Antenna-by-Antenna and Joint-over-Antenna MIMO Signal Detection Techniques for Turbo-Coded SC/MMSE Frequency Domain Equalization

Juha Karjalainen, Kimmo Kansanen, Nenad Veselinović, and Tad Matsumoto
Centre for Wireless Communications
University of Oulu, Finland
phone:+358 8 553 7637, fax: +358 8 553 2845
e-mail: {juha.karjalainen, kimmo.kansanen, nenad.veselinovic, tadashi.matsumoto}@ee.oulu.fi

***Abstract*— This paper investigates iterative frequency domain techniques for the reception of spatially multiplexed single carrier signals transmitted over frequency-selective multiple input multiple output (MIMO) channels. The investigated equalizers are based on the soft-cancellation (SC) and minimum mean square error (MMSE) filtering technique for turbo-coded single carrier point-to-point MIMO systems. We consider two different transmit antenna separation techniques in the frequency domain: (1) antenna-by-antenna (AA) and (2) joint over antenna techniques (JA). (1) aims to separate signals in different layers by MMSE filtering antenna-by-antenna, whereas (2) aims to detect the composite signal comprised of the signals transmitted from the multiple antennas. In (2) the composite received signal is decomposed by using the spatial maximum aposteriori probability (MAP) algorithm. We evaluate performances in terms of frame error rate (FER) and troughput in point-to-point MIMO frequency-selective fading channels. Impacts of spatial correlation on performance of the two detectors are also investigated in this paper.**

***Index Terms*— Frequency domain equalization,Multiple input multiple output (MIMO), Single carrier, Spatial correlation**

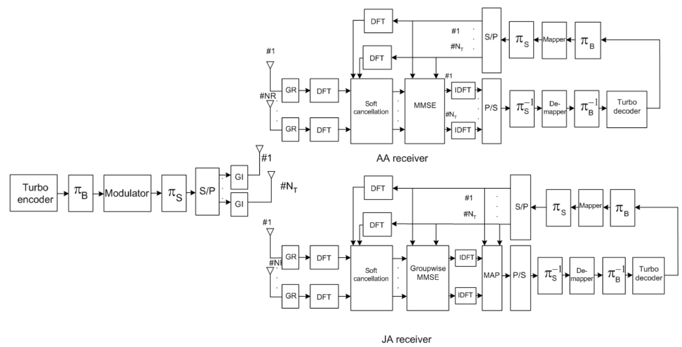

Fig. 1. System model and structures of AA receiver and JA receiver.

I. Introduction

In recent years, multiple input multiple output (MIMO) systems have attracted much attention due to the large capacity gains over single input single output (SISO) channel capacity. However, it is well known that MIMO channel capacity largely depends on spatial correlation properties of channels. Therefore, it is important to consider the effects of spatial correlation when designing new MIMO signal detection algorithms for future broadband communication systems. The spatial correlation properties are determined by antenna spacing, antenna arrangement, and angular spreads at transmitter and receiver sides.

Broadband single carrier point-to-point MIMO communication requires receiver to be able to reduce distortions caused by intersymbol interference (ISI) and co-antenna interference (CAI). One of the most promising techniques that can achieve excellent performance is soft-cancellation (SC) and minimum mean squeare error (MMSE) filtering based iterative receiver (SC/MMSE)[1]. In the last couple of years, it has been intensively researched, of which aim is the reduction of its computational complexity. The original version of the algorithm requires a cubic order of complexity due to the matrix inversion for symbol-by-symbol MMSE. Recursive matrix inversion updating is proposed in [1] and its time-averaging version by [2].

Recently, several frequency-domain processing techniques for SC/MMSE have been proposed, which are known to be able to to significantly reduce the complexity [3], [4], [5]. The frequency domain techniques proposed by [3] and [5] aim to detect signals transmitted from the multiple antennas on an antenna-by-antenna (AA) basis. Therefore, it is expected that their perfomances are degraded when the channel is spatially correlated. Recently, [6] proposed a joint-over antenna (JA) signal detection technique based on SC/MMSE for space-time trellis coded multiuser MIMO systems. However, its computation is in the time domaim, for which the cubic order complexity of due to the matrix inversion for MMSE is still required. In this paper, we investigate spatially multiplexed (SM) point-to-point-MIMO transmission with the frequency domain AA and JA detection techniques. Impact of the spatial correlation on performances of the both schemes are evaluated and compared through simulations in frequency selective MIMO channels with spatial correlation.

II. System Model

Figure 1 shows a system model of the considered SM MIMO system with N_T transmit and N_R receive antennas for cylic-prefix single carrier burst transmission. Since the cyclic-prefix burst transmission technique is very well known [7],[8], details are not described in this paper. Figure 1 depicts structures of two investigated receivers (AA and JA). After guard period removal, a space-time repsentation of the signal, $\tilde{\mathbf{r}}$, $\tilde{\mathbf{r}} \in \mathbb{C}^{N_R K \times 1}$, received by N_R received antennas is given by,

$$\tilde{\mathbf{r}} = \tilde{\mathbf{H}}_{ISI}\mathbf{b} + \mathbf{v}, \tag{1}$$

where $\mathbf{v}$, $\mathbf{v} \in \mathbb{C}^{N_R M \times 1}$, is white additive i.i.d Gaussian noise vector with variance σ^2, $\mathbf{b}$, $\mathbf{b} \in \mathbb{C}^{N_T K \times 1}$, is the transmitted layered signal vector for the frame considered, channel matrix, $\tilde{\mathbf{H}}_{ISI}$, $\tilde{\mathbf{H}}_{ISI} \in \mathbb{C}^{N_R K \times N_T K}$, is a block matrix that contains circulant submatrices $\tilde{\mathbf{H}}_{ISI}^{i,j}$, $\tilde{\mathbf{H}}_{ISI}^{i,j} \in \mathbb{C}^{K \times K}$, $i = 1, ..., N_R$, $j = 1, ..., N_T$, and K is

