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Description	一般論文

1C04 Identification of the Optimal R&D Investment Trajectory

—Theoretical Analysis and Empirical Demonstration—

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Abstract

In line with the increasing significance of the identification of firm's optimal R&D investment strategy, an attempt to introduce optimal theory into techno-economics analysis is conducted.

On the basis of the theoretical analysis and empirical demonstration, identification of firm's optimal R&D investment strategy taking profitability, new innovation products and optimal trajectory of R&D intensity is attempted.

1. Introduction

The governing factors of industry R&D interrelate with each other to construct a metabolic system. The increasing global competition inevitably highlights the importance and difficulty of R&D investment decision for policy-makers.

Given the above, the identification of optimal trajectory has become a crucial subject. To date, a number of studies were concerned with the classic problems of economic growth and optimal allocation of resources.[1] None, however, have treated the dynamic systems and utilized the optimal theory into techno-economics.

This paper, on the basis of the optimal R&D investment model developed by A. Tarasyev and C. Watanabe,[4] with the support of other relative researches in the general framework shown as **Fig. 1**, attempts to identify firm's optimal investment strategy taking profitability, new innovation products and optimal trajectories of R&D intensity. **Section 2** provides the theoretic analysis of optimal R&D investment control problem. **Section 3** demonstrates the economic implication of analytic solution of the test optimal R&D investment control problem. **Section 4** provides the empirical demonstration of optimal R&D trajectory for Japanese manufacturing industry. **Section 5** briefly summarizes the concluding remarks.

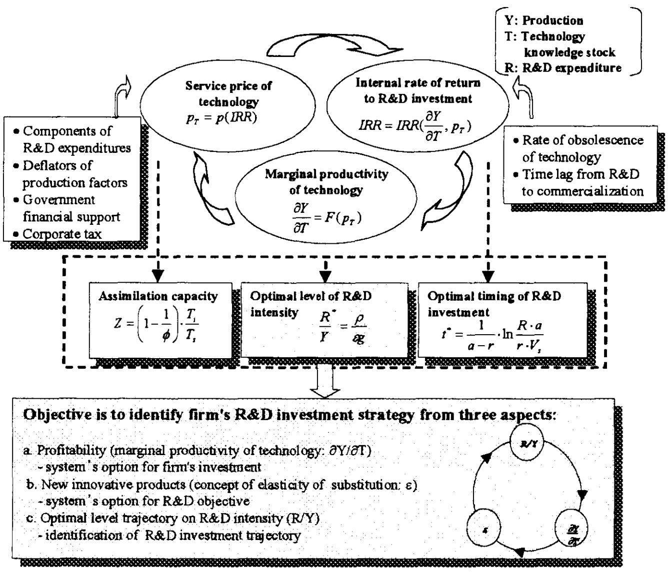

Fig. 1 General Framework of R&D Investment Analysis

2. Optimal control problem of R&D investment

To construct the dynamic model of manufacturing and R&D investment, the following variables are used:

$y = y(t)$ - manufacturing, production
t: - time trend
$\dot{y}/y$ - production rate
$T = T(t)$ - accumulated R&D investment, technology
$\dot{T} = r = r(t)$ - change in technology
r/y - R&D intensity
X (=L, K, M, E) - production factors (labor, capital, materials and energy), involved in manufacturing and R&D
X_T (=L_T, K_T, M_T, E_T) - production factors input, directed to R&D

The classical production function is used to construct dynamics as follows.

$$y = F(t,(L - L_T),(K - K_T),(M - M_T),(E - E_T),T) \tag{1}$$

Assume that the functional dependence between the L_T, K_T, M_T, E_T and the accumulated R&D investment T is given by the function of the substitution type:

$$T = T(L_T,K_T,M_T,E_T) = \min\{h_1(L_T),h_2(K_T),h_3(M_T),h_4(E_T)\} \tag{2}$$

and the inverse relations exist

$$L_T = L_T(T) = h_1^{-1}(T), \qquad K_T = K_T(T) = h_2^{-1}(T)$$
$$M_T = M_T(T) = h_3^{-1}(T), \qquad L_T = L_T(T) = h_4^{-1}(T) \tag{3}$$

