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The Role of Market Learning as a Coordinator of Techno-Countervailing Power

— An Empirical Analysis of Canon Printers based on Optimal Theory

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Abstract

This analysis attempts to identify the optimal R&D investment level of the firms and corresponding optimal level of learning efforts. By means of the optimal theory postulated by Pontryagin, mathematical equations identifying firms optimal R&D intensity level and corresponding optimal levels of learning coefficient and marginal productivity of technology were developed. Taking Canon's printers development trajectories, their learning efforts were examined.

1. Introduction

Under a new paradigm characterized by a shift from an industrial society to an information society, it is important for high-tech firms to improve their competitive power simply depends on continual R&D investment which guarantees them to increase technology stock.

The purpose of this research is to demonstrate a dynamic model, which includes increase growth and decrease trends of R&D investment, and try to solve optimal problem for trajectories of firms' technology growth. Learning efforts of Canon printers will be examined by empirical analysis.

2. Analytical Framework

2.1 Assumption

For constructing a dynamic model for optimal R&D investment, sales of innovative goods $S(t)$ can be expressed as Cobb-Douglas production function of technology stock $T(t)$ as follows:

$$S(t) = AT(t)^{\alpha} \qquad (1)$$

where A: scale factor and α : constant elasticity of sales to technology,. $0< \alpha <1$.

Take logarithms of equation (1):

$$\ln S(t) = \ln A + \alpha \ln T(t) \qquad (2)$$

Differentiate equation (2) in time t,

$$\frac{\dot{S}(t)}{S(t)} = \alpha \frac{\dot{T}(t)}{T(t)}$$

Technology intensity is defined as $x(t) = \dfrac{T(t)}{S(t)}$

Therefore, dynamic process for technology intensity can be expressed as follows:

$$\dot{x}(t) = \left(\frac{T(t)}{S(t)}\right)' = \frac{\dot{T}(t)S(t) - T(t)\dot{S}(t)}{S(t)^2} = \frac{\dot{T}(t)}{S(t)} - \frac{T(t)}{S(t)}\frac{\dot{S}(t)}{S(t)} = \frac{\dot{T}(t)}{S(t)}(1-\alpha) \quad (3)$$

where $\mu(t) = \dfrac{\dot{T}(t)}{S(t)}$ $= R(t)/S(t)$: **R&D intensity.**

Main control parameter R&D intensity $\mu(t)$ is decided as

$\mu(t) = \dfrac{\dot{T}(t)}{S(t)}$, $0 \le \mu(t) \le \mu^0 < 1$ .The finial form of dynamics for technology intensity can be expressed as:

$$\dot{x}(t) = \mu(t)(1-\alpha) \qquad (4)$$

Logarithmic index of consumption in time t:

$$D(t) = \ln(1-\mu(t)S(t) = \ln(1-\mu(t)) + \ln S(t) \qquad (5)$$

Based on equation (1), technology stock $T(t)$ can be expressed as: $T(t) = \dfrac{S(t)^{1/\alpha}}{A^{1/\alpha}}$.

Therefore, technology intensity $x(t)$ and sales of innovative goods $S(t)$ can be expressed as follows:

$$x(t) = \frac{T(t)}{S(t)} = \frac{\frac{S(t)^{1/\alpha}}{A^{1/\alpha}}}{S(t)} = \left(\frac{1}{A}\right)^{1/\alpha} S(t)^{\left(\frac{1-\alpha}{\alpha}\right)}$$

$$S(t) = A^{\frac{1}{1-\alpha}} x(t)^{\frac{\alpha}{1-\alpha}}$$

Equation (5) can be developed as:

$$\ln(1-\mu(t)S(t) = \ln(1-\mu(t)) + \ln S(t) = \ln(1-\mu(t)) + \frac{\alpha}{1-\alpha}\ln x(t) + \frac{1}{1-\alpha}\ln A \qquad (6)$$

2.2 Utility Function

Utility function J for optimal problem of R&D intensity can be expressed by a discount rate r according to Grossman and Helpman (1991) as follows:

$$J = \int_0^{+\infty} e^{-rt} D(t)dt$$

$$= \int_0^{+\infty} e^{-rt}\left(\ln(1-u(t)) + \frac{\alpha}{1-\alpha}\ln x(t)\right)dt + \frac{\ln A}{(1-\alpha)r}$$

where: e^{-rt} , discount multiplier.

