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| Description  |  |

# Collision Prevention Platform for a Dynamic Group of Asynchronous Cooperative Mobile Robots

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**Abstract**—This paper presents a fail-safe platform on which cooperative mobile robots rely for their motion. The platform consists of a collision prevention protocol for a dynamic group of cooperative mobile robots with asynchronous communications. The collision prevention protocol is time-free, in the sense that it never relies on physical time, which makes it extremely robust for timing uncertainty common in wireless networks. It guarantees that no two robots ever collide, regardless of the respective activities of the robots. The protocol is based on a fully distributed path reservation system.

It assumes a mobile ad hoc network formed by the robots themselves, and takes advantage of the inherent locality of the problem in order to reduce communication. The protocol requires neither initial nor complete knowledge of the composition of the group.

A performance analysis of the protocol provides insights for a proper dimensioning of system parameters in order to maximize the average effective speed of the robots.

**Index Terms**—collision prevention, mobility, fail-safe, autonomous cooperative robots, wireless ad hoc networks, distributed algorithms.

## I. INTRODUCTION

There is a marked trend in distributed systems research toward studying problems in which hosts are mobile and their physical location can no longer be abstracted out. While most efforts are still aimed at mobile ad hoc networks and sensor networks, there is also a gradual realization that cooperative robotics raises many interesting new challenges with respect to distributed systems, and particularly in relation to mobility. Indeed, unlike traditional distributed systems and even more so than ad hoc or sensor networks, mobility becomes an essential part of the problems to address.

Many interesting applications are envisioned that rely on groups of cooperating mobile robots. Tasks may be inherently too complex (or impossible) for a single robot

to accomplish, or performance benefits can be gained from using multiple robots [2].

As a simple and peaceful illustration, consider the following example. A decentralized team of tiny autonomous mobile robots cooperate to maintain a small oriental garden. Based on the needs of the garden, the team must carry on with many tasks concurrently, such as looking after the moss, picking undesired weeds, nourishing flowers, trimming the trees, bringing dead leaves to the compost, distributing water, etc. These tasks require the robots to almost constantly roam the same limited space, namely the garden, while moving along according to their respectively assigned tasks. Due to the nature of the system, the robots cannot share exact knowledge of each other's location, speed, or even current intention. Nevertheless, an important challenge is to ensure that robots will not collide against each other, regardless of their respective activities.

*Context and problem statement:* We consider that the robots have the ability to communicate wirelessly and also that they can query their own position according to a common referential, as given by a positioning system (e.g., GPS, landmarks). However, the robots do not have the ability to detect each other's position in the environment, and they are not synchronized. In addition, communication delays are unpredictable, and actual robot motion speed is unknown.

In that context, our goal is to ensure *safe* motion, in the sense that, regardless of the respective activities of the robots, no two robots ever collide. The *safety* of the system must never be compromised, regardless of the uncertainty of the underlying system. However, the *performance* of the system may possibly degrade as the result of badly unstable network characteristics or erratic robot speed.

*Related work:* There are several approaches to address the problem of avoiding collisions between robots.

First, a widespread approach consists in using proximity sensors (e.g., infrared, sonar, laser) in the same way fixed obstacles are detected [3]–[5]. This approach is, however, sensitive to the robots respecting planned speeds, and normally requires an unbroken line-of-sight.

A second approach consists in relying on global motion planning, specifying the respective timing of robots as

A preliminary version of this paper appeared in *Proc. 8th IEEE Intl. Symp. on Autonomous Decentralized Systems ISADS'07* (IEEE CS press, pp. 188–195) [1].

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well as the path to follow [6], [7]. This approach is even more sensitive to the speed of the robots, and normally requires much synchronization between the robots.

A third approach uses wireless communication as a means to synchronize the robots and their motion [8], [9]. To do so, communication is extended to satisfy strict real-time guarantees, or at least probabilistic ones [8].

With all three approaches, protocols must rely on explicit time and the speed of the robots. This is fine as long as both robots and communication meet their timing assumptions. However, if a robot happens to move too slowly or too fast, or a few messages are delayed for too long, then there is a risk of collision. In contrast, our aim is to rely as little as possible on the respect of timing assumptions from the underlying system.

*Contribution:* We present a fail-safe platform on which cooperative mobile robots rely for their motion, thus ensuring that no (physical) collision ever occurs between robots. Its core consists of a collision prevention protocol for a dynamic group of cooperative mobile robots with asynchronous communications. The protocol is time-free, in the sense that it never relies on physical time (as given by a clock), thus making it extremely robust for timing uncertainty common in wireless networks. Furthermore, although the communication range may be limited, no routing is needed as robots only communicate with their direct neighborhood.

This paper extends work that we presented recently at an international conference [1]. The main additions are that we present a more rigorous specification for the collision prevention problem, and include proofs of correctness of the protocol and its properties.

*Structure of the paper:* The remainder of the paper is structured as follows. Section II describes the system model, definitions, and terminology. Section III defines the collision prevention problem and specification. In Section IV, we present the collision prevention protocol. Section V explains in detail the deadlock and starvation problem. Section VI provides the proofs that the protocol achieves the safety and the liveness properties. Section VII presents a performance analysis and provides insights to maximize the average effective speed of the robots. Section VIII discusses other related work, and Section IX concludes the paper.

## II. SYSTEM MODEL AND DEFINITIONS

### A. Model

We consider a dynamic distributed system of mobile robots  $S = \{R_i\}$  in which each robot has a unique identifier. The total composition of the system, of which robots have only a partial knowledge, can change dynamically. Robots have access to a global positioning device that, when queried by a robot  $R_i$ , returns  $R_i$ 's position with a bounded error  $\varepsilon_{gps}$ . The robots communicate using wireless communication with a limited range  $D_{tr}$ . If the distance between two robots  $R_i$  and  $R_j$  is less than  $D_{tr}$ , then the two robots can communicate with each other. Communications assume retransmission mechanisms such

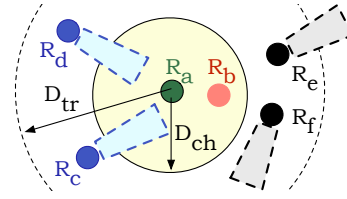


Figure 1. The neighborhood discovery primitive.  $D_{tr}$  is the transmission range and  $D_{ch}$  is the reservation range. The neighbors of  $R_a$  are:  $\{R_b, R_c, R_d\}$ .

that communication channels are reliable. The system is asynchronous in the sense that there is no bound on communication delays, processing speed and on robots' speed of movement. Each robot has access to a neighborhood discovery primitive<sup>1</sup> named *NDiscover*.

*Neighborhood discovery (NDiscover):*

*Characteristics :* The neighborhood discovery primitive called *NDiscover* is a function that enables a robot to detect its local neighbors. These neighbors are within one communication hop and satisfy a certain known predefined condition.

*Implementation :* *NDiscover* can be implemented as the traditional neighborhood discovery primitive of mobile ad hoc networks. An implementation of *NDiscover* primitive can be performed by Geocasting<sup>2</sup> a ping message in a geographical region centered on the robot at the time of calling *NDiscover* with a radius within the transmission range. All the robots that receive the message and satisfy the predefined condition acknowledge the caller of *NDiscover*.<sup>3</sup>

In wireless environments, the delays in delivering messages are very difficult to anticipate. There are several reasons for the asynchrony of communications in wireless environments, such as the delays required to access the shared medium, due to competition with other nodes. The competition to access the wireless medium causes message loss due to interference, collisions between messages, and fading. Therefore, a retransmission mechanism is needed to ensure message delivery in wireless environments.

Figure 1 illustrates the *NDiscover* primitive. The robot  $R_a$  starts *NDiscover*, the set of robots returned by *NDiscover* is the set  $\{R_b, R_c, R_d\}$ . The robot  $R_b$  is located within the reservation range  $D_{ch}$ , the robots  $R_c$  and  $R_d$  request zones that intersect with the reservation circle which is the circle centered on  $R_a$  with radius  $D_{ch}$ . The robots  $R_e$  and  $R_f$  request zones that do not intersect with the reservation circle of  $R_a$ .