Differentiating (1) by time t and taking into account (3), we obtain the following equation:

$$\frac{\dot{y}}{y} = \frac{\partial F}{\partial t}\frac{1}{y} + \sum\frac{\partial F}{\partial X}\frac{X}{y}\frac{\dot{X}}{X} - \sum\frac{\partial F}{\partial X}\frac{\partial X_T}{\partial T}\frac{\dot{T}}{y} + \frac{\partial F}{\partial T}\frac{\dot{T}}{y} \tag{4}$$

Rewrite (4) in the form of:

$$\frac{\dot{y}}{y} = f - p\frac{r}{y} + q\frac{r}{y} \tag{5}$$

where terms related to the production factors X (=L, K, M, E), learning and scale effects are combined into function f

$$f = \frac{\partial F}{\partial t}\frac{1}{y} + \sum\frac{\partial F}{\partial X}\frac{X}{y}\frac{\dot{X}}{X} \tag{6}$$

decrease in manufacturing due to R&D spending X_T (=L_T, K_T, M_T, E_T) is collected into function p

$$p = p(t) = \sum\frac{\partial F}{\partial X}\frac{\partial X_T}{\partial T} \tag{7}$$

increase of R&D knowledge stock is described by the marginal productivity of technology q

$$q = q(t) = \frac{\partial F}{\partial T} \tag{8}$$

the control parameter r stands for the current change $\dot{T}$ in technology

$$\dot{T} = r \tag{9}$$

Collecting the terms $(r/y)p$, $(r/y)q$ which depend on the control parameter r into the net contribution by R&D intensity $(r/y)g$, we obtain the first equation for the dynamic control process

$$\frac{\dot{y}}{y} = f - g\frac{r}{y} \qquad (g = g(t) = p(t) - q(t) > 0) \tag{10}$$

In the general case function f depends on the accumulated R&D investment T.

$$f = f_1 + f_2\left(\frac{T}{y}\right)^{\gamma}, f_1 = f_1(t), f_2 = f_2(t) \tag{11}$$

Combining (9), (10) and (11), the dynamic process described by the system of differential equations is obtained.

$$\begin{cases} \dfrac{\dot{y}}{y} = f_1 + f_2\left(\dfrac{T}{y}\right)^{\gamma} - g\dfrac{r}{y} \\ \dot{T} = r \end{cases} \tag{12}$$

The production y and the accumulated R&D investment T stand for the phase parameters in dynamics (12). The current change r in technology T is the control parameter. The control parameter $r = r(t)$ is not fixed and can be chosen for obtained "good" properties of trajectories of dynamics (12).

To formalize the goal for designing the control parameter $r = r(t)$ and indicate the profit of R&D investment in the long-run, the utility function represented by the integral with the discount coefficient ρ is considered.

$$U_t = \int_t^{\infty} e^{-\rho(s-t)} \ln D(s)ds \tag{13}$$

$$D = D(s) = \left(\int_0^n x^{\beta}(j)dj\right)^{1/\alpha}, n = n(s) \quad (14)$$

$$x(j) = \frac{y}{n}, y = y(s), n = n(s) \quad (15)$$

$$n = n(s) = be^{\kappa s}T^{\beta_1}r^{\beta_2}, T = T(s), r = r(s) \quad (16)$$

Here $D(s)$ is the consumption index, s is the running time, t is the initial time, j is the current index of invented products, $x(j)$ is the quantity of production of the brand with index j, n is the quantity of available (invented) products, α is the parameter of elasticity (ε, $\varepsilon = 1/(1-\alpha)$) of substitution between any two products. Note that D depends on production, technology and current investment.

Combining (13)-(16) and omitting constant term independent on y, T and r, the following expression for the utility function can be obtained:

$$U = \int_t^{\infty} e^{-\rho(s-t)}(\ln y + a_1 \ln T + a_2 \ln r)ds \quad (17)$$

where $a_1 = A\beta_1$, $a_2 = A\beta_2$, $A = (1-\alpha)/\alpha$.