This work was financially supported by Nokia, Elektrobit, Instrumentointi, Finnish Air Forces and National Technology Agency of Finland (Tekes)

0-7803-8887-9/05/$20.00 (c)2005 IEEE

the size of the discrete fourier transform (DFT). The circulant block matrix denoted as

$$\tilde{\mathbf{H}}_{ISI} = \begin{bmatrix} \tilde{\mathbf{H}}_{ISI}^{1,1} & \cdots & \tilde{\mathbf{H}}_{ISI}^{1,N_T} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{H}}_{ISI}^{N_R,1} & \cdots & \tilde{\mathbf{H}}_{ISI}^{N_R,N_T} \end{bmatrix}, \quad (2)$$

is given by

$$\tilde{\mathbf{H}}_{ISI} = \mathbf{F}_{N_R}^{-1}\mathbf{\Gamma}\mathbf{F}_{N_T}, \quad (3)$$

where $\mathbf{\Gamma}$, $\mathbf{\Gamma} \in \mathbb{C}^{N_RK\times N_TK}$, is its corresponding diagonal block matrix. $\mathbf{\Gamma}$ has diagonal submatrices whose elements are the eigenvalues of the circulant submatrix given by (2). $\mathbf{F}_{N_R}^{-1} = \frac{1}{K}\mathbf{F}_{N_R}^{\dagger}$, $\mathbf{F}_{N_R}^{-1} \in \mathbb{C}^{N_RK\times N_RK}$, is the unitary block Inverse Discrete Fourier Transform (IDFT) matrix that has submatrices $\mathbf{F}$, $\mathbf{F} \in \mathbb{C}^{K\times K}$, [9] with the elements $f_{m,l} = \exp^{\frac{j2\pi ml}{K}}$, $m, l = 0, ..., K-1$. † indicates Hermitian transpose, $\mathbf{F}_{N_R}$, $\mathbf{F}_{N_R} \in \mathbb{C}^{N_RK\times N_RK}$, is a block-diagonal DFT matrix given by

$$\mathbf{F}_{N_R} = \mathbf{I}_{N_R} \otimes \mathbf{F} \quad (4)$$

for N_R receive antennas, and $\mathbf{F}_{N_T}$, $\mathbf{F}_{N_T} \in \mathbb{C}^{N_TK\times N_TK}$, given by

$$\mathbf{F}_{N_T} = \mathbf{I}_{N_T} \otimes \mathbf{F} \quad (5)$$

for N_T transmit antennas. $\mathbf{I}_{N_R}$ and $\mathbf{I}_{N_T}$, are the identity matrices with $\mathbf{I}_{N_T} \in \mathbb{R}^{N_R\times N_R}$ and $\mathbf{I}_{N_T} \in \mathbb{R}^{N_T\times N_T}$, respectively.

III. ITERATIVE FREQUENCY DOMAIN EQUALIZATION

The both AA and JA frequency domain receivers depicted in Fig. 1 exchange iteratively extrinsic information between two soft input soft output (SfISfO) stages separated by (inverse) discrete fourier transforms (IDFT)/DFT and symbol/bit de/interlerleavers. The extrinsic information exchange follows the turbo principle. The first SfISfO stage is the equalization stage that aims to mitigate ISI and CAI in the frequency domain by using received signal and a priori information given by each coded symbol. The second SfISfO stage is the channel decoder that generates soft decisions of the decoded bits in the time domain based on the a priori information for the coded bits and the trellis structure of the consituent codes.

A. Antenna-by-Antenna Detection

Signal processing for the mitigation of ISI and CAI is performed by soft cancellation in the frequency domain. After the soft cancellation, the objective of frequency domain MMSE filtering is to suppress the residual ISI and CAI. Moroeover, it also aims to decompose transmitted layers each other. The detailed derivation of this frequency domain technique can be found in [5]. The frequency domain filter coefficients $\mathbf{W}^i$, $\mathbf{W}^i \in \mathbb{C}^{N_RK\times K}$, for the i^{th} transmitted layer are determined so that the following MMSE criterion is satisfied:

$$\mathbf{W}^i = \arg \min_{\mathbf{W}^i} E\left\{\left\|\mathbf{F}^{-1}\mathbf{W}^{i\dagger}\hat{\mathbf{r}} - \mathbf{S}(n)\mathbf{b}^i\right\|^2\right\}, \quad (6)$$

where $\hat{\mathbf{r}}$, $\hat{\mathbf{r}} \in \mathbb{C}^{N_RK\times 1}$, is given by

$$\hat{\mathbf{r}} = \tilde{\tilde{\mathbf{r}}} + \mathbf{F}_R\tilde{\mathbf{H}}^i\mathbf{S}(n)\tilde{\mathbf{b}}^i \quad (7)$$