*Node presence detector:* Detecting the presence of nodes in an asynchronous system where there are

<sup>1</sup>The wormhole of our platform is encapsulated in the primitive Neighborhood Discovery which is available through most wireless communication devices.

<sup>2</sup>Geocast is defined by the transmission of a message to a predefined geographical region.

<sup>3</sup>An implementation of *NDiscover* requires timing property for transmitting and processing the ping messages. *NDiscover* relies on very lightweight ping messages carrying only the position coordinates of the caller.

no bounds on communication delays, cannot be solved *deterministically* [10]. The impossibility is based on the fact that, in such systems, a very slow node can not be distinguished from an absent one. Thus, a timing property is required to implement the primitive *NDiscover*.

*Timing property:* The primitive *NDiscover* relies on the following timing property: there exists a known upper bounded time delay called  $T_{nd}$  such that the following property holds:

For any robot  $R_i$ , if  $R_i$  starts *NDiscover* at some time  $t$ , then  $R_i$  receives an acknowledgment from every robot  $R_j$  located in its communication range and satisfying a certain predefined condition, at a time  $t'$  such that:  $t' - t \leq T_{nd}$ .

The protocol is based on a Neighborhood Discovery service, which is the only synchronous part of the system. The delay  $T_{nd}$  is specified large enough to cover the necessary retransmissions and hence ensure the delivery of messages related to the Neighborhood Discovery service.

### B. Definitions and terminology

*Paths :* We denote by *chunk* a line segment along which a robot moves. A path of a robot is a continuous route composed of a series of contiguous chunks.

*Errors :* There are three sources of geometrical incertitude concerning the position and the motion of a robot. An error related to the position information provided by the positioning system is denoted  $\varepsilon_{gps}$ . In addition, the motion of a robot creates two additional sources of errors. The first error is related to the translational movement, denoted:  $\varepsilon_{tr}$ . The second error is related to the rotational movement, denoted:  $\varepsilon_{\theta}$ .

*Zones :* A *zone* is defined as the area needed by a robot to move safely along a chunk. This includes provisions for the shape of the robot, positioning error, and imprecision in the moving of the robot. The zone must be a convex shape and contain the chunk the robot is following. Figure 2 shows the zone  $Z_i$  for a robot  $R_i$  moving along a chunk  $AB$ , where  $d$  represents the radius of the geometrical shape of  $R_i$ . The zone  $Z_i$  is composed of the following three parts, illustrated in Figure 2: the first part, named *pre-motion zone* and denoted  $pre(Z_i)$ , is the zone that robot  $R_i$  possibly occupies while waiting (before moving). The second part, named *motion zone* and denoted  $motion(Z_i)$ , is the zone that robot  $R_i$  possibly occupies while moving. The third part, named *post-motion zone* and denoted  $post(Z_i)$ , is the zone that robot  $R_i$  possibly reaches after the motion.

### III. COLLISION PREVENTION: SPECIFICATION

The basic idea is essentially a mutual exclusion on geographical zones. The algorithm consists of a distributed path reservation system, such that a robot must reserve a zone before it moves. When a robot reserves a zone, it can move *safely* inside the zone.

All robots run the same protocol. When a robot wants to move along a given chunk, it must reserve the zone

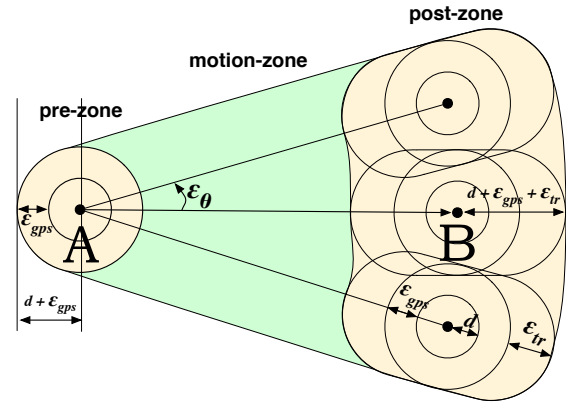


Figure 2. Reservation Zone.

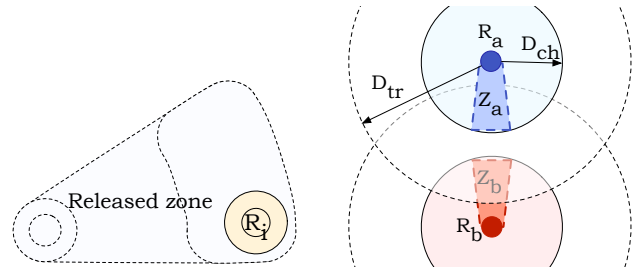


Figure 3. A robot  $R_i$  releases the previous zone and keeps only the place that may occupy  $pre(Z_i)$

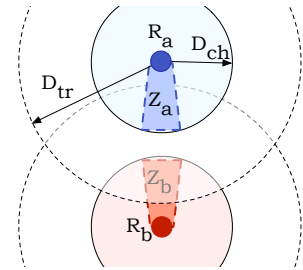


Figure 4.  $D_{tr}$ : transmission range.  $D_{ch}$ : reservation range.  $D_{ch} \leq \frac{D_{tr}}{2}$ , if  $R_a$  and  $R_b$  can not communicate, then they would not collide.

that surrounds this chunk. When this zone is reserved, the robot moves along the chunk. Once the robot reaches the end of the chunk, it releases the zone except for the area that the robot occupies. When moving along a path, the robot repeats this procedure for each chunk along the path.

We say that a robot  $R_i$  is the *owner* of a zone  $Z_i$  ( $Z_i$  is *granted* to  $R_i$ ), if  $R_i$  reserves  $Z_i$  and did not release it yet. A robot  $R_i$  *releases* the zone  $Z_i$  that it has owned and keeps only a part of  $post(Z_i)$  under its reservation. The part of the zone that has been released by  $R_i$  is denoted:  $RelZone_i$ . Figure 3 shows that the *pre-motion zone*  $pre(Z_i)$  is entirely included within the previous *post-motion zone*, and presents also the current and the previous positions of  $R_i$ .

$RelZone_i = pre(Z_i) \cup motion(Z_i) \cup SubPost(Z_i)$ , where:  $SubPost(Z_i) \subset post(Z_i)$   
 $pre(Z_i) \subset PREVIOUS(post(Z_i))$

The relationship between robots and zones changes in time. A zone is said to be free if it is not owned by any robot. In order to resolve the collision prevention problem, and to keep the system of mobile robots always in progress towards its final goal, certain properties of *safety* and *liveness* must hold. If a robot requests a zone, then eventually it owns this zone or receives an exception. We say that the robot owns the zone and all the points contained in this zone. A given point can be owned by only one robot. If a robot owns a zone, it eventually releases that zone.

### A. Reservation range property

Robots have a limited wireless transmission range. It follows that a reserved zone by a robot must be entirely within a circle centered on the robot with a radius within half of the transmission range. The motivation behind this maximal value is that each robot can communicate with all the robots that it might collide with. Figure 4 illustrates the reservation range property.

The collision prevention protocol provides a parameter named *reservation range* and denoted  $D_{ch}$ , that is within half of the transmission range ( $D_{ch} \leq \frac{D_{tr}}{2}$ ), such that a reserved zone by a robot is entirely within a circle centered on the robot with a radius equals to the *reservation range*.

### B. Properties

**Property 1 (Mutual exclusion):** If a zone  $Z_i$  of a robot  $R_i$  intersects with a zone  $Z_j$  of a robot  $R_j$ , then either  $R_i$  or  $R_j$  (but not both) is the owner of its zone. Consequently, a point in the plane can be owned by only one robot.

**Property 2 (Liveness):** If a robot  $R_i$  requests a zone  $Z_i$ , then eventually ( $R_i$  owns  $Z_i$  or an exception is raised).

Exception is potentially raised by the protocol only if a deadlock or a starvation situation occurs.

The following property must hold to ensure the *integrity* of the system. If a robot owns a zone, then eventually it leaves that zone. If a robot leaves a zone, then it releases that zone.