2.3 Optimal Control Problem

$$J(x(t), u(t)) = \int_0^{+\infty} e^{-rt}\left(\ln(1-u(t)) + \frac{\alpha}{1-\alpha}\ln x(t)\right)dt$$

Under system's dynamics,

$\dot{x}(t) = \mu(t)(1-\alpha)$,

Control constraints: $0 \le \mu(t) \le \mu^0 < 1$ and initial condition for the technology intensity: $x(0) = x^0$.

2.4 Maximum Principle of Pontryagin

The Hamiltonian function for the stationary problem has the following function:

$$H(x(t), \mu, \psi(t)) = \frac{\alpha}{1-\alpha} \ln x(t) + \ln(1-\mu) + \psi(t)\mu(1-\alpha) \quad (8)$$

Ajoint variables $\psi = \psi(t)$ is defined as shadow price for x(t), $\psi(t) > 0$.

According to maximize principle of Pontryagin, maximum of the Hamiltonian function in control parameter should be:

$$\frac{\partial H}{\partial \mu(t)}(x(t), \mu(t), \psi(t)) = \frac{1}{1-\mu(t)}(-1) + \psi(t)(1-\alpha) = 0$$

$$\mu(t) = 1 - \frac{1}{\psi(t)(1-\alpha)}$$

For adjoint differential equation,

$$\dot{\psi} = r\psi(t) - \frac{\partial H}{\partial x} = r\psi(t) - \frac{\alpha}{1-\alpha}\frac{1}{x(t)}$$

2.5 Hamiltonian System

According to the Hamiltonian system in prime and adjoint variables, the following equations can be obtained:

$$\dot{x}(t) = (1 - \frac{1}{\psi(t)(1-\alpha)})(1-\alpha) = (1-\alpha) - \frac{1}{\psi(t)} = (1-\alpha) - \frac{x}{z}$$

Because costs of technology intensity are expressed as: $z = \psi(t)x(t)$.

For the dynamics of costs,

$$\dot{z} = \dot{\psi}(t)x(t) + \psi(t)\dot{x}(t) = r\mu(t)x(t) - \frac{\alpha}{1-\alpha} + \psi(1-\alpha) - 1$$

$$= rz + \frac{z}{x}(1-\alpha) - 1 = rz + \frac{z}{x}(1-\alpha) - \frac{1}{1-\alpha}$$

According to algebraic equation for steady state,

$$rz + \frac{z}{x}(1-\alpha) - \frac{1}{1-\alpha} = 0 \quad (9)$$

$$(1-\alpha) - \frac{1}{xz} = 0 \quad (10)$$

Based on equation (9) and (10), solution for steady states state should be:

$x^* = \frac{\alpha}{r}$ and $z^* = \frac{\alpha}{r(1-\alpha)}$.

Optimal control at the steady states should be $\mu^* = 0$.

2.6 Saddle Character of the steady State

Jacobin matrix for the Hamiltonian system at the steady state

$$\begin{cases} \dot{x}(t) = (1-\alpha) - \frac{x}{z} = F_1(x, z) \\ \dot{z} = \rho z + \frac{z}{x}(1-\alpha) - \frac{1}{1-\alpha} = F_2(x, z) \end{cases}$$

