of P_{b2} can be calculated by averaging $p_E(\gamma)$ over the pdf of the SNR associated with the K erasure bits. The pdf for the K erasures can be derived in a way similar to that in which the pdf of the SNR for the N-K remainders is derived. P_{b2} is given by

$$P_{b2} = \frac{N}{K} \sum_{i=1}^{K} \cdot \sum_{r=0}^{K-i} (-1)^r \cdot \frac{{}_{N-1}C_{N-K+i-1} \cdot {}_{K-i}C_r}{2(N-K+i+r)+\Gamma}. \qquad \cdots(11)$$

Finally, $\mathrm{Prob}(\gamma_e \geqq \gamma_c)$ can be calculated by numerical double integration with respect to the pdfs of γ_e and γ_c. The pdfs of γ_e and γ_c are derived using the characteristic function approach.

C. Discussion

The Golay code ($N=23$ and $2d+1=7$) is considered. The calculated upper and lower bounds of the WER are shown in Fig. 4. The parameters of K and C are set at $K=3$, $C=3$ and $K=3$, $C=2$. The WER's with minimum distance decoding with 2- and 3-bit error correction ($K=0$, $C=2$ and $K=0$, $C=3$) are also shown for comparison. Bound estimation is very tight: when $K=3$ and $C=2$, the lower bound is almost equal to the upper bound; when $K=3$ and $C=3$, the difference between the upper and lower bounds is only about 1.5dB. Soft decision decoding with $K=3$ and $C=2$ requires an average SNR of 14 dB for a WER of 10^{-3}, which is about 4dB lower than that for minimum distance decoding with 3-bit error correction ($K=0$, $C=3$). When $K=3$ and $C=3$, it requires an average SNR of 13 dB. If errors less than or equal to 6 bits can be corrected for Golay code, a WER of 10^{-3} is achieved at an average SNR of about 11 dB, which is the lower bound given by Chase [4]. The WER estimation for $K=3$ and $C=3$ more closely approximates actual WER performance than that given by Chase's lower bound.

4. Effect of Bit Interleaving

In Section 3, the envelope variations were assumed to be statistically independent. However, the envelope variation is in many cases so slow that the signal envelope samples in the received block are statistically correlated. It is anticipated that envelope correlation degrades the WER performance of the soft decision decoding. The bit interleaving technique can be introduced to randomize the envelope variations. This section investigates the effect of the bit interleaving through computer simulation for Golay (23, 12, 7) code. In the simulation, the bit interleaving technique with degree M_i was employed: write M_i code words as rows of an $N \times M_i$ bit array in a memory and transmit the bits by reading the columns sequentially.

A time-varying Rayleigh envelope is generated and sampled with a normalized sampling period $f_D T$, where f_D is the maximum Doppler frequency given by (vehicle speed/carrier wave length) and $T=M_i T_b$, with bit rate T_b^{-1}.

Simulation results for the WER are plotted in Fig. 4 for $f_D T=1$, where the envelope variations are considered statistically independent. These results agree well with the theoretical ones. The values of average SNR required for a WER of 10^{-2} are plotted in Fig. 5 for both soft decision decoding and minimum distance decoding with K and C as parameters. The WER performance degrades as $f_D T$ decreases, e.g., the envelope correlation between any two bits becomes large. When $f_D T \geqq 0.2$, the required SNR is almost the same as that when $f_D T=1$. For example, $M_i T_b \geqq 5\times10^{-3}$ is necessary to obtain the full advantage of using soft decision decoding when $f_D=40$ Hz.

5. Laboratory Experimental Results

Laboratory experiments were conducted for the Golay (23, 12, 7) code. A 2.4kb/s bit stream was interleaved, Manchester-coded, and fed to the 900MHz FSK-modulator with a frequency deviation of 2.0kHz. The fading FSK signal was generated by a Rayleigh fading simulator. The maximum Doppler frequency f_D of the fading simulator was set at 40Hz, corresponding to a typical vehicle speed of 48 km/h for the 900 MHz band. From the computer simulation results in Section 4, bit interleaving degree M_i was set at 64 (i.e., $f_D T=1.06$) so that the envelope variations could be regarded as statistically independent.

A limiter-discriminator type receiver was used*. An approximately Gaussian-shaped ceramic filter with a center frequency of 455 kHz and a 3 dB bandwidth of 6 kHz was adopted for the pre-detection bandpass filter. The frequency discriminator output was lowpass-filtered by a 4-pole Butterworth filter with 3 dB bandwidth of 2.4 kHz for post-detection noise reduction. The filter output was fed to a decision circuit, and data stream was regenerated.