## IV. COLLISION PREVENTION: LOCALITY-PRESERVING PROTOCOL

All robots run the same distributed algorithm. When a robot  $R_i$  requests a zone  $Z_i$ ,  $R_i$  must determine all the robots  $R_j$  that conflict with  $R_i$ . The robots  $R_j$  are located within one communication hop with respect to  $R_i$ , because the reservation range of the robots must be within half of the transmission range. The Neighborhood Discovery primitive returns the set of neighbors  $Neighbor_i$  within one communication hop with respect to  $R_i$ . Therefore,  $R_i$  can determine the set of robots  $R_j$  that conflict with  $R_i$ .  $R_i$  multicasts  $Z_i$  to the list of neighbors  $Neighbor_i$ , then  $R_i$  waits until it receives the response messages. Consequently,  $R_i$  determines the set of robots that it conflicts with. Intuitively,  $R_i$  performs a pair-wise negotiation with each of the robots that  $R_i$  conflicts with. Therefore,  $R_i$  and each robot  $R_j$  decide consistently about the scheduling of their requests. So, a dynamic scheduling for the conflicting requests takes place. When  $R_i$  receives a release message from all the robots that  $R_i$  waits for, it reserves  $Z_i$  and becomes the owner of  $Z_i$ . After  $R_i$  has reached the *post-motion* zone,  $R_i$  releases  $Z_i$  except for the area occupied by  $R_i$ .

### A. Protocol variables

We present the variables used by the collision prevention protocol.

- $Z_i$  is the zone currently requested or owned by robot  $R_i$ .
- $Neighbor_i$  represents the set of robots that may possibly conflict with robot  $R_i$  (i.e., the output of the neighborhood discovery primitive  $NDiscover$ ).
- $G_i$  is a set of  $\{(R_j, Z_j)\}$  such that  $R_j$  belongs to  $Neighbor_i$ , and  $Z_j$  is the requested or the owned zone of  $R_j$ .  $Z_j$  intersects with  $Z_i$ .
- $After_i$  is the list of robots waiting for  $R_i$  until it releases its zone.
- $Before_i$  is the list of robots that  $R_i$  waits for.
- $Depend_i$  is the *dependency* set. If a robot  $R_i$  requests  $Z_i$  then it conflicts with a set of robots each of which conflicts with another set of robots and so on. The *dependency* set is the union of  $G_k$  for each robot  $R_k$  related to  $R_i$  by the *transitive closure* of the relation *conflict*.
- $Dag$  is a *wait-for* graph such that the vertices represent robots and a directed edge from  $R_i$  to  $R_j$  represent that  $R_i$  waits for  $R_j$  to release  $Z_j$ .
- $msg$  is a message exchanged during the run of the protocol. Each  $msg$  message consists of three fields, the first is the type of the message which belongs to the set {REQUEST, RELEASE, WAITFORME, ACK, PROHIBITED}, the second field is the identifier of the robot sending the message, and the third field is the body of the message which consists of the specifications and the parameters of the requested (or owned) zone. The type REQUEST denotes a request message, RELEASE denotes a release message, and the type WAITFORME means that the receiver of the message must wait for the sender. The type ACK indicates the sender and receiver do not conflict. The type PROHIBITED indicates that the receiver requests a zone that intersects with the pre-motion zone of the stationary sender, which does not move any more.
- $Request\_pending_i$  is a boolean indicates that robot  $R_i$  has a requested zone and that  $R_i$  has determined its wait-for graph  $Dag_{wait}$ .

### B. Protocol phases

We explain the phases of the protocol with respect to a robot  $R_i$ . The robot  $R_i$  is located in the *pre-motion* zone  $pre(Z_i)$ . When robot  $R_i$  requests a new zone  $Z_i$ , it proceeds as follows.

- 1) Discovery phase:  $R_i$  calls the neighborhood discovery primitive  $NDiscover$ , to determine the set  $Neighbor_i$ . This set consists of robots  $R_j$ , that may possibly come in conflict with  $R_i$  for  $Z_i$ , since  $Z_j$  intersects with the circle centered on  $R_i$  with radius equals to the *reservation range*.
- 2) Negotiation phase: The Negotiation phase of  $R_i$  starts by the determination of the set  $G_i$  which consists of the robots of  $Neighbor_i$  that *conflict* with  $R_i$ . The output of the Negotiation phase is the *wait-for* graph,  $Dag_{wait}$ . Thus,  $R_i$  determines the set of robots that it waits for. If  $R_i$  receives a request from a robot  $R_k$  ( $Z_k$  intersects with  $Z_i$ ) and

$R_k$  does not belong to  $G_i$ , then  $R_k$  must wait for  $R_i$ . The Negotiation phase proceeds as follows.

- $R_i$  multicasts  $msg_i = (\text{REQUEST}, i, Z_i)$  indicating that  $R_i$  requests  $Z_i$  to all the members of  $Neighbor_i$  carrying the parameters of  $Z_i$ . This multicast does not require any routing because the neighbors are located within one communication hop with respect to  $R_i$ .
- $R_i$  waits until it receives a response message  $msg_j$  from each member  $R_j \in Neighbor_i$ .
- After  $R_i$  has received the messages  $msg_j$ ,  $R_i$  determines the set of robots  $G_i$  that conflict with  $R_i$ . ( $G_i$  is obtained from the received messages  $msg_j$  after discarding the release messages  $msg_j = (\text{RELEASE}, j, Z_j)$ , and discarding also the request messages  $msg_j = (\text{REQUEST}, j, Z_j)$  such that  $Z_j$  does not intersect with  $Z_i$ ). The set  $G_i$  contains two disjoint subsets of robots: the first subset denoted  $(G1)_i$  is composed of robots  $R_j$  such that  $R_i$  does not belong to  $G_j$  (i.e.,  $R_i$  must wait for  $R_j$ ,  $R_j$  has sent the message  $msg_j = (\text{WAITFORME}, j, Z_j)$ ). The second is the complementary subset denoted  $(G2)_i$ , which is composed of robots  $R_j$  such that  $R_i$  belongs to  $G_j$ . Thus  $R_i$  must wait for all the robots of  $(G1)_i$ , in addition to a subset of  $(G2)_i$ . (This subset would be determined by the Conflict\_Resolver and the Pathologic\_Detector).
- $R_i$  determines the dependency set  $Depend_i$  by applying an Echo algorithm inspired from [11]. The Echo algorithm is explained as follows.  $R_i$  multicasts a token message to each robot that belongs to  $G_i$ . Upon receipt of the first message of  $R_i$  by a robot  $R_k$  from  $R_j$  ( $R_j$  is called the father of  $R_k$ ), it multicasts the message of  $R_i$  to all the robots of  $G_k$  except its father  $R_j$ . When a robot  $R_k$  has received the token message of  $R_i$  from all the robots of  $G_k$ ,  $R_k$  adds the contents of  $G_k$  to the token message and sends it (echo) to the father  $R_j$ . When  $R_i$  has received the token message from all the robots of  $G_i$ , it obtains the dependency set<sup>4</sup>. The motivation for building the dependency set is to enable the conflicting robots to build the wait-for graph  $Dag_{wait}$  in a consistent manner and so to avoid cyclic wait-for relations.
- $R_i$  uses the dependency set  $Depend_i$  to construct  $Dag_{wait}$ . The vertices represent the robots of the set  $Depend_i$  and a directed edge from  $R_i$  to  $R_j$  means that  $R_i$  waits for  $R_j$ .  $Dag_{wait}$  is built as follows.  $R_i$  starts by establishing the imposed wait-for relations (Subsection. IV-C), and then it breaks ties for the remainder of the conflicting robots by the

Conflict\_Resolver. (Subsection IV-E) At first,  $R_i$  builds the WAITFORME graph, denoted  $Dag_{wm}$ . This graph corresponds to the relation between  $R_i$  and  $R_j$  from the set  $(G1)_i$ . The next step,  $R_i$  builds  $Dag_{pg}$  by adding the directed edges imposed by the pathological intersection situations, explained in section V. After having established the imposed wait-for relations,  $R_i$  adds the directed edges that result from resolving the conflicts according to a specified policy. The conflicting robots build the directed acyclic graph  $Dag_{wait}$  in a consistent manner.

