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Author(s)	Hiraishi, Kunihiko
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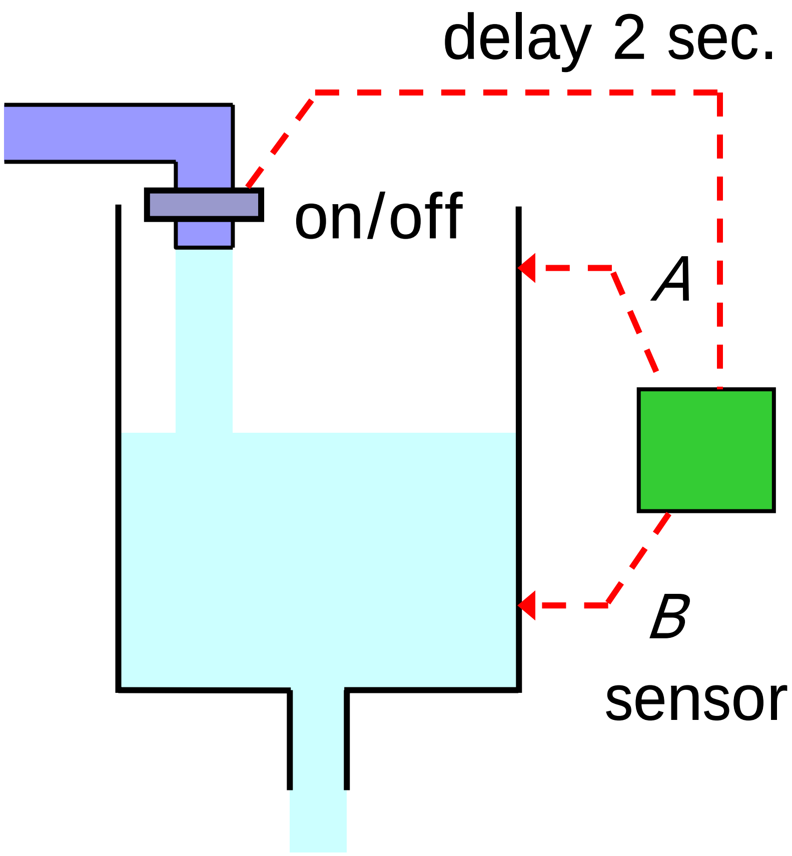
Solving Problems on Hybrid Systems by Constraint Satisfaction

Kunihiko Hiraishi