The logarithmically compressed IF signal was envelope-detected. The envelope-detector output was lowpass-filtered by a 2-pole Butterworth filter with a 3 dB bandwidth of 2 kHz. The filter output was sampled using an 8-bit A/D converter. The regenerated data stream and the envelope samples were delivered to an 8-bit micro-computer, which carried out the soft decision decoding. The envelope samples were anti-logarithmically expanded using a look-up-table on a ROM. The experimental and theoretical WER's versus the channel BER's are shown in Fig. 6 for soft decision decoding ($K=3$, $C=3$) and minimum distance decoding ($K=0$, $C=3$). The experimental results agree well with the theoretical ones.

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* Limiter-discriminator detection is widely used for reception of an FSK signal. The BER for FSK with limiter-discriminator can be approximated by $(1/2)exp(-a\gamma)$, in which parameter a is experimentally obtained. Hence, this algorithm can also be applied.

6. Conclusion

WER performance with Chase's second algorithm using received signal envelope was theoretically investigated for noncoherent FSK in a Rayleigh fading channel. The theoretical upper and lower bounds of the WER were derived assuming independent signal envelope variations in a received block. When Golay (23, 12, 7) code is used, soft decision decoding with 3-bit erasure and 3-bit error correction required an average SNR about 5dB lower than that for minimum distance decoding with 3-bit error correction for a WER of 10^{-3}. The effects of bit interleaving on the WER performance when fading envelope variation is slow compared with the bit rate were investigated through computer simulations. When $23\times M_i$ bit interleaving is used for transmitting M_i Golay code words, the simulation results show that $M_i \approx 0.2\times$ (bit rate/fading maximum Doppler frequency) is sufficient. The theoretical analysis was supported by laboratory experiments.

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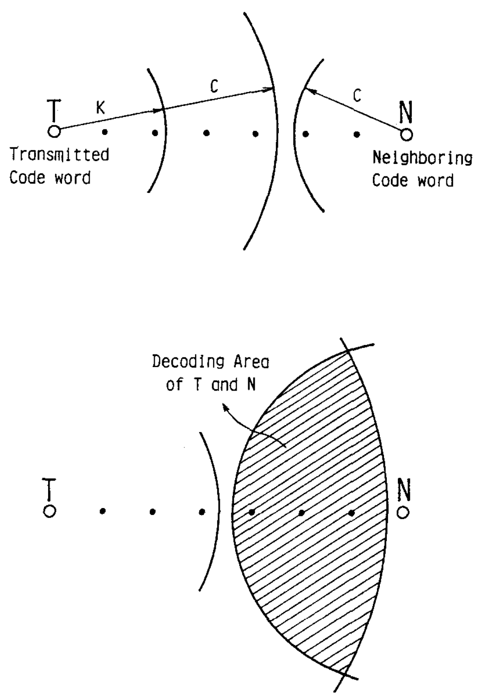

Fig. 1 Geometric Sketch for Decoding.

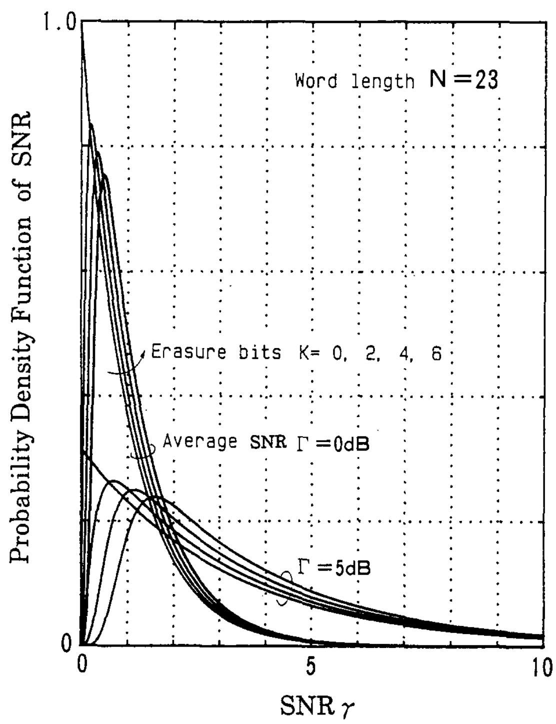

Fig. 2 Probability Density Functions of SNR for Remainder Bits.