with $\mathbf{b}^i$, $\mathbf{b}^i \in \mathbb{C}^{K\times 1}$, being the desired i^{th} transmitted layer, and $\tilde{\mathbf{b}}^i$, $\tilde{\mathbf{b}}^i \in \mathbb{C}^{K\times 1}$, being its soft estimate. $\mathbf{F}^{-1}$ is the unitary IDFT matrix that is the inverse of DFT matrix $\mathbf{F}$, $\tilde{\mathbf{H}}^i$, $\tilde{\mathbf{H}}^i \in \mathbb{C}^{N_RK\times K}$, is a circulant block matrix that contain multipath components between the desired i^{th} transmit antenna and all receiver antennas, $\mathbf{S}(n)$, $\mathbf{S}(n) \in \mathbb{R}^{K\times K}$, is a time-varying sampling matrix having ones at main diagonal at a time of interest, and $\tilde{\tilde{\mathbf{r}}}$, $\tilde{\tilde{\mathbf{r}}} \in \mathbb{C}^{N_RK\times 1}$, is the output of the soft canceller in the frequency domain denoted as

$$\tilde{\tilde{\mathbf{r}}} = \mathbf{F}_R\tilde{\mathbf{r}} - \mathbf{\Gamma}\mathbf{F}_{N_T}\tilde{\mathbf{b}}. \quad (8)$$

$\tilde{\mathbf{b}}$, $\tilde{\mathbf{b}} \in \mathbb{C}^{N_TK\times 1}$, represents the soft-estimates of the transmitted layers given as

$$\tilde{\mathbf{b}} \quad = \quad [\tilde{\mathbf{b}}^{1^T}, ..., \tilde{\mathbf{b}}^{N_T T}]^T \quad (9)$$

with

$$\tilde{\mathbf{b}}^i \quad = \quad [\tilde{b}_0^i, ..., \tilde{b}_{K-1}^i]^T, \quad (10)$$

where $\tilde{b}_0^i$ is the soft estimate of the first coded symbol of the i^{th} layer in the frame and ^{T} indicates the transpose operation. Computational complexity to obtain the solution to (6) is high due to symbol-by-symbol matrix inversion. However, by following the algorithm derivation in [4],[5], significant complexity reduction is possible. In the derivation of MMSE filter coefficients it is assumed that channel is static over the frame, and that interferce powers at each layers to be detected are constant. Therefore, equalizer coefficients have to be computed only once per frame for each transmitted layer to be detected. It is shown in [4] that only a neglible performance loss is incurred with this approximation compared to the exact solution. As a result of derivation the output $\mathbf{z}^i$, $\mathbf{z}^i \in \mathbb{C}^{K\times 1}$, of equalizer for the i^{th} layer to be detected is given as [5]

$$\mathbf{z}^i = (1+\overline{\gamma}_i\delta_i)^{-1}\left(\overline{\gamma}_i\tilde{\mathbf{b}}_d^i + \mathbf{F}^{-1}\mathbf{\Gamma}_d^{\dagger i}\left(\mathbf{\Gamma}\mathbf{\Delta}\mathbf{\Gamma}^{\dagger} + \sigma^2\mathbf{I}\right)^{-1}\tilde{\tilde{\mathbf{r}}}\right) \quad (11)$$

with $\overline{\gamma}_i$ being a scalar given by

$$\overline{\gamma}_i = \frac{1}{K}\mathbf{Tr}\left\{\mathbf{\Gamma}^{\dagger i}\left(\mathbf{\Gamma}\mathbf{\Delta}\mathbf{\Gamma}^{\dagger} + \sigma^2\mathbf{I}\right)\mathbf{\Gamma}^i\right\}, \quad (12)$$

where $\mathbf{\Gamma}^i$, $\mathbf{\Gamma}^i \in \mathbb{C}^{N_RK\times K}$, is obtained by applying (3) to $\mathbf{H}^i$ and the scalar δ_i is given by

$$\delta_i = \frac{1}{K}\sum_{j=1}^{K} E\left\{\left|\tilde{b}_i(j)\right|^2\right\}. \quad (13)$$

The $\mathbf{\Delta}$, $\mathbf{\Delta} \in \mathbb{C}^{N_TK\times N_TK}$, matrix representing the residual interference energy after soft-cancellation is approximated by

$$\mathbf{\Delta} \approx diag\left\{\mathbf{F}_{N_T}\mathbf{\Lambda}\mathbf{F}_{N_T}^{\dagger}\right\}, \quad (14)$$

where $\mathbf{\Lambda}$, $\mathbf{\Lambda} \in \mathbb{C}^{N_TK\times N_TK}$, is given by

$$\mathbf{\Lambda} = diag\left\{E\left\{|\mathbf{b}|^2\right\} - \overline{\tilde{\mathbf{b}}}\right\} \quad (15)$$

with $\overline{\tilde{\mathbf{b}}}$, $\overline{\tilde{\mathbf{b}}} \in \mathbb{C}^{N_TK\times 1}$, being

$$\overline{\tilde{\mathbf{b}}} = [\delta_i\mathbf{I}\cdots\delta_K\mathbf{I}]^T. \quad (16)$$