- According to the graph  $Dag_{wait}$ ,  $R_i$  determines  $Before_i$  the set of robots that  $R_i$  waits for.  $Before_i = (G1)_i \cup \text{subset}((G2)_i)$ .  $R_i$  dynamically updates the set  $After_i$  by adding robots  $R_k$  that does not belong to  $G_i$  and whose requested zone  $Z_k$  intersects with  $Z_i$ . ( $R_i$  sends to  $R_k$  the message  $msg_i = (\text{WAITFORME}, i, Z_i)$ ).  $R_i$  keeps updating the set  $After_i$  until  $R_i$  releases  $Z_i$ .
  - $R_i$  waits until it receives a release message from each robot in the set  $Before_i$ .
- 3) Reservation phase: When  $R_i$  has received a release message from all the robots of the set  $Before_i$ , or (when the set  $Before_i$  is empty),  $R_i$  reserves  $Z_i$  and becomes the owner of  $Z_i$ .
  - 4) Release phase: When  $R_i$  reaches the post-motion zone  $post(Z_i)$ , it releases  $Z_i$  except the place that  $R_i$  occupies.  $R_i$  multicasts a release message to all the robots that belong to the set  $After_i$ . These robots are within one communication hop with respect to  $R_i$ , due to the reservation range property. Therefore, the robots of the set  $After_i$  can receive the release message of  $R_i$ .

### C. Imposed wait-for relations

The imposed wait-for relations are the WAITFORME relations in addition to the wait-for relations imposed by the pathological intersection situations.

**WAITFORME\_Handler:** The input of the WAITFORME Handler is the dependency set  $Depend_i$ , and the output is the directed acyclic graph  $Dag_{wm}$ . This handler generates  $Dag_{wm}$  by establishing the imposed wait-for directed edges that correspond to the situation where  $R_i$  must wait for  $R_j$  because  $R_j$  is a member of the set  $(G1)_i$ . The relation WAITFORME is transitive, so if a robot  $R_i$  must wait for  $R_j$  and the robot  $R_j$  must wait for  $R_k$ , then  $R_i$  must wait for  $R_k$ . Therefore, no cycles can be created in the graph  $Dag_{wm}$ .

### D. Pathologic\_Detector

The pathological intersection situations discussed in section V lead to a deadlock situation or potentially to a starvation situation. Consequently, there exist pathological intersection situations between two zones  $Z_i$  and  $Z_j$  that

<sup>4</sup>The dependency set is piggybacked with the messages of type WAITFORME.  $R_i$  computes the dependency set when it does not receive any WAITFORME message.



**Algorithm 1** Collision prevention protocol (Code for robot  $R_i$ )

```

1: Initialization:  $G_i := \emptyset$ ;  $Before_i := \emptyset$ ;  $After_i := \emptyset$ ;  $Neighbor_i := \emptyset$ ;
    $Requested\_zone_i := \perp$ ;  $Request\_pending_i := false$ ;

2: when receive (REQUEST,  $k$ ,  $Z_k$ ) from  $R_k$ 
3:   if  $Request\_pending_i$  then
4:     if  $R_k \notin Neighbor_i$  and  $Z_k \cap Z_i \neq \emptyset$  then
5:       Send(WAITFORME,  $i$ ,  $Z_i$ ) to  $R_k$     $\{R_i$  piggybacks the
         wait-for graph  $Dag_{wait}\}$ 
6:        $After_i = After_i \cup \{R_k\}$     $\{R_i$  keeps updating the set
          $After_i$  until  $R_i$  releases  $RelZone_i\}$ 
7:     end if
8:     if  $Z_k \cap Z_i = \emptyset$  then
9:       Send(ACK,  $i$ ) to  $R_k$ 
10:    end if    $\{If R_k \in Neighbor_i$  and  $Z_k \cap Z_i \neq \emptyset$ , then the
      Pathologic_Detector or the Conflict_Resolver handles it.}
11:   else
12:     if  $Requested\_zone_i = \perp$  then
13:       if  $Z_k \cap pre(Z_i) \neq \emptyset$  then
14:         Send(PROHIBITED,  $i$ ,  $pre(Z_i)$ ) to  $R_k$ 
15:       else
16:         Send(ACK,  $i$ ,  $\perp$ ) to  $R_k$ 
17:       end if
18:     end if
19:   end when

21: procedure Request( $Z_i$ )
22:    $Requested\_zone_i := Z_i$ 
23:   Phase 1:
24:      $Neighbor_i := NDiscover()$     $\{Neighborhood Discovery\}$ 

25:   Phase 2:
26:     multicast (REQUEST,  $i$ ,  $Z_i$ ) to  $Neighbor_i$     $\{Negotiation\}$ 
27:     wait until receive response from all  $R_j \in Neighbor_i$ 
28:     build the set  $G_i := (R_j, Z_j)$  such that  $R_j \in Neighbor_i$  and
        $Z_j$  intersects with  $Z_i$ .
29:      $Depend_i := Dependency$  set    $\{Depend_i$  is received with a
       WAITFORME message.}
        $\{If R_i$  does not receive any WAITFORME message, then
        $Depend_i$  is computed using the echo algorithm.}

30:      $Dag_{wm} := WAITFORME\_Handler(Depend_i)$ 
31:      $Dag_{pg} := Pathologic\_Detector(Dag_{wm}, Depend_i)$ 
32:      $Dag_{wait} := Conflict\_Resolver(Dag_{pg}, Depend_i, policy)$ 
33:      $Request\_pending_i := true$ 
34:     build the set  $Before_i$  and update the set  $After_i$  according to
       the directed acyclic graph  $Dag_{wait}$ 
35:     if  $Before_i \neq \emptyset$  then
36:       when receive (RELEASE,  $j$ ,  $Z_j$ ) from  $R_j \in Before_i$ 
37:          $G_i := G_i \setminus \{R_j, Z_j\}$     $\{R_i$  removes the entry of  $R_j$ 
           from the set  $G_i\}$ 
38:       end when  $\{receive the release message from all  $R_j$  of the
         set  $Before_i\}$$ 
39:     end if
40:   end Request

41: Phase 3:
42:   reserve( $Z_i$ )    $\{R_i$  reserves the zone  $Z_i\}$ 

43: procedure Release( $Z_i$ )
44:   Phase 4:
45:     when  $R_i$  reaches the post-motion zone  $post(Z_i)$ 
46:       if  $After_i \neq \emptyset$  then
47:         multicast( $RelZone_i$ ) to  $After_i$     $\{release(R_i,$ 
            $RelZone_i)\}$ 
            $\{R_i$  multicasts a release message to all  $R_j$  of the set
            $After_i\}$ 
48:       end if
49:     end when
50:   end Release

```

**Algorithm 2** WAITFORME\_Handler algorithm

```

1: function WAITFORME_Handler ( $Depend_i$ )
2:   for all  $(R_x, R_y) \in Depend_i$  do
3:     if  $R_x$  must wait for  $R_y$  (WAITFORME) then
4:        $Dag_{wm} := Dag_{wm} \cup DirEdge(R_x, R_y)$     $\{R_x$  must wait
         for  $R_y$ , because  $R_y \in G_x$  but  $R_x \notin G_y\}$ 
5:     end if
6:     if  $DirEdge(R_x, R_y)$  and  $DirEdge(R_y, R_z)$  then
7:        $Dag_{wm} := Dag_{wm} \cup DirEdge(R_x, R_z)$     $\{The relation$ 
         WAITFORME is transitive}
8:     end if
9:   end for
10:  return  $Dag_{wm}$ 
11: end

```

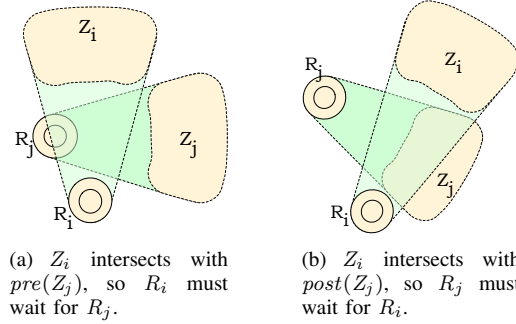


Figure 5. Pathological intersection situations impose wait-for relations between  $R_i$  and  $R_j$ .

impose certain wait-for relations between the requesting robots. The imposed wait-for relations are presented as follows.