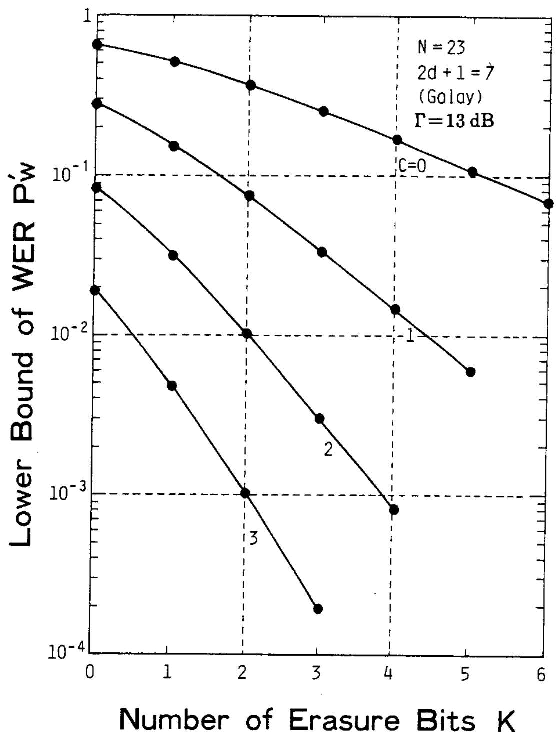

Fig. 3 Lower Bound of Word Error Rate for SNR=13 dB.

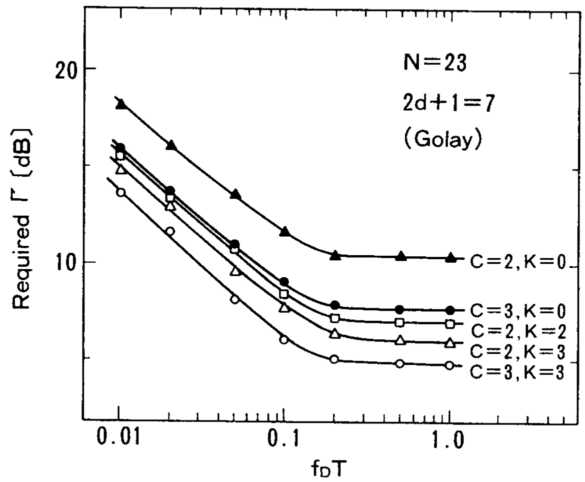

Fig. 5 Required SNR for Obtaining WER=10^{-2} vs. Normalized Doppler Frequency $f_D T$.

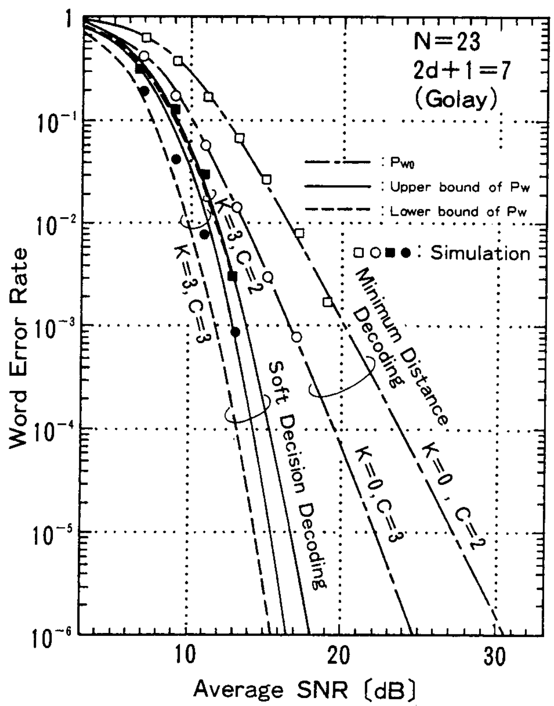

Fig. 4 Word Error Rate.

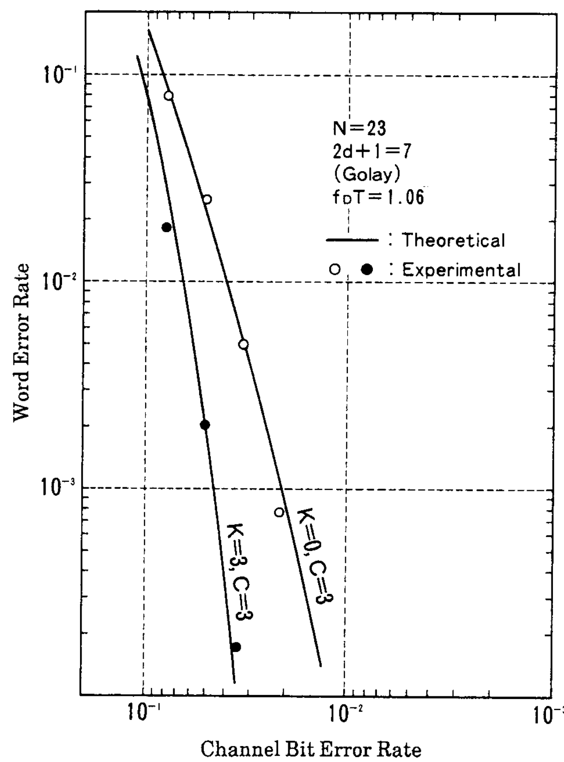

Fig. 6. Word Error Rate vs. Channel Error Rate.