By assuming that the MMSE filter output, $\mathbf{z}^i$, can been seen as the output of an equivalent Gaussian channel [1], and by defining the filter output as

$$\mathbf{z}^i = \mathbf{\Phi}_{AA}^i\mathbf{b}_d^i + \mathbf{\Psi}_{AA}^i \quad (17)$$

with the diagonal elements $\mathbf{\Phi}_{AA}^i$, $\mathbf{\Phi}_{AA} \in \mathbb{R}^{K\times K}$, of equivalent channel matrix being

$$\mathbf{\Phi}_{AA}^i = diag\left\{(1+\overline{\gamma}_i\delta_i)^{-1}\overline{\gamma}_i\right\}, \quad (18)$$

the elements $\mathbf{\Psi}_{AA}^i$, $\mathbf{\Psi}_{AA}^i \in \mathbb{R}^{K\times K}$, of equivalent channel noise covariance matrix are obtained as

$$\mathbf{\Psi}_{AA}^i = \mathbf{\Phi}_{AA}^i - \mathbf{\Phi}_{AA}^{2i}. \quad (19)$$

B. Joint-Detection-over-Antenna

In the JA detector signal detection is divided into two stages. The first stage performs the frequency domain groupwise MMSE-based filtering. In the groupwise filtering the residual ISI is supressed within the same layered group, and CAI is suppressed between different layered groups. The second stage performs joint detection over each layered group. The joint detection MAP detector separates trasmitted layers within the group to be jointly detected using spatial MAP technique [6]. The frequency domain groupwise filter coefficients $\mathbf{\Omega}$, $\mathbf{\Omega} \in \mathbb{C}^{N_R K \times N_G K}$, for antenna group with a size of N_G and virtual antenna matrix $\mathbf{A}$, $\mathbf{A} \in \mathbb{C}^{N_G K \times N_G K}$, of equivalent Gaussian channel are determined according to the following MMSE criterion:

$$[\mathbf{\Omega}, \mathbf{A}] = \arg \min_{\mathbf{\Omega},\mathbf{A}} E\left\{\left\|\mathbf{F}_{N_G}^{-1}\mathbf{\Omega}^\dagger\dot{\mathbf{r}} - \tilde{\mathbf{S}}(n)\mathbf{A}^\dagger\beta\right\|^2\right\} \quad (20)$$

where $\mathbf{F}_G$, $\mathbf{F}_G \in \mathbb{C}^{N_G \times N_G}$, is defined as $\mathbf{F}_{N_G} = \mathbf{I}_{N_G} \otimes \mathbf{F}$. $\dot{\mathbf{r}}$ is given by

$$\dot{\mathbf{r}} = \tilde{\tilde{\mathbf{r}}} + \mathbf{F}_R\mathbf{A}_d\tilde{\mathbf{S}}(n)\tilde{\beta}, \quad (21)$$

β, $\beta \in \mathbb{C}^{N_G K \times 1}$, is group vector for the layers within the group to be jointly detected, and $\tilde{\beta}$, $\tilde{\beta} \in \mathbb{C}^{N_G K \times 1}$ is its soft estimate. The block-circulant channel matrix $\mathbf{A}_d$, $\mathbf{A}_d \in \mathbb{C}^{N_R K \times N_G K}$, corresponds to transmit antennas to be detected jointly, and $\tilde{\mathbf{S}}(n)$, $\tilde{\mathbf{S}}(n) \in \mathbb{R}^{N_G K \times N_G K}$, is a time-varying sampling matrix having ones at main diagonal at a time of intrest with rest of the elements being zeros. The virtual antenna matrix $\mathbf{A}$ is given by

$$\mathbf{A}^\dagger = \begin{bmatrix} \mathbf{A}^{1,1\dagger} & \dots & \mathbf{A}^{1,N_G\dagger} \\ \vdots & \ddots & \vdots \\ \mathbf{A}^{N_G,1\dagger} & \dots & \mathbf{A}^{N_G,N_G\dagger} \end{bmatrix} \quad (22)$$

with submatrix $\mathbf{A}^{u,v\dagger}$, $\mathbf{A}^{u,v\dagger} \in \mathbb{C}^{K \times K}$, being the virtual channel matrix between the u^{th} virtual transmit and the v^{th} transmit antenna pair, given by

$$\mathbf{A}^{u,v\dagger} = \begin{bmatrix} a_0^{i,j\ddagger} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & \dots & a_0^{i,j\ddagger} \end{bmatrix}. \quad (23)$$

$a_0^{i,j}$ is the first path of the virtual equivalent channel matrix and $\ddagger$ indicates complex conjugation. The optimization problem of (20) can be rewritten as

$$[\mathbf{\Omega}, \mathbf{A}] = \arg \min_{\mathbf{\Omega},\mathbf{A}} E\left\{\left\|\mathbf{G}^\dagger\mathbf{y}\right\|^2\right\}, \quad (24)$$

where $\mathbf{G}^\dagger$, $\mathbf{G}^\dagger \in \mathbb{C}^{N_G K \times (N_R+N_G)K}$, is given by

$$\mathbf{G}^\dagger = \begin{bmatrix} \mathbf{F}_T^{-1}\mathbf{\Omega}^\dagger & -\mathbf{S}(n)\mathbf{A}^\dagger \end{bmatrix}, \quad (25)$$

and $\mathbf{y}$, $\mathbf{y} \in \mathbb{C}^{(N_R+N_G)K \times 1}$, by

$$\mathbf{y} = \begin{bmatrix} \dot{\mathbf{r}} \\ \beta \end{bmatrix}. \quad (26)$$