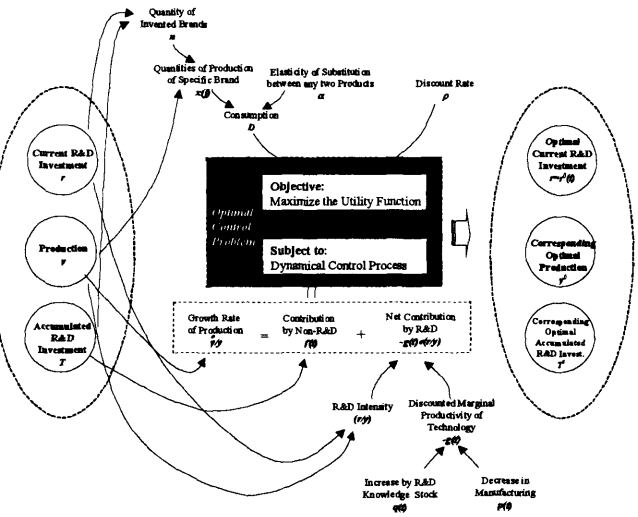

Fig. 2 Schematic Structure of the Optimal Problem

The considered problem shown as **Fig. 2** is a classical problem of the optimal control theory. The Pontryagin's maximum principle[2] is a key instrument in the theory. The main elements in the analysis are Hamiltonian H and the adjoint variable ψ_1 and ψ_2. The Hamiltonian has the form

$$H(y,T,r,\psi_1,\psi_2) = \ln y + a_1 \ln T + a_2 \ln r + \psi_1(f_1 y + f_2 T^{(1-\gamma)} - gr) + \psi_2 r \quad (18)$$

and represents the utility flow. The adjoint variables act as marginal prices. The maximum value of the Hamiltonian is attained at the current optimal technology rate

$$r^0 = a_2 \frac{1}{g\psi_1 - \psi_2} \quad (19)$$

The optimal dynamics is given by the following differential equations:

$$\begin{cases} \dot{x}_1 = f_1 x_1 + f_2 x_1^{1-\gamma} - \dfrac{a_2(x_1 + g)x_1}{(gx_2 - x_1 x_4)}, \\ \dot{x}_2 = \rho x_2 + \gamma f_2 x_2 \dfrac{1}{x_1^{\gamma}} - 1 - \dfrac{a_2 g x_2}{(gx_2 - x_1 x_4)} \\ \dot{x}_3 = -\dfrac{a_2 x_1 x_3}{(gx_2 - x_1 x_4)} \\ \dot{x}_4 = \rho x_4 - \gamma f_2 x_2 \dfrac{1}{x_1^{\gamma}} - a_1 + \dfrac{a_2 x_1 x_4}{(gx_2 - x_1 x_4)} \end{cases} \quad (20)$$

where x_1, x_2, x_3, x_4 are new variables connected with the original and adjoint variables:

$$x_1 = \frac{y}{T}, x_2 = \psi_1 y, x_3 = \frac{1}{T}, x_4 = \psi_2 T \quad (21)$$

3. Economic implication of analytic solution of the test optimal control problem

The nonlinear system (20) is rather complicated and at the first glance does not have the analytic solution expressed in the explicit function. In order to obtain explicit solutions we consider first the reduced version - the test optimal control problem. In the test optimal control problem, it is assumed that function f doesn't depend on the technology parameter T, and utility function U_t doesn't depend on the accumulated R&D investment T. According to these assumptions, $\gamma=0$ in (11) and $\beta_1=0$ and $\beta_2=1$ in (16). Utilizing the Pontryagin principle of maximum into the reduced nonlinear system, finally we can find the analytic solution. The obtained relation between the optimal investment r and the optimal production y can be demonstrated as

$$r = \frac{(1-\alpha)\rho}{g} y \qquad (22)$$

It means that the optimal R&D investment r increases proportionally to the growth of the optimal production y with coefficient $((1-\alpha)\rho/g)$.

For R&D intensity r/y we have the following formula

$$\frac{r}{y} = \frac{(1-\alpha)\rho}{g} = \frac{(1-\alpha)\rho}{p-q} \qquad (23)$$

It describes the dependence of the optimal R&D intensity on the substitution parameter α, the subjective discounted rate ρ and the discounted marginal productivity of technology g. When the cost p for sustaining the accumulated R&D investment T is high, then the R&D intensity r/y is low. Increasing marginal productivity of technology, q, leads to the growth of R&D intensity r/y.

4. Empirical demonstration of optimal R&D trajectory

4.1 Empirical measurement of discounted marginal productivity of technology *(g)*

Implication of discounted marginal productivity of technology *(g)* can be demonstrated as **Fig. 3**.

(1) Measurement of q(t)

Using the method provided by C. Watanabe and K. Wakabayashi,[6] a practical computer program is used to measure simultaneously the marginal productivity of technology as well as the service price of technology and rate of internal return to technology investment.