Linearization at the steady state,

$$\frac{\partial F_1}{\partial x} = -\frac{1}{z}\Big|_{\substack{x=x^* \\ z=z^*}} = -\frac{1}{z^*} = -\frac{r(1-\alpha)}{\alpha}$$

$$\frac{\partial F_1}{\partial z} = \frac{x}{z^2}\Big|_{\substack{x=x^* \\ z=z^*}} = -\frac{x^*}{(z^*)^2} = \frac{r(1-\alpha)^2}{\alpha}$$

$$\frac{\partial F_2}{\partial x} = -\frac{z}{x^2}(1-\alpha)\Big|_{\substack{x=x^* \\ z=z^*}} = -\frac{z^*}{(x^*)^2}(1-\alpha) = -\frac{r}{\alpha}$$

$$\frac{\partial F_2}{\partial z} = r + \frac{(1-\alpha)}{x}\Big|_{\substack{x=x^* \\ z=z^*}} = r + \frac{(1-\alpha)}{x^*} = \rho + \frac{r(1-\alpha)}{\alpha}$$

The Jacobi matrix at the steady state:

$$A = \begin{pmatrix} -\frac{r(1-\alpha)}{\alpha} & \frac{r(1-\alpha)^2}{\alpha} \\ -\frac{r}{\alpha} & r + \frac{r(1-\alpha)}{\alpha} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$$

Trace of the Jacobin matrix as:

$t = T_r A = a_{11} + a_{22} = r$

Deter mint of the Jacobin matrix as follows:

$$d = def(A) = a_{12}a_{22} - a_{11}a_{21}$$

$$= -\frac{r(1-\alpha)}{\alpha}(r + \frac{r(1-\alpha)}{\alpha}) + \frac{r}{\alpha}\frac{r}{\alpha}(1-\alpha)^2 = -\frac{r^2(1-\alpha)}{\alpha}$$

Characteristic equation for eigenvalues

$$\lambda^2 - t\lambda + d = 0, \quad \lambda^2 - r\lambda - \frac{r^2(1-\alpha)}{\alpha} = 0$$

$$\lambda_{1,2} = \frac{r \pm \sqrt{r^2 + \frac{4r^2(1-\alpha)}{\alpha}}}{2} = \frac{r \pm \sqrt{r^2 + \frac{4r^2(1-\alpha)}{\alpha}}}{2} = \frac{r \pm r\sqrt{1 + \frac{4(1-\alpha)}{\alpha}}}{2}$$

Therefore,

$$\lambda_1 = \frac{r + r\sqrt{1 + \frac{4(1-\alpha)}{\alpha}}}{2} > r, \text{and } \lambda_2 = \frac{r - r\sqrt{1 + \frac{4(1-\alpha)}{\alpha}}}{2} < 0$$

Equation for an eigenvector corresponding to the negative eigenvalue as follows:

$$\begin{pmatrix} \lambda_2 - a_{11} & -a_{12} \\ -a_{22} & \lambda_2 - a_{22} \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0$$

$$\frac{y_2}{y_1}=\frac{\lambda_2-a_{11}}{a_{12}}=\frac{\dfrac{r(1-\sqrt{1+\frac{4(1-\alpha)}{\alpha}})+\frac{r}{\alpha}(1-\alpha)}{2}}{\frac{r}{\alpha}(1-\alpha)^2}$$

$$=\frac{2-\alpha-\alpha\sqrt{1+\frac{4(1-\alpha)}{\alpha}}}{2(1-\alpha)^2}=\frac{z-z^*}{x-x^*}=\alpha$$