- 1)  $[Z_i \cap pre(Z_j) \neq \emptyset]$  and  $[post(Z_i) \cap post(Z_j) = \emptyset] \Rightarrow R_i$  must wait-for  $R_j$ .
- 2)  $[Z_i \cap post(Z_j) \neq \emptyset]$  and  $[post(Z_i) \cap post(Z_j) = \emptyset] \Rightarrow R_j$  must wait-for  $R_i$ .

Figure 5 illustrates the imposed wait-for relations due to pathological intersection situations. Figure 5(a) illustrates that  $R_i$  must wait for  $R_j$ , and Figure 5(b) illustrates that  $R_j$  must wait for  $R_i$ .

The input of the Pathologic\_Detector is the dependency set  $Depend_i$  and the graph  $Dag_{wm}$ . It outputs the graph  $Dag_{pg}$  by adding the directed edges according to the imposed wait-for relations due to the two previous pathological intersection situations.

If a cycle is created by adding a directed edge, then the Pathologic\_Detector calls the Deadlock\_Resolution policy. The cycle is created because of the two possible pathological situations.

- A pathological intersection case which leads to a deadlock situation between  $n$  robots ( $n > 2$ ). In case of three robots, for example,  $R_a$ ,  $R_b$ , and  $R_c$ .  $Z_a$  intersects with  $pre(Z_b)$ ,  $Z_b$  intersects with  $pre(Z_c)$ , and  $Z_c$  intersects with  $pre(Z_a)$ . It is the general form of the deadlock situation.
- $Z_i$  intersects with  $post(Z_j)$  and  $R_i$  must wait for  $R_j$  because of an imposed wait-for relation.

When the Pathologic\_Detector detects a deadlock or a starvation situation, then it calls the Deadlock\_Resolution policy, which handles the deadlock or the starvation situation. If the Deadlock\_Resolution policy does not find

a solution then an exception is raised by the protocol. The Deadlock\_Resolution policy is discussed in subsection IV-F. Therefore, the graph  $Dag_{pg}$  has no cycles.

### E. Conflict\_Resolver

The *Conflict\_Resolver* breaks ties and determines the *wait-for* relation between two conflicting robots according to a conflict resolution *policy*, if there is no imposed *wait-for* relation between the two robots. A conflict resolution policy can be as follows.  $R_i$  *waits-for*  $R_j$  if the number of the previous requested zones by  $R_i$  is higher than that of  $R_j$ . The conflict resolution policy is specified by the robotic application. For example, the robot farther to the intersection zone *waits-for* the closer one, and in case of an equidistance situation, their identifiers are used to break the symmetry. The Conflict\_Resolver generates the graph  $Dag_{wait}$  by breaking ties between each pair of the robots of the dependency set  $Depend_i$ . The graph  $Dag_{wait}$  is generated in a consistent manner, such that each robot of the set  $Depend_i$  generates the *same* graph  $Dag_{wait}$  starting from the graph  $Dag_{pg}$  by adding the directed edges representing the *wait-for* relations after resolving the conflict between each pair of the conflicting robots. The dependency set is scanned according to the increasing order of the identifiers of robots and the conflict resolution policy is applied. If adding a directed edge creates a cycle, then the new directed edge is reversed to break the cycle.

---

#### Algorithm 3 Conflict\_Resolver algorithm

---

```

1: function Conflict_Resolver ( $Dag_{pg}$ ,  $Depend_i$ , policy)
2:    $Dag_{wait} := Dag_{pg}$ 
3:   for each robot's identifier  $x$  from MINID to MAXID such that
      $R_x \in Depend_i$  do
4:     for each robot's identifier  $y > x$  to MAXID such that  $R_y$ 
        $\in Depend_i$  do
5:       if Conflict( $R_x$ ,  $R_y$ ) and no edge ( $R_x$ ,  $R_y$ ) then
6:         DirEdge( $R_x$ ,  $R_y$ ) := policy( $R_x$ ,  $R_y$ ) {apply the conflict
           resolution policy}
7:          $Dag_{wait} := Dag_{wait} \cup$  DirEdge( $R_x$ ,  $R_y$ )
8:         if DetectCycle then
9:           DirEdge( $R_x$ ,  $R_y$ ) := DirEdge( $R_y$ ,  $R_x$ ) {If a cycle is
             detected, then inverse the direction of the edge}
10:        end if
11:      end if
12:    end for
13:  end for
14:  return  $Dag_{wait}$ 
15: end

```

---

### F. Deadlock\_Resolution policy

The Deadlock\_Resolution policy handles deadlock and starvation situations detected by the Pathologic\_Detector. The policy used to resolve a deadlock or a starvation situation is based on a *Request Preemption* strategy. For a deadlock situation between two requests ( $R_i$ ,  $Z_i$ ) and ( $R_j$ ,  $Z_j$ ), the Deadlock\_Resolution policy preempts one of the two requests in a deterministic manner. If the request ( $R_i$ ,  $Z_i$ ) is preempted, then  $R_i$  cancels its request of the zone  $Z_i$ .

$R_i$  retries its request ( $R_i$ ,  $Z_i$ ) at a later time, by restarting the protocol for the same zone  $Z_i$ . If the request is still preempted after a certain number of retrials, then the protocol raises an exception.

The upper layer (e.g., motion planning layer) may then catch the exception and applies some alternative strategy. For instance, in the case of motion planning, the strategy may simply consist in retrying with an alternate route. Of course, if no alternative strategy is available, then that upper layer may itself propagate the exception to higher layers.

### G. Optimizations and discussion

Pipelining of requests. Actually, in the collision prevention protocol, a robot  $R_i$  starts to request the next zone when it reaches the post-motion zone  $post(Z_i)$ . A possible optimization of the protocol performance can be achieved as follows.  $R_i$  starts to request the next zone when it reserves  $Z_i$ . So,  $R_i$  performs the negotiation for the next zone in parallel while moving inside the reserved zone  $Z_i$ .

The Deadlock\_Resolution policy applies an application-based strategy in order to resolve deadlock and starvation situations. The performance of the protocol depends on the number of retrials of the same zone and also on the time durations between the retrials.

A robot can know, without any additional cost, the parameters of the requested zones in its local neighborhood region due to the multicast and antenna properties in wireless networks. So, if possible, the motion planning layer can plan an alternative chunk of a robot's path that avoids highly contended areas.

## V. DEADLOCK AND STARVATION SITUATIONS

There are pathological intersection cases between two zones  $Z_i$  and  $Z_j$ , such that neither  $R_i$  nor  $R_j$  can be granted its requested zone (deadlock situation). If a robot  $R_i$  could not be granted its requested zone  $Z_i$ , we say that robot  $R_i$  starves. (starvation situation)

Certain pathological intersection cases constitute necessary conditions but not sufficient for a potential starvation situation.

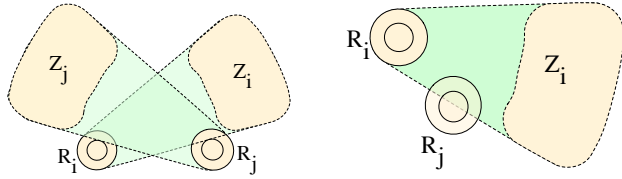
*Definition 1 (Deadlock situation):* We say that robot  $R_i$  and robot  $R_j$  are in a deadlock situation when neither  $R_i$  can be granted  $Z_i$  nor  $R_j$  can be granted  $Z_j$ , due to a pathological intersection between the two zones  $Z_i$  and  $Z_j$ . This pathological intersection occurs if the requested zone  $Z_i$  intersects with the *pre-motion zone*  $pre(Z_j)$  and the requested zone  $Z_j$  intersects with the *pre-motion zone*  $pre(Z_i)$ .

The pathological intersection case which leads to a deadlock situation between two robots  $R_i$  and  $R_j$  is the following.

Pathological situation 1 ( $Z_i$ ,  $Z_j$ ): [ $Z_i \cap pre(Z_j) \neq \emptyset$ ] and [ $Z_j \cap pre(Z_i) \neq \emptyset$ ].