$\mathbf{\Omega}$ and $\mathbf{A}$ are subject to constraint in order to avoid the trivial solution $[\mathbf{\Omega}, \mathbf{A}] = [0, 0]$. The path constraint [6] is imposed to (24) in the same way as in [6]. By introducing a Lagrange multiplier the cost function to be minimized corresponding to the optimazation problem of (24) is

$$\Im = \mathbf{G}^\dagger\mathbf{R_{yy}}\mathbf{G} - \lambda(\mathbf{G}^\dagger\mathbf{Q} - \mathbf{I}) \quad (27)$$

where λ is a scalar, $\mathbf{R}_{yy}$, $\tilde{\mathbf{R}}_{yy} \in \mathbb{C}^{(N_R+N_G)K \times (N_R+N_G)K}$, is correlation matrix of $\mathbf{y}$, and the constraint matrix $\mathbf{Q}$, $\mathbf{Q} \in \mathbb{C}^{(N_R+N_G)K \times (N_G)K}$, that imposes the path constraint

$$\mathbf{G}^\dagger\mathbf{Q} = \mathbf{I} \quad (28)$$

is given as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0_{N_R K \times K}} & \dots & \mathbf{0_{N_R K \times K}} \\ \mathbf{I_{K \times K}} & \vdots & \vdots \\ \mathbf{0_{K \times K}} & \ddots & \vdots \\ \mathbf{0_{K \times K}} & \dots & \mathbf{I_{K \times K}} \end{bmatrix}. \quad (29)$$

By differentiating (27) with respect to $\mathbf{G^H}$ the optimal weights are obtained as

$$\mathbf{G} = \lambda\mathbf{R}_{yy}^{-1}\mathbf{Q}, \quad (30)$$

where $\mathbf{R}_{yy}^{-1}$ is obtained by applying the block-matrix inversion Lemma, which results in

$$\mathbf{R}_{yy}^{-1} = \begin{bmatrix} \mathbf{\Sigma}^{-1} & -\mathbf{\Sigma}^{-1}\mathbf{F}_R\mathbf{A}_d\tilde{\mathbf{S}}(n) \\ -\tilde{\mathbf{S}}(n)^\dagger\mathbf{A}_d^\dagger\mathbf{F}_R^\dagger\mathbf{\Sigma}^{-1} & (\mathbf{I} + \tilde{\mathbf{S}}(\mathbf{n})^\dagger\mathbf{A_d}^\dagger\mathbf{F_R}^\dagger\mathbf{\Sigma^{-1}}\mathbf{F_R}\mathbf{A_d}\tilde{\mathbf{S}}(\mathbf{n})) \end{bmatrix} \quad (31)$$

with $\mathbf{\Sigma}^{-1}$,$\mathbf{\Sigma}^{-1} \in \mathbb{C}^{N_R K \times N_R K}$, defined as

$$\mathbf{\Sigma}^{-1} = (\mathbf{\Gamma}\mathbf{F_T}\mathbf{\Lambda}\mathbf{F_T}^\dagger\mathbf{\Gamma}^\dagger + \sigma^2\mathbf{I} + \mathbf{F}_R\mathbf{A}_d\tilde{\mathbf{S}}(n)(\mathbf{\Upsilon} - \mathbf{I})\tilde{\mathbf{S}}(n)^\dagger\mathbf{A}_d^\dagger\mathbf{F}_R^\dagger)^{-1} \quad (32)$$

and $\mathbf{\Upsilon}$, $\mathbf{\Upsilon} \in \mathbb{C}^{N_G K \times N_G K}$, being

$$\mathbf{\Upsilon} = \begin{bmatrix} |\tilde{\beta}(0)|^{2\ (1,1)} & 0 & 0 \\ 0 & \ddots & \vdots \\ 0 & \dots & |\tilde{\beta}(K)|^{2\ (N_G,N_G)} \end{bmatrix}. \quad (33)$$

$|\beta(0)|^{2\ (u,u)}$ is the power of soft-estimate of the first symbol at the u^{th} layer which is jointly detected. The optimal filter weights are then obtained by using (25), (28) and (30) as

$$\mathbf{F}_{N_G}^{-1}\mathbf{\Omega}^\dagger = \frac{\tilde{\mathbf{S}}(n)^\dagger\mathbf{A}_d^\dagger\mathbf{F}_R^{-1}\mathbf{\Sigma}^{-1}}{\mathbf{I} + \tilde{\mathbf{S}}(n)^\dagger\mathbf{A}_d^\dagger\mathbf{F}_R^\dagger\mathbf{\Sigma}^{-1}\mathbf{F}_R\mathbf{A}_d\tilde{\mathbf{S}}(n)} \quad (34)$$

The equalizer output $\mathbf{z}$, $\mathbf{z} \in \mathbb{C}^{N_G K \times 1}$ is then obtained by using (21) and (34), as

$$\mathbf{z} = \frac{\tilde{\mathbf{S}}(n)^\dagger\mathbf{A}_d^\dagger\mathbf{F}_R^{-1}\mathbf{\Sigma}^{-1}}{\mathbf{I} + \tilde{\mathbf{S}}(n)^\dagger\mathbf{A}_d^\dagger\mathbf{F}_R^\dagger\mathbf{\Sigma}^{-1}\mathbf{F}_R\mathbf{A}_d\tilde{\mathbf{S}}(n)}\dot{\mathbf{r}}. \quad (35)$$