Direction of an eigenvalue from the steady state,

$z=z^*+a(x-x^*)$

Sub optional control

$$\mu=1-\frac{x}{(z^*+a(x-x^*))(1-\alpha)}$$

For condition 1 of covertness of the solution,

$$\frac{x}{(z^*+a(x-x^*))(1-\alpha)}\ >\ 0,\ z^*+a(x-x^*)\ >\ 0$$

Because $\alpha\ >0,\ 2-\alpha-\alpha\sqrt{1+\frac{4(1-\alpha)}{\alpha}}\ >\ 0$

$$2-\alpha\ >\ \alpha\sqrt{1+\frac{4(1-\alpha)}{\alpha}}$$

$$4-4\alpha+\alpha^2\ >\ \alpha^2+\alpha^2\frac{4(1-\alpha)}{\alpha}$$

$$4-4\alpha\ >\ 4\alpha-4\alpha^2$$

$$-(\alpha-1)^2\ <\ 0 \qquad 0\ <\ \alpha\quad 1$$

$$x=0 \quad \Longrightarrow z^*-ax^*\ >\ 0$$

$$\frac{\alpha}{r(1-\alpha)}-a\frac{\alpha}{r}\ >\ 0$$

$$\frac{1}{(1-\alpha)}-\frac{2-\alpha-\alpha\sqrt{1+\frac{4(1-\alpha)}{\alpha}}}{2(1-\alpha)^2}\ >\ 0$$

$\sqrt{1+\frac{4(1-\alpha)}{\alpha}}\ >\ 1$, which coincides the condition of the covertness.

For condition 2 of covertness of the solution,

$$\frac{x}{(z^*+a(x-x^*))(1-\alpha)}\ <\ 1$$

$$\frac{\alpha}{r}+\frac{(2-\alpha-\alpha\sqrt{1+\frac{4(1-\alpha)}{\alpha}}(x-\frac{\alpha}{r})}{2(1-\alpha)}\ >\ x$$

$$2-\alpha-\alpha\sqrt{1+\frac{4(1-\alpha)}{\alpha}}\ <\ 2-2\alpha$$

$\alpha-\alpha\sqrt{1+\frac{4(1-\alpha)}{\alpha}}\ <\ 0$, which also coincides condition 2 of covertness of the solution.

3. Optimal R&D Investment and Learning

3.1 Effect of Obsolescence of Technology

Change rate of technology stock $\dot{T}(t)$ can be expressed as follows:

$$\dot{T}(t)=-\rho T(t)+R(t) \tag{11}$$

where ρ : Coefficient of obsolescence and $R(t)$: real R&D investment.

Therefore, real R&D investment can be expressed as:

$$R(t)=\dot{T}(t)+\rho T(t)$$

$$\frac{R(t)}{S(t)}=\frac{\dot{T}(t)}{S(t)}+\rho\frac{T(t)}{S(t)}$$

where $\frac{R(t)}{S(t)}$: real R&D intensity, $\frac{\dot{T}(t)}{S(t)}$: intensity of technology change

and $\frac{T(t)}{S(t)}$: technology intensity.

$$\frac{R(t)}{S(t)}=\frac{\dot{T}(t)}{S(t)}+\rho\frac{T(t)}{S(t)}=\rho x+(1-\frac{x}{(z^*+\alpha(x-x^*))}\frac{1}{(1-\alpha)})$$

3.2 Optimal R&D Intensity

Because at the steady state, $z^*=\frac{\alpha}{r(1-\alpha)}$ and $x^*=\frac{\alpha}{r}$,

Therefore, optimal R&D intensity $\frac{R(t)}{S(t)}=\rho x^*=\frac{\rho\alpha}{r}$

In case when production function is depicted as follows,

$$S=Ae^{kt}T^{\alpha} \tag{1'}$$

optimal R&D intensity can be $\mu^*=(\rho+k)\alpha/r$.

3.3 Optimal Learning

Learning process can be depicted by prices of innovative goods P_v decrease by technology stock T as follows:

$P_v=AT^{-\phi}$

Differentiate by time t,

$$\Delta P_V / P_V = -\phi \Delta T / T$$

$$-\phi = (\Delta P_V / P_V) / R / T$$
$$= (\Delta P_V / P_V) / ((R/S)/(S/T)) = (\Delta P_V / P_V)/(x/\mu)$$

When in optimal steady state, $x^{*} = \frac{\alpha}{r}$, $\mu^{*} = (\rho + k)\alpha / r$.

