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Reasoning in Nonclassical Logics

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Formal reasoning

- Logics as **formal systems of reasoning**.
- Logics as **languages** for describing structures and relations, and for modeling.

Thus, logic gives us a basic framework for formal description and verification.

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First-order classical logic

- strong **expressive power** and **generality**, while **undecidable**.
- suitable as a language for describing mathematics, and rigorous arguments in general.

What are **nonclassical logics**, and what for?

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Nonclassical reasoning

- constructive reasoning, relevant reasoning.
- resource-sensitive reasoning.
- reasoning with uncertain or incomplete information.
- reasoning about time and space.
- dynamic aspects, logic of action.
- epistemic aspects.
- reasoning about communication.

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Nonclassical logics

- intuitionistic logic, relevant logics, linear logic.
- substructural logics.
- fuzzy logics, many-valued logics.
- temporal logics, logics of space.
- dynamic logics, belief revision.
- epistemic logics, common knowledge logics.

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Why nonclassical logics?

- usually **propositional logics**.
- specific purpose** vs **all-purpose**.
- trade-off between **decidability (low complexity)** and **expressiveness**.

Usually, these nonclassical logics are decidable. Sometimes, their expressive power is comparable with or even stronger than that of first-order classical logic.

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Temporal logics

Examples

- It will snow sometime in the future.
- He got a job but lost it later.
- Since I found a gap in my proof, I have always been nervous.

We introduce temporal operators, **F** at some future time, **P** at some past time, **S** since, **U** until and so on.

We will focus mainly on:

- computational aspects.
- richness of expressive power.

- temporal logics.
- epistemic logics and common knowledge.

Temporal connectives and models

- F** snow.
- P** (he-loses-job **P** (he-gets-job)).
- S** (I-find-gap, I-am-nervous).

Each temporal model is defined by using a **flow of time** $T, <$, where $<$ is irreflexive, transitive and often linear.

Integers $\mathbb{Z}, <$, Reals $\mathbb{R}, <$, Rationals $\mathbb{Q}, <$.

Decidability, Complexity

PTL: propositional temporal logic (with linear flows of time)

- PTL is finitely axiomatizable.
- Validity and satisfiability are decidable.

The complexity of satisfiability is:

- NP-complete if **U**, **S** omitted (Ono-Nakamura 1980 etc).
- PSPACE-complete if **U**, **S** included (Sistla-Clarke 1985 etc).

Expressive power

How expressive is temporal logic?

Each temporal formula can be translated into first-order logic. $S(\cdot, \cdot)$ is translated as:

$$w (w < t_0 \quad (w) \quad u (w < u < t_0 \quad (u)))$$

How much of first-order logic can be covered by temporal formulas?

Expressive completeness

- Essentially, all of first-order formulas are expressed by temporal formulas with **U** and **S** in any linear flow of time **without gap** (Kamp 1968).

For example, $\mathbb{Z}, <$, $\mathbb{R}, <$, but not $\mathbb{Q}, <$.

Second-order properties

Temporal formulas (and modal formulas in general) can define some of second-order properties of flows of time.

For instance, it is known that **well-orderedness** (i.e. no infinite descending chains, or existence of the minimum element in each nonempty subset) is not a first-order property. Venema (1993) gave a finite axiom system for well-ordered flows of time, using the following formula:

$$F \cup (, \neg).$$

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Decidable fragments of predicate temporal logics.

Applications:

- specification and verification of programs, reactive systems.
- knowledge representation, etc.

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Epistemic logics

An example (due to J. van Benthem)

Question. Is this the road to Kanazawa? (I)

Answer. Yes. (You)

- What information has passed?

I know it, but I didn't know it before.

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Epistemic statements

Moreover,

You know that I know it.

I know that you know I know it.

Various epistemic states can be described.

- I know your password.
- I know that you don't know I know your password.

$$K_I \text{ vs } K_I \neg K_U K_I$$

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Common knowledge

Now, both **this is the road to Kanazawa** and **I know this is the road to Kanazawa** are common knowledge between you and I.

Common knowledge is more than what all of us know.

- $K_I \quad K_U$.
- $K_U K_I, K_I K_U K_I, K_U K_I K_U K_I, \dots$

Common knowledge logics are decidable.

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Logic and communication

- From logic of single agent to logic of multi-agents

Many agents, and various kinds of information flow and action in computational process.

- computer science.
- game theory, economics.

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Research ongoing

- modal, temporal and epistemic logics — with King's College, London etc.
- theorem provers for basic modal logics — with "Logic and Computation" of NICTA.
- substructural logics, including many-valued logics, fuzzy logics and relevant logics — Project: \mathbf{A}_L (Algebra & Substructural Logics).
- algebraic methods in nonclassical logics — Barcelona Meeting 2005.