After minor mathematical manipulations the equalizer output (35) can be rewritten for the i^th composite signal $\widehat{\mathbf{z}}^i$, $\widehat{\mathbf{z}}^i \in \mathbb{C}^{K \times 1}$, as follows:

$$\begin{aligned} \widehat{\mathbf{z}}^i \quad &= \quad \frac{1}{\Xi^i}\left(\tilde{\mathbf{S}}(n)^{i,i}\mathbf{A_d^{i\dagger}}\mathbf{F}_R^\dagger\Sigma^{-1}\tilde{\tilde{\mathbf{r}}}\right) \quad (36) \\ &+ \quad \frac{1}{\Xi^i}\left(\tilde{\mathbf{S}}(n)^{i,i}\mathbf{A_d^{i\dagger}}\mathbf{F}_R^\dagger\Sigma^{-1}\mathbf{F}_R\sum_{j=1}^{N_G}\mathbf{A}_d^j\tilde{\mathbf{S}}(n)^{j,j}\tilde{\beta}^j\right) \end{aligned}$$

where $\tilde{\beta}^j$, $\tilde{\beta}^j \in \mathbb{C}^{K \times 1}$, is the j^{th} layer which is jointly detected, and a scaling factor Ξ^i, $\Xi^i \in \mathbb{C}^{K \times K}$, for the i^{th} composite signal is given as

$$\Xi^i = \mathbf{I} + \tilde{\mathbf{S}}(n)^{i,i\dagger}\mathbf{A}_d^{i\dagger}\mathbf{F}_R^\dagger\mathbf{\Sigma}^{-1}\mathbf{F}_R\mathbf{A}_d^i\tilde{\mathbf{S}}(n)^{i,i} \quad (37)$$

with $\tilde{\mathbf{S}}(n)^{i,i}$, $\tilde{\mathbf{S}}(n)^{i,i} \in \mathbb{C}^{K \times K}$, being the i^{th} submatrix of the sampling matrix and $\mathbf{A}_d^i$, $\mathbf{A}_d^i \in \mathbb{C}^{N_R K \times K}$, being the i^{th} circulant block submatrix of $\mathbf{A}_d$. By using the same assumptions as in the case of AA equalizer derivation, the sampling matrix $\tilde{\mathbf{S}}(n)^{i,i}$ can be neglected, since DFT is not affected by symbol timing. As a result, the filter output (35) can be rewriten by using (3) and (12) as

$$\widehat{\mathbf{z}}^i = \frac{1}{1 + \alpha^{i,i}}\left(\mathbf{F^{-1}}\mathbf{\Gamma}^{\dagger i}\tilde{\Sigma}^{-1}\tilde{\tilde{\mathbf{r}}} + \sum_{j=1}^{N_G}\alpha^{i,j}\tilde{\beta}^j\right), \quad (38)$$

where the scaling factor $\alpha^{i,j}$ is defined as

$$\alpha^{i,j} = \frac{1}{K}\mathbf{Tr}\left\{\mathbf{\Gamma}^{\dagger i}\tilde{\Sigma}^{-1}\mathbf{\Gamma}^{j}\right\} \quad (39)$$

with

$$\tilde{\Sigma}^{-1} = \left(\mathbf{\Gamma}\mathbf{\Delta}\mathbf{\Gamma}^{\dagger} + \sigma^{2}\mathbf{I} + \mathbf{\Gamma}_{A}\nabla\mathbf{\Gamma}_{A}^{\dagger}\right)^{-1}. \quad (40)$$

$\mathbf{\Gamma}_A$, $\mathbf{\Gamma}_A \in \mathbb{C}^{N_G K \times N_G K}$, is obtained by applying (3) to $\mathbf{A}_d$, and ∇, $\nabla \in \mathbb{C}^{N_G K \times N_G K}$, is a diagonal matrix given by

$$\nabla = diag\left\{\overline{\overline{\beta}} - \mathbf{I}\right\}, \quad (41)$$

where $\overline{\overline{\beta}}$, $\overline{\overline{\beta}} \in \mathbb{C}^{N_G K \times 1}$, is

$$\overline{\overline{\beta}} = [\eta_i \mathbf{I} \cdots \eta_K \mathbf{I}]^T \quad (42)$$

with

$$\eta_i = \frac{1}{K}\sum_{j=1}^{K} E\left\{\left|\tilde{\beta}_i(j)\right|^2\right\}. \quad (43)$$

Now, it is noticed that the output of the groupwise MMSE filter (34) is a composite signal comprised of the desired trasmitted layers plus residual CAI and ISI. The amount of residual CAI and ISI depends on the quality of the soft feedback. The equalizer coefficients $\tilde{\mathbf{\Omega}}$, $\tilde{\mathbf{\Omega}} \in \mathbb{C}^{N_R K \times N_G K}$ in (34) can be rewritten as

$$\tilde{\mathbf{\Omega}}^{\dagger} = \frac{\mathbf{\Gamma}_A^{\dagger}\tilde{\Sigma}^{-1}}{\mathbf{I} + \mathbf{C}} \quad (44)$$

where $\mathbf{C}$, $\mathbf{C} \in \mathbb{C}^{N_G K \times N_G K}$ is defined as

$$\mathbf{C} = \begin{bmatrix} \alpha^{1,1}\mathbf{I} & \dots & \alpha^{1,N_G}\mathbf{I} \\ \vdots & \ddots & \vdots \\ \alpha^{N_G,1}\mathbf{I} & \dots & \alpha^{N_G,N_G}\mathbf{I} \end{bmatrix} \quad (45)$$