(2) Measurement of p(t)

Using the method presented by B. Zhu,[10] $p(t)$ can be empirically measured as follows under the assumptions of linear homogeneous and cost-minimization.

$$p(t) = \left(\sum \frac{Mx}{REXS} \cdot DX\right)\frac{Y}{T} \qquad (24)$$

$$= \left(\frac{\left(\frac{GLC}{GC}\right)}{\left(\frac{GTCl}{GTC}\right)} \cdot \left(\frac{L_T}{L}\right) + \frac{\left(\frac{GCC}{GC}\right)}{\left(\frac{GTCk}{GTC}\right)} \cdot \left(\frac{K_T}{K}\right) + \frac{\left(\frac{GMC}{GC}\right)}{\left(\frac{GTCm}{GTC}\right)} \cdot \left(\frac{M_T}{M}\right) + \frac{\left(\frac{GEC}{GC}\right)}{\left(\frac{GTCe}{GTC}\right)} \cdot \left(\frac{E_T}{E}\right) \right)\frac{Y}{T}$$

where *Mx: (=Ml, Mk, Mm, Me) Mx=GXC/GC,* Cost share of labor, capital, materials and energy in gross cost; *REXS: (=RELS, REKS, REMS, REES) REXS=GTCx/GTC,* R&D expenditure share of respective R&D components; *DX: (=DL, DK, DM, DE)* $DX=X_T/X$, Ratios of duplication of technology to labor, capital, material and energy; *GC:* Gross cost; *GXC (=GLC, GCC, GMC, GEC)*: gross cost for labor, capital, material and energy; *GTC*: R&D expenditure; *GTCx (=GTCl, GTCk, GTCm, GTCe)*: R&D expenditure for labor capital, material and energy.

(3) Analysis result of g(t)

Discounted marginal productivity of technology $g(t)$ is the balance of $p(t)$ and $q(t)$. Here the analysis result of $g(t)$ for Japanese manufacturing industry during 1960-1992 is illustrated in **Fig. 4**.

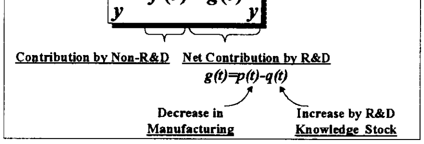

Fig. 3 Economic Implication of *g(t)*

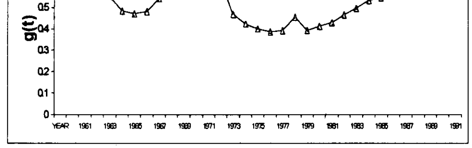

Fig. 4 Discounted Marginal Productivity of Technology in Japan's Manufacturing Industry (1960-1992)

4.2 A suggestive idea to measure elasticity of substitution

(1) Definition of substitution elasticity

α is the parameter of elasticity (ε) of substitution between any pair of innovative goods. $\varepsilon = 1/(1-\alpha)$. The definition of elasticity of substitution between any two innovative goods:

$$\varepsilon = \frac{\partial(\frac{x_1}{x_2})}{\partial\left(\frac{f_2}{f_1}\right)} \cdot \frac{\frac{x_2}{x_1}}{\left(\frac{f_1}{f_2}\right)} \qquad (f_1 = \frac{\partial Y}{\partial x_1},\ f_2 = \frac{\partial Y}{\partial x_2}) \qquad (25)$$

where x_1, x_2 are the amounts of the various innovative goods

(2) Analysis of the measurement of substitution elasticity in this optimal control problem

Under the condition of the equilibrium between demand and supply, elasticity of substitution measured by demand-side factors (e.g. substitution between innovative goods) could be interpreted by elasticity of substitution measured by supply-side factors (e.g. substitution among production factors).

Based on the CES (Constant Elasticity of Substitution) function, under the assumptions of homogenous and cost-minimization,

$$\varepsilon = \frac{f_1 f_2}{f f_{12}} = \frac{d \ln \frac{x_2}{x_1}}{d \ln \frac{P_1}{P_2}} \qquad (f_{12} = f_{21} = \frac{\partial^2 f}{\partial x_1 \partial x_2}) \qquad (26)$$

where P_1, P_2 are the prices of the various innovative goods or production factors. For the supply-side, two pairs of innovative goods should be substituted by two pairs of factors.

Given that production is represented by GDP (value added), the substitution elasticity should be between labor, capital and technology.

By using Vintage model to treat technology T embodied in labor and capital, the considered substitution elasticity will be only for that between labor and capital $\varepsilon(K^*(T), L^*(T))$.