Therefore, $-\phi = (\Delta P_V / P_V)/(x/\mu) = (\Delta P_V / P_V)/(\rho + k)$ (12)

3.4 Optimal Marginal Productivity of Technology (MPT)

Under the optimal R&D investment condition,

$R/S = (\rho + \kappa)\alpha / r$, $\alpha = MPT \cdot T/S = P \cdot T/S$

$R/S = (\rho + \kappa) \cdot P \cdot (T/S) / r$

$P = (R/T) \cdot r / (\rho + \kappa) = (\rho + g) \cdot r / (\rho + \kappa)$,

$P_t = P \cdot P_V$

where P: relative prices of technology; P_t: prices of technology and P_V: prices of innovative goods.

Given that $(\rho + g)/(\rho + \kappa) = 1/B$, then $r = BP$, since

$1 + mr = P/(\rho + r)$, $1 + mBP = P/(\rho + BP)$

From this equation, P can be identified as follows:

$$P = \frac{-(m\rho B + B - 1) + \sqrt{(m\rho B + B - 1)^2 - 4mB^2\rho}}{2mB^2}$$

4. Impacts on Learning Coefficient – *A Case of Canon Printers*

Table 1 summarizes the analytical results of Canon printers over the period 1975-1999 based on the following equation:

$$\ln(S/T^{\alpha}) = a + \kappa_1 Dt + \kappa_2 (1 - D)t + cD_2$$

Table 1 Analytical Results of Canon printers (1975-1999)

α	a	κ_1	κ_2	c	DW	AIC	$adj.R^2$
0.6	0.005	0.147	0.285	1.132	1.25	40.69	0.961
	(0.02)	(3.10)	(17.38)	(5.41)			
0.7	**-0.042**	**0.129**	**0.266**	**1.126**	**1.27**	**40.21**	**0.978**
	(-0.14)	**(2.74)**	**(16.43)**	**(5.43)**			
0.8	0.501	0.096	0.220	1.197	1.23	41.75	0.938
	(1.68)	(1.98)	(13.16)	(5.60)			

where dummy variables D: 1975-84 = 1, D_2: 1983-90 = 1, others = 0.

By comparison of AIC, under optimal state of Canon printers, $\alpha = 0.7$, κ is 0.129 over 1975-1984 and 0.266 over 1985-1999 after the introduction of LBP in the market.

Since BJ has full fledged introduction in 1993 resulting in the structural change in the competition structure in the printers market, the period 1985-1999 should be divided to 1985-1992 and 1993-1999. Based on the result in Table 1, by means of equation (12) learning coefficient with optimal R&D investment condition in respective period can be computed as 1.335 (1985-1992) and 0.366 (1993-1999).

While the actual learning coefficient can be estimated as follows:

$$\ln P_V = \underset{(53.42)}{6.608} - \underset{(-35.71)}{1.083} \ln T - \underset{(-3.40)}{0.149} D \quad DW \ 1.29 \quad adj.R^2 \ 0.990$$

This suggests that Canon made intensive efforts in learning thereby, effective utilization of potential resources in innovation.

5. Conclusions

In light of the increasing significance of the effective utilization of the potential resources in innovation for firms competition in a global megacompetition, this analysis attempts to identify the optimal R&D investment level of the firms and corresponding optimal level of learning efforts.

By means of the optimal theory postulated by Pontryagin, mathematical equations identifying firms optimal R&D intensity level and corresponding optimal levels of learning coefficient and marginal productivity of technology were developed.

Taking Canon's printers development trajectories, their learning efforts were examined and identified that while their efforts were maintained optimal level as far as LBP development in the later half of the 1980s to the beginning of the 1990s are concerned, their efforts dramatically declined correspond to the full fledged development of the BJ in the early 1990s.This decline resulted in the decrease in marginal productivity of printers technology in the late 1990s.

These results suggest the significance of the integrated approach in both indigenous R&D investment and market learning efforts for firms optimal R&D strategy for their competitiveness.

Further analyses should be focused on the empirical analysis of the more comprehensive optimal strategies including marginal productivity of technology and functionality development.

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