By assuming that the MMSE filter output, (35), can been seen as the output of an equivalent Gaussian channel [1], and by defining the filter output as

$$\mathbf{z} = \mathbf{\Phi}_{JA}\beta + \mathbf{\Psi}_{JA} \quad (46)$$

with equivalent channel matrix, $\mathbf{\Phi}_{JA}$, $\mathbf{\Phi}_{JA} \in \mathbb{C}^{N_G K \times N_G K}$, being

$$\mathbf{\Phi}_{JA} = \tilde{\mathbf{\Omega}}^{\dagger}\mathbf{\Gamma}_A \quad (47)$$

the equivalent channel noise covariance matrix, $\mathbf{\Psi}_{JA}$, $\mathbf{\Psi}_{JA} \in \mathbb{C}^{N_G K \times N_G K}$ is obtained as

$$\mathbf{\Psi}_{JA} = \tilde{\mathbf{\Omega}}^{\dagger}\tilde{\Sigma}\tilde{\mathbf{\Omega}} \quad (48)$$

The second stage performs joint detection over each layered group. The joint detection MAP detector separates trasmitted layers within the group to be jointly detected using the spatial MAP technique [6]. The spatial MAP computes distance metric for each possible bit candidate, d_p^l, with $p = \{0,1\}$, $l = \{0,\cdots,2^{N_T m} - 1\}$, and m being the number of bits per symbol at the instant n as follows:

$$\zeta(d_{p,k}^{l}(n)) = (\mathbf{z}(\mathbf{n})_k - \mathbf{\Phi}_n\mathfrak{M}\{\mathbf{d}\})^{\dagger}\mathbf{\Psi}_n^{-1}(\mathbf{z}(\mathbf{n})_k - \mathbf{\Phi}_n\mathfrak{M}\{\mathbf{d}\}) \quad (49)$$

where $\mathbf{z}(\mathbf{n})$, $\mathbf{z}(\mathbf{n}) \in \mathbb{C}^{N_G \times 1}$, is the n^{th} time instant of Eq. (46), $\mathbf{\Phi}_n$, $\mathbf{\Phi}_n \in \mathbb{C}^{N_G \times N_G}$, is the n^{th} time instant of Eq. (47), $\mathbf{\Psi}_n$, $\mathbf{\Psi}_n \in \mathbb{C}^{N_G \times N_G}$, is the n^{th} time instant of Eq. (48), $\mathfrak{M}\{\}$ is mapping function that maps bits to symbols and $\mathbf{d}$ is the bit vector. A Log-likelihood ratio of the extrinsic probability of each bit d_p is given as

$$\mathfrak{L}^l(n) = ln\frac{\sum\limits_{d\in\mathbb{D},1} e^{-\zeta(d_1^l(n))}\exp\sum\limits_{t\neq l}\mathfrak{L}_a(d^t)}{\sum\limits_{d\in\mathbb{D},0} e^{-\zeta(d_0^l(n))}\exp\sum\limits_{t\neq l}\mathfrak{L}_a(d^t)}. \quad (50)$$

where $\mathbb{D},1$ is set of bit vectors having $d^l = 1$, $\mathbb{D},0$ is set of bit vectors having $d^l = 0$ and $\mathfrak{L}_a(d^t)$ is a priori Log-likelihood information of coded bits omitting the l^{th} bit.

IV. Numerical Results

Parameters used in the simulations are as follows: Quadrature Phase Shift Keying (QPSK) modulation, frame length M = 1152, DFT length K = 1024, guard period length P= 128 and symbol time period = 10 ns. Power delay profile of the channel is exponentially decaying and RMS delay spread of the channel is 256 ns. In modelling the spatial correlation, we refer to the cases defined in a 3GPP report [10]: The spatial correlation matrices for the case 2-4 for transmitter, and the matrix for the case 4 for receiver (the cases defined by [10]). The turbo encoder uses two recursive systematic component codes with a generator polynomial (15,13), and the coded bit sequence are adequately punctured so that overall code rate is 1/2. The MAP algorithm is used in the SfISfO decoder. The number of iterations with the turbo decoder is 8, and number of iterations with equalizer is 2. The lengths of the random bit interleaver and the semi-random symbol interleaver are 1792 bits and 896 symbols,respectively. Perfect channel state information as well as perfect syncronization are assumed in the simulations. Figures 2 and 3 present FER and normalized spectral efficiency results for N_T = N_R = 2 and N_T = N_R = 4, respectively. The matched filter bound (MFB) provides an upper bound of performance. The MFB curves were simulated assuming perfect feedback. The loss caused by spatial correlation is around 0.5 dB with the MFB receiver with N_T = N_R = 2, N_T = N_R = 4, respectively. The results for AA show that a significant perfomance degradation is caused when antennas are spatially correlated: The degradation is around 2 dB with N_T = N_R = 2 and around 3.5 dB with N_T = N_R = 4. Results clearly show that the effect of spatial correlation to the perfomance of the AA receiver becomes larger when number of transmitter/receiver antennas is increased. It is expected that in the presence of spatial correlation the performance degradation with the JA detection technique is made significantly smaller compared to AA technique.