4.3 Qualitative identity of optimal trajectory via real data

As proved in the paper of A. Tarasyev and C. Watanabe,[4] the system (20) has a first function

$$z = \psi_1 y + \psi_2 T = p^0 = \frac{a_1 + a_2 + 1}{\rho} \qquad (27)$$

Thanks to this first integral the four-dimensional system (20) is reduced to the next three-dimensional system whose two variables, x_1, x_2, do not depend on the third one, x_3:

$$\begin{cases} x_1 = f_1 x_1 + f_2 x_1^{1-\gamma} - \dfrac{a_2(x_1 + g)x_1}{((x_1 + g)x_2 - p^0 x_1)} \\ \dot{x}_2 = \rho x_2 + \gamma f_2 x_2 \dfrac{1}{x_1^{\gamma}} - 1 - \dfrac{a_2 g x_2}{((x_1 + g)x_2 - p^0 x_1)} \\ \dot{x}_3 = -\dfrac{a_2 x_1 x_3}{((x_1 + g)x_2 - p^0 x_1)} \end{cases} \qquad (28)$$

Under several natural assumptions, system (20) has the unique equilibrium point $x^0 = (x_1^0, x_2^0, x_3^0)$, which is a saddle point. All optimal trajectories converge to x^0. The nonlinear system (28) is provided for the empirical demonstration of optimal R&D trajectory.

We select the Japanese manufacturing techno-economics database as our real econometric data input of identity of optimal R&D trajectory. The first task is to identify the relative parameters and intermediate variables in the nonlinear system. In 4.1, we provide a measurement method of discounted marginal productivity of technology (g) as a classical example of measurement of relative parameters via real data. In 4.2, we provide a suggestive idea to measure the parameter of elasticity of substitution (α).

On the basis of the measurements of relative parameters and intermediate variables, substituting the real techno-economics data of Japanese manufacturing industry into program RATE developed by R. Sergey [3], we can get the optimal trajectory for the nonlinear system (25). Because of $T(t) = 1/x_3(t)$ and $y(t) = x_1(t)/x_3(t)$, the optimal trajectories in technology and production are naturally obtained. Changing the substitution parameter α results in different clusters of optimal trajectories. **Fig. 5** shows the relationship between empirical data and optimal synthetic trajectories in production, technology and technology productivity. It is found that the optimal synthetic trajectories agree well with the empirical time series and are robust with respect to the substitution parameter α.

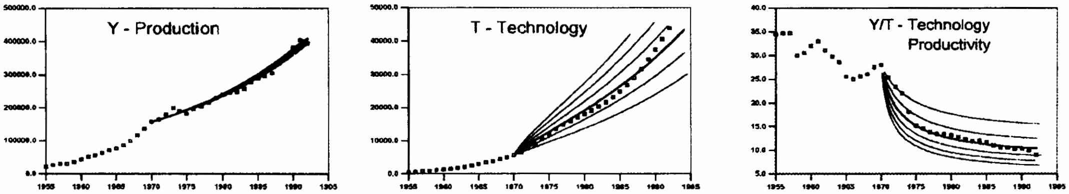

Fig. 5 Optimal Trajectories for the Japanese Manufacturing Industry: (α=0.85, ..., 0.95)

5. Concluding remarks

In this paper, we have conducted theoretical analysis and empirical demonstration to identify the optimal R&D investment trajectory. In the theoretical analysis, optimal theory is used to construct and solve a nonlinear model of optimal allocation of resources - R&D investment in a techno-metabolic system, which describes behavior of production and technology rates with respect to R&D investment. The analytic solution of test optimal problem provides an interesting result demonstrating the possible governing factors on optimal R&D intensity. This result provides an important reference to make R&D investment decision for a policy-maker. In the empirical demonstration, firstly we find the measuring method of discounted marginal productivity of technology, secondly we present a suggestive direction to measure elasticity of substitution, finally on the basis of measurements of relative key parameters, we demonstrate the optimal trajectories in technology, production and technology productivity in the Japan's manufacturing industry. The agreement of optimal trajectory with the empirical time series proves the rationality of the optimal R&D investment model developed by A. Tarasyev and C. Watanabe.

There are several ways to extend the analysis: (1) develop a precise measurement of the substitution elasticity; (2) identify the correlation among substitution elasticity, R&D intensity and marginal productivity of technology; (3) analyze the sensitivity of optimal R&D investment dynamics.

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