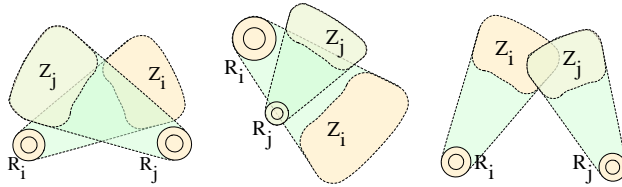
The deadlock situation is illustrated in Figure 6(a).





(a) Pathological situation 1, so a deadlock situation between robots  $R_i$  and  $R_j$ .  $Z_i \cap pre(Z_j) \neq \emptyset$  and  $Z_j \cap pre(Z_i) \neq \emptyset$ .  
 (b) Pathological situation 5,  $Z_i \cap pre(Z_j) \neq \emptyset$  and  $R_j$  does not request a zone, so  $R_i$  starves.

Figure 6. Deadlock and starvation situations.



(a) Pathological situation 2 : Each zone intersects with the post-motion of the other zone.  
 (b) Pathological situation 3 :  $Z_i$  intersects with both  $pre(Z_j)$  and  $post(Z_j)$ .  
 (c) Pathological situation 4 : The post-motion zones intersect.

Figure 7. Pathological intersection situations. The necessary conditions that potentially lead to a starvation.

**Definition 2 (Starvation situation):** We say that robot  $R_i$  starves when  $Z_i$  cannot be granted to  $R_i$ . The starvation of  $R_i$  is due to a pathological intersection case. This pathological intersection occurs if  $Z_i$  intersects with the pre-motion zone  $pre(Z_j)$  and  $R_j$  does not request a zone.

Figure 6(b) illustrates the starvation situation for robot  $R_i$ .

The pathological intersection situations that *potentially* lead to a starvation situation are the following.

- Pathological situation 2 ( $Z_i, Z_j$ ):  
 $[Z_i \cap post(Z_j) \neq \emptyset]$  and  $[Z_j \cap post(Z_i) \neq \emptyset]$  and  $[post(Z_i) \cap post(Z_j) = \emptyset]$   
 Pathological situation 3 ( $Z_i, Z_j$ ):  
 $[Z_i \cap pre(Z_j) \neq \emptyset]$  and  $[Z_i \cap post(Z_j) \neq \emptyset]$  and  $[post(Z_i) \cap post(Z_j) = \emptyset]$   
 Pathological situation 4 ( $Z_i, Z_j$ ):  
 $post(Z_i) \cap post(Z_j) \neq \emptyset$

The pathological situation that leads to a starvation situation for  $R_i$  is the following.

Pathological situation 5:  $Z_i \cap pre(Z_j) \neq \emptyset$  and  $R_j$  does not request a zone.

## VI. PROOF OF CORRECTNESS

We prove that the collision prevention protocol satisfies the *Safety* and the *Liveness* properties.

**Lemma 1:** The wait-for graph  $Dag_{wait}$  has no cycles.

*Proof:* The wait-for graph  $Dag_{wait}$  is based on  $Dag_{wm}$  and  $Dag_{pg}$ .

- The graph  $Dag_{wm}$  is a directed acyclic graph, since the WAITFORME relation is transitive. if a robot  $R_x$  must wait for  $R_y$  and the robot  $R_y$  must wait for  $R_z$ ,

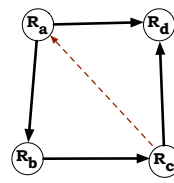


Figure 8. A directed edge ( $R_c, R_a$ ) is added.

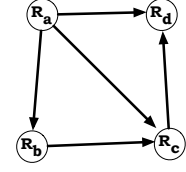


Figure 9. The direction is reversed, so the directed edge is replaced by ( $R_a, R_c$ ).

then  $R_x$  must wait for  $R_z$ . (Algorithm 2, Line 7). Therefore, the graph  $Dag_{wm}$  has no cycles.

- The graph  $Dag_{pg}$  has no cycles, because if a cycle is created, then the Deadlock.Resolution policy is called to break the cycle. So, the graph  $Dag_{pg}$  has no cycles.
- We prove that the graph  $Dag_{wait}$  is a directed acyclic graph.  $Dag_{wait}$  is generated starting from  $Dag_{pg}$  which is a directed acyclic graph. If adding a directed edge to  $Dag_{wait}$  creates a cycle, then the direction is reversed. We prove that reversing the direction of the edge does not create a cycle and hence the wait-for graph  $Dag_{wait}$  is a directed acyclic graph.

For a graph with no cycles that consists of three vertices  $\{R_a, R_b, R_c\}$ , if adding a directed edge creates a cycle, then it is obvious that reversing the direction of the edge does not create a cycle. For a graph with no cycles that consists of more than three vertices, the proof proceeds by contradiction.

Figure 8 shows that when the directed edge ( $R_c, R_a$ ) is added to  $Dag_{wait}$ , it creates the cycle ( $R_a, R_b, R_c, R_a$ ). So, the graph has already the directed edges: ( $R_a, R_b$ ) and ( $R_b, R_c$ ). Figure 9 shows that the edge ( $R_c, R_a$ ) is replaced by ( $R_a, R_c$ ).

Let us assume that the directed edge ( $R_a, R_c$ ) creates a cycle ( $R_a, R_c, R_d, R_a$ ). So, the graph has already the directed edges: ( $R_c, R_d$ ) and ( $R_d, R_a$ ). Consequently, the graph has already the cycle ( $R_a, R_b, R_c, R_d, R_a$ ) (the graph has already a cycle), which leads to a contradiction. Therefore, if adding a directed edge creates a cycle, then reversing the direction would not create a cycle in  $Dag_{wait}$ .

Therefore, the wait-for graph  $Dag_{wait}$  has no cycles.

**Lemma 2:** The wait-for relations between the robots related by the transitive closure of the relation *conflict*, are generated consistently, so the robots build the same wait-for graph  $Dag_{wait}$ .

*Proof:* The set  $Depend_i$  consists of the union of  $G_k$  for each robot  $R_k$  related to  $R_i$  by the transitive closure of the relation *conflict*, so  $Depend_i$  equals to  $Depend_k$ .

The robots that  $R_i$  conflicts with, belong to  $G_i$  or to  $After_i$ . We prove that the set  $G_i$  is sufficient to build the wait-for graph  $Dag_{wait}$  consistently.

Let us consider three conflicting robots  $R_a, R_b$  and  $R_c$ , such that each zone intersects with the two other zones. Let us assume that the set  $G_a$  contains  $R_b$ , but does not contain  $R_c$ , ( $R_c \in After_a$ ). Assume that the set  $G_b$  contains both  $R_a$  and  $R_c$ . When the dependency

set  $Depend_b$  is determined, then  $R_b$  deduces the wait-for relation between  $R_a$  and  $R_c$  and that  $R_c$  waits for  $R_a$ , since the  $Z_a$  intersects with  $Z_c$  and  $R_c \notin G_a$ .

If  $R_a$  receives the set  $G_b$  (due to the dependency set  $Depend_a$ ) before  $R_a$  receives the request message of  $R_c$ , then  $R_a$  deduces that a request message of  $R_c$  eventually arrives, and that  $R_c$  belongs to the set  $After_a$ , since  $Z_a$  intersects with  $Z_c$ .

If  $R_c \notin G_a$  and  $R_c \notin G_b$ , then  $R_c \in After_a$  and  $R_c \in After_b$ , so  $R_c$  waits for both  $R_a$  and  $R_b$ .

The wait-for graph  $Dag_{wait}$  is generated based on the set  $Depend_i$ , by applying a sequence of deterministic functions. The graph  $Dag_{wm}$  is generated according to the imposed wait-for relation WAITFORME. The graph  $Dag_{pg}$  is generated starting from the graph  $Dag_{wm}$  according to the imposed wait-for relations of the pathological situations.

The Conflict.Resolver defines a deterministic function (*policy*) to break ties between two conflicting robots, starting from the graph  $Dag_{pg}$  and the set  $Depend_i$  which is scanned according to the increasing order of the robots identifiers. Therefore, the wait-for graph  $Dag_{wait}$  is generated consistently, and the robots that are related by the transitive closure of the relation conflict build the same wait-for graph.

**Theorem 1 (Mutual Exclusion):** If a zone  $Z_i$  of a robot  $R_i$  intersects with a zone  $Z_j$  of a robot  $R_j$ , then either  $R_i$  or  $R_j$  (but not both) is the owner of its zone.

*Proof:* If  $Z_i$  intersects with  $Z_j$ , then  $R_i$  and  $R_j$  are within the transmission range of each other, (reservation range property), thus  $R_i$  and  $R_j$  can communicate.

Let us assume that  $R_j \in G_i$ .

- If  $R_i \notin G_j$ , then  $R_i$  must wait for  $R_j$  (WAITFORME relation). If  $Z_i$  intersects with  $post(Z_j)$ , then this situation is detected by the Pathologic\_Detector and the request  $(R_i, Z_i)$  is preempted.
- If  $R_i \in G_j$  and there is no deadlock situation between  $R_i$  and  $R_j$ , then the wait-for relation is determined either by the Pathologic\_Detector if there is an imposed wait-for relation due to pathological situations, or by the Conflict.Resolver. If there is a deadlock situation between  $Z_i$  and  $Z_j$ , then one of the requests is deterministically selected and preempted.

Consequently, there is a wait-for relation between  $R_i$  and  $R_j$ . According to Lemma 2 the wait-for relations between conflicting robots are generated consistently, so  $R_i$  and  $R_j$  establish the same wait-for relation and either  $R_i$  waits for  $R_j$  or  $R_j$  waits for  $R_i$ .

Let us assume that  $R_i$  waits for  $R_j$ . When  $R_j$  releases  $RelZone_j$ , then  $R_i$  owns  $Z_i$ . When the robot  $R_i$  is the owner of  $Z_i$ , the robot  $R_j$  is deprived from the ownership of  $Z_j$ . The robot  $R_j$  just keeps a part of  $post(Z_j)$  under its reservation.  $Z_i$  does not intersect with the part of  $post(Z_j)$  that is still owned by  $R_j$ , because:

- 1)  $pre(Z_i) \cap post(Z_j) = \emptyset$  (Proof by contradiction).  
If  $pre(Z_i)$  intersects with  $post(Z_j)$ , then  $R_j$  had to wait for  $R_i$  according to the imposed wait-for

relations. This leads to a contradiction, since we assume that  $R_i$  has wait for  $R_j$ .

- 2)  $motion(Z_i) \cap post(Z_j) = \emptyset$  (Proof by contradiction). If the *motion* zone of  $R_i$  intersects with the *post-motion* zone of  $R_j$ , then  $R_j$  had to wait for  $R_i$ .
- 3)  $post(Z_i) \cap post(Z_j) = \emptyset$  (Proof by contradiction). If the *post-motion* zones intersect, then the situation is the pathological situation 4. This leads to a contradiction.

Consequently, the *Safety* property holds.

**Theorem 2 (Liveness):** If a robot  $R_i$  requests a zone  $Z_i$ , then eventually ( $R_i$  owns  $Z_i$  or an exception is raised).

*Proof:* If a robot  $R_i$  requests a zone  $Z_i$ , then:

- 1) If  $Z_i$  does not intersect with a zone  $Z_j$ , then  $R_i$  owns  $Z_i$ .
- 2) If  $Z_i$  intersects with a zone  $Z_j$ , then a wait-for relation is established between  $R_i$  and  $R_j$  and a directed edge is added to the wait-for graph  $Dag_{wait}$ . According to Lemma. 1 the graph  $Dag_{wait}$  has no cycles. Therefore,  $R_i$  eventually owns  $Z_i$ .
- 3) If a deadlock or a starvation situation is detected, then the Deadlock\_Resolution policy is called. If the Deadlock\_Resolution policy fails to resolve the situation, then an exception is raised.
- 4) Robot  $R_j \in After_i$  eventually receives the release message of  $R_i$  when  $R_i$  reaches  $post(Z_i)$ . Because,  $R_i$  multicasts a release message to all the robots that belong to the set  $After_i$  and to robots  $R_a$  such that  $R_i \in Neighbor_a$  and the request message  $msg_a = (REQUEST, a, Z_a)$  of  $R_a$  has not yet been received by  $R_i$ . When  $R_i$  reaches the *post-motion* zone, the robots of the set  $After_i$  and the robots  $R_a$  are within one communication hop with respect to  $R_i$  (due to the reservation range property). Hence the robots of the set  $After_i$  and the robots  $R_a$  can receive the release message of  $R_i$ .

Consequently, the *liveness* property holds.

## VII. PERFORMANCE ANALYSIS

We study the performance of our protocol in terms of the time needed by a robot  $R_i$  to reach a given destination when robots are active (robots do not sleep). We compute the average effective speed of robots executing our collision prevention protocol. We provide insights for a proper dimensioning of system parameters in order to maximize the average effective speed of the robots. For simplicity, we assume in this section that the physical dimensions of the robots are very small, such that a robot can be considered as a point in the plane. The geometrical uncertainty related to the positioning system, translational and rotational movements are neglected. Consider a set of robots, each one moving along a chunk (line segment) of length equal to the reservation range  $D_{ch}$ .

A. Time needed to reserve and move along a chunk

The average physical speed of a robot is denoted:  $V_{mot}$ . We calculate<sup>5</sup> the average time required for a robot  $R_i$  to reserve and move along a chunk of length  $D_{ch}$  with a physical speed  $V_{mot}$ .

*Number of robots to wait for:* The total number of robots  $n_{avg}$  that  $R_i$  waits for to reserve a chunk is:

$$n_{avg} = \frac{1}{1 - \frac{n_{reg}}{\pi(\pi+2)}} - 1, \quad n_{reg} < \pi(\pi+2) \approx 16 \quad (1)$$

where,  $n_{reg}$  is the number of robots in the region  $reg$ , which is the region of possible collisions for a robot  $R_i$  that moves along a line segment of length  $D_{ch}$ .

*Communication delays:* In order to evaluate the performance of the protocol, we need to consider the average of communication delays in the system, although the protocol is time-free. The average communication delays is denoted:  $T_{com}$ . The delay of the neighborhood discovery primitive  $NDiscover$  is denoted:  $T_{nd}$ . The time  $T_{ch}$  needed to reserve and move along a chunk is the following:

$$T_{ch} = T_{nd} + 2n_{avg}T_{com} + n_{avg}\left(T_{com} + \frac{D_{ch}}{V_{mot}}\right) + \frac{D_{ch}}{V_{mot}} \quad (2)$$

The optimal time  $T_{ch}$  for a robot  $R_i$  is when it is alone, so there are no robots in the region  $reg$ . In this case, the time  $T_{ch}$  is:  $T_{ch}(alone) = T_{nd} + \frac{D_{ch}}{V_{mot}}$ .

B. Optimal reservation range

We compute the average effective speed  $V$  of a robot  $R_i$  as a function of the reservation range and the density of robots in the system (the density is denoted:  $s$ ), then we determine an optimal value of the reservation range that maximizes the average effective speed of  $R_i$  for a given value of the density of robots. In our protocol the reservation range is a constant parameter given by the system.

$$V = \frac{-sD_{ch}^3 + \pi D_{ch}}{(3T_{com} - T_{nd})sD_{ch}^2 + \frac{\pi}{V_{mot}}D_{ch} + \pi T_{nd}}, \quad D_{ch} < \frac{\sqrt{\pi}}{\sqrt{s}} \quad (3)$$

The previous relation shows that the effective speed is a function of the reservation range and the density of robots, and also that the average effective speed depends on some system-based fixed parameters such as the average communication delays and the physical speed of robots. Figure 10(a) presents the relationship between the speed and the reservation range for different densities. The values of density extend from zero ( $R_i$  is alone) to  $3[robots/m^2]$ . Figure 10(a) shows the optimal reservation range for a given density. The value of the optimal reservation range maximizes the average effective speed of the robots. The curve that corresponds to the density zero (when robot is alone), in Figure 10(a), shows that the effective speed always increases as the reservation range increases, until the effective speed  $V$  approaches

the physical speed  $V_{mot}$  when the value of the reservation range becomes very large. The curve has a horizontal asymptote at  $V = V_{mot} = 1[m/s]$ . The effective speed of  $R_i$  depends on the reservation range, even in the case when  $R_i$  is alone, because it needs to perform a certain number of steps to reach a destination, and the number of steps is a function of the reservation range. When the reservation range increases, the number of steps decreases. The relation between the effective speed and the reservation range (when  $R_i$  is alone), is the following:  $V = \frac{D_{ch}}{\frac{D_{ch}}{V_{mot}} + T_{nd}}$ . In each step,  $R_i$  needs  $T_{nd}$  time units to discover that it is alone. If  $D_{ch}$  approaches infinity, then  $V$  approaches  $V_{mot}$ .

*Numerical values.:* The values of the fixed system parameters are:  $T_{com} = 10[ms]$ ,  $T_{nd} = 1[s]$ , the physical speed  $V_{mot} = 1[m/s]$ . For a density  $s = 0.3[robot/m^2]$ , the optimal reservation range is  $\approx 1.53 [m]$ , which gives a maximal speed  $\approx 0.51 [m/s]$ .

C. Speed vs density of robots

The average effective speed always decreases when the density of robots increases for a given reservation range. Figure 10(b) presents the relationship between the average effective speed and the density for different values of the reservation range, (from  $0.7[m]$  to  $2[m]$ ).

VIII. RELATED WORK

Martins et al. [8] demonstrated the avoidance of collisions between three cars, elaborated in the CORTEX project. They rely on the coexistence of two networks, as defined in the Timely Computing Base of Verissimo and Casimiro [13]. One network, the payload network, is asynchronous and carries the information payload of the application. The second network, the control network or wormhole, enforces strict real-time guarantees and is used sparingly by the protocol. Although the system allows for deadlines to be adapted dynamically (called time-elastic), their approach differs from ours because the use of time remains explicit in their protocol. A second difference is that we assume that robots do not know the existence of other robots that are not in their local neighborhood. This said, our neighborhood discovery primitive has a role similar to the wormhole of Martins et al. in that it encapsulates the synchrony required by the protocol.

Nett et al. [9] presented a protocol for cooperative mobile systems in real-time applications. They considered a traffic control application in which a group of mobile robots share a specified predetermined space. Communication is done through WiFi (802.11) with a base station. All robots can communicate directly with each other, and the system assumes the existence of a known upper bound on communication delays. Needless to say that the protocol relies on the strict enforcement of timing assumptions.

Based on the ad hoc protocol presented in this paper, we have recently developed a simpler version [14] aimed at small groups of mobile robots, the composition of

<sup>5</sup>The details and the proofs are not presented in the paper due to space limitations; see our research report [12].

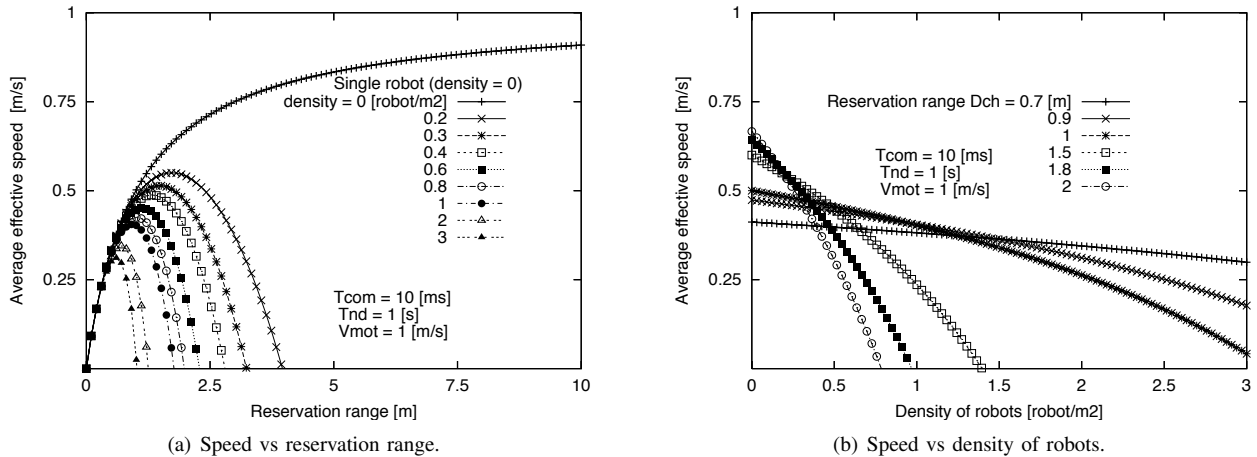


Figure 10. Average effective speed as a function of the reservation range and the density of robots.

which is static and known to all. In that simpler variant, all robots can communicate directly. In contrast, the protocol presented in this paper is more challenging as it relies on *ad hoc* communication and supports *dynamic* groups of robots.

Clark et al. [15] presented a collision avoidance based on a motion planning framework by combining centralized with decentralized motion planning techniques. When robots become within communication range of each other, they dynamically establish a network. Their protocol ensures that at any time, robots in each network share a common world model by accessing sensing information of all other robots in the same network. Robots avoid collisions by re-planning their paths. Their approach relies on proper timing of communications and robots speed.

Jager et al. [7] presented a decentralized collision avoidance mechanism based on motion coordination between robots. When the distance between two robots goes below a certain threshold, they exchange information about their respective planned paths and determine whether there is a risk of collision. If a collision is possible, then they monitor each other's movements and may change their speed to avoid the collision. The approach is highly dependent on the proper timing of communication and, to some extent, on the proper control of robots' speed. Besides, the composition of the system is static and known to all robots.

Similarly, Azarm et al. [6] presented an on line distributed motion planning. When a conflict is detected between two robots, they exchange their information and determine their respective priorities. The robot with the highest priority keeps its original path while other robots must re-plan their motion.

The problem of robots' collision avoidance has also been handled using different strategies which are sensor-based motion planning methods. The detailed information about motion planning strategies is inspired from [3].

Minguez et al. [3] compute collision-free motion for a robot operating in dynamic and unknown scenarios. Motion planning algorithms compute a collision-free path between a robot's location and its destination. Robots

involve sensing directly within the motion planning by sensing periodically at a high rate.

Some of these approaches (e.g., [4]) apply mathematical equations to the sensory information and the solutions are transformed into motion commands. Another group of methods (e.g., [5]) computes a set of suitable motion commands to select one command based on navigation strategy. Finally, other methods (e.g., [3]) compute a high-level information description (e.g., entities near obstacles, areas of free space) from the sensory information, and then apply different techniques simplifying the difficulty of the navigation to obtain a motion command in complex scenarios.

Sensor-based approaches depend on real-time guarantees for processing the sensory information. Furthermore, the information provided by proximity sensors is unreliable and much more limited in range than most wireless network interfaces.

## IX. CONCLUSION

We have presented a fail-safe platform on which cooperative mobile robots rely for their motion. This platform consists of a collision prevention protocol for a dynamic group of cooperative mobile robots with asynchronous communications. The platform ensures that no (physical) collision ever occurs between robots regardless of the respective activities of the robots.

The protocol is time-free, in the sense that it never relies on physical time, which makes it extremely robust with regard to timing uncertainty common in wireless networks. It requires neither initial nor complete knowledge of the composition of the group, and it relies on a neighborhood discovery primitive which is readily available through most wireless communication devices. Furthermore, although the transmission range may be limited, no routing is needed as robots only communicate with their local neighborhood.

We have also presented a performance analysis of the protocol, which provided insights for a proper dimensioning of system parameters in order to maximize the average

effective speed of the robots. The design of the protocol yields scalability due to its locality-preserving property. Therefore, the protocol can handle a large-sized dynamic group of cooperative mobile robots, provided with limited energy resources and limited transmission range.

If a robot crashes, then the local neighbors that are located within one communication hop with respect to the crashed robot at the time of the crash are blocked waiting for the crashed robot. The two-hop and farther neighbors that do not *conflict* with any of the blocked robots, are not affected at the time of the crash. Therefore, the impact of a crash is limited in space and affects only a part of the system for a period of time, however, the *Snowball* effect takes place with the progress of time.

In the future, we intend to further investigate and optimize the performance of the protocol, and consider the problem of a crash of a certain number of robots. In particular, we will analyze the performance of the resulting protocols using simulations and experimentation.

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