V. Conclusions

The concept of a new frequency domain joint over-antenna (JA) SC/MMSE detector has been proposed for broadband frequency-selective MIMO channels in this paper. Mathematical expressions for the AA and JA frequency domain algorithms have been provided in detail. In the time domain processing matrix inversion for the group-wise filtering dominates computational. However, with freuquency domain technique, the MMSE filtering is no longer a major part of the complexity, and for JA the heaviest part in computation is the spatial MAP. The size of the group determines the MAP complexity, which is still far lower than the time domain approact as a whole.

References

[1] X. Wang and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1046–1061, July 1999.

[2] K. Kansanen and T. Matsumoto, "A computationally efficient MIMO turbo-equaliser," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Jeju, Korea, Apr. 22–25 2003, vol. 1.

[3] M.S.Yee, M.Sandell, and Y.Sun, "Comparison study of single-carrier and multi-carrier modulation using iterative based receiver for MIMO system," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Milan, Italy, May 17–19 2004.

[4] M. Tüchler and J. Hagenauer, "Linear time and frequency domain turbo equalisation," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, oct 7-11 2001, vol. 4, pp. 2773–2777.

[5] R. Wohlgenannt, K. Kansanen, D. Tujkovic, and T. Matsumoto, "Outage -based LDPC code design for SC/MMSE turbo equalization," in *Proc. IEEE Veh. Technol. Conf. (VTC), in press*, Stockholm, Sweden, Apr. 22–25 2005.
[6] N. Veselinović, T.Matsumoto, and C.Schneider, "Interference suppression and joint detection for reduction of sensitivity to timing offset and spatial correlation in space-time coded mimo turbo equalization," in *Proc. Joint COST 273/284 Workshop on Antennas and Related System Aspects in Wireless Communications*, June 2004, vol. 1, pp. 99–104.
[7] D. Falconer, S.L Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol. 40, no. 4, pp. 58–66, 2002.
[8] J. Coon, J. Siew, M. Beach, A. Nix, S. Armour, and J. McGeehan, "A comparison of MIMO-OFDM and MIMO-SCFDE in WLAN environments," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, San Francisco, USA, Dec. 1–5 2003, vol. 6, pp. 3296–3301.
[9] P.J. Davis, *Circulant Matrices*, Chelsea Publishing, New York, USA, 2nd edition, 1994.
[10] 3rd Generation Partnership Project (3GPP), "TSGR1#23 R1-01-1179," Tech. Rep., 3rd Generation Partnership Project (3GPP), 2001.

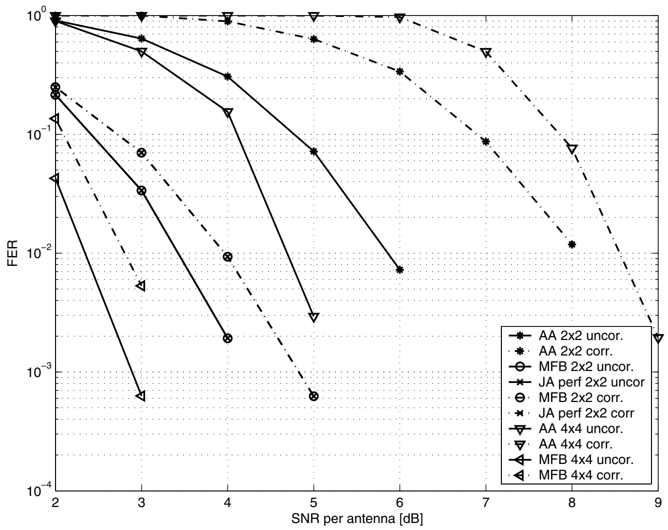

Fig. 2. FER results for $N_T = N_R = 2$ and $N_T = N_R = 4$

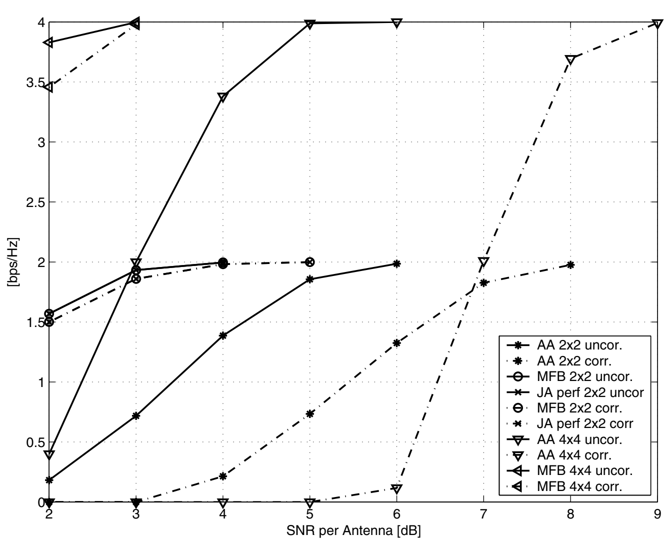

Fig. 3. Throughput results for $N_T = N_R = 2$ and $N_T = N_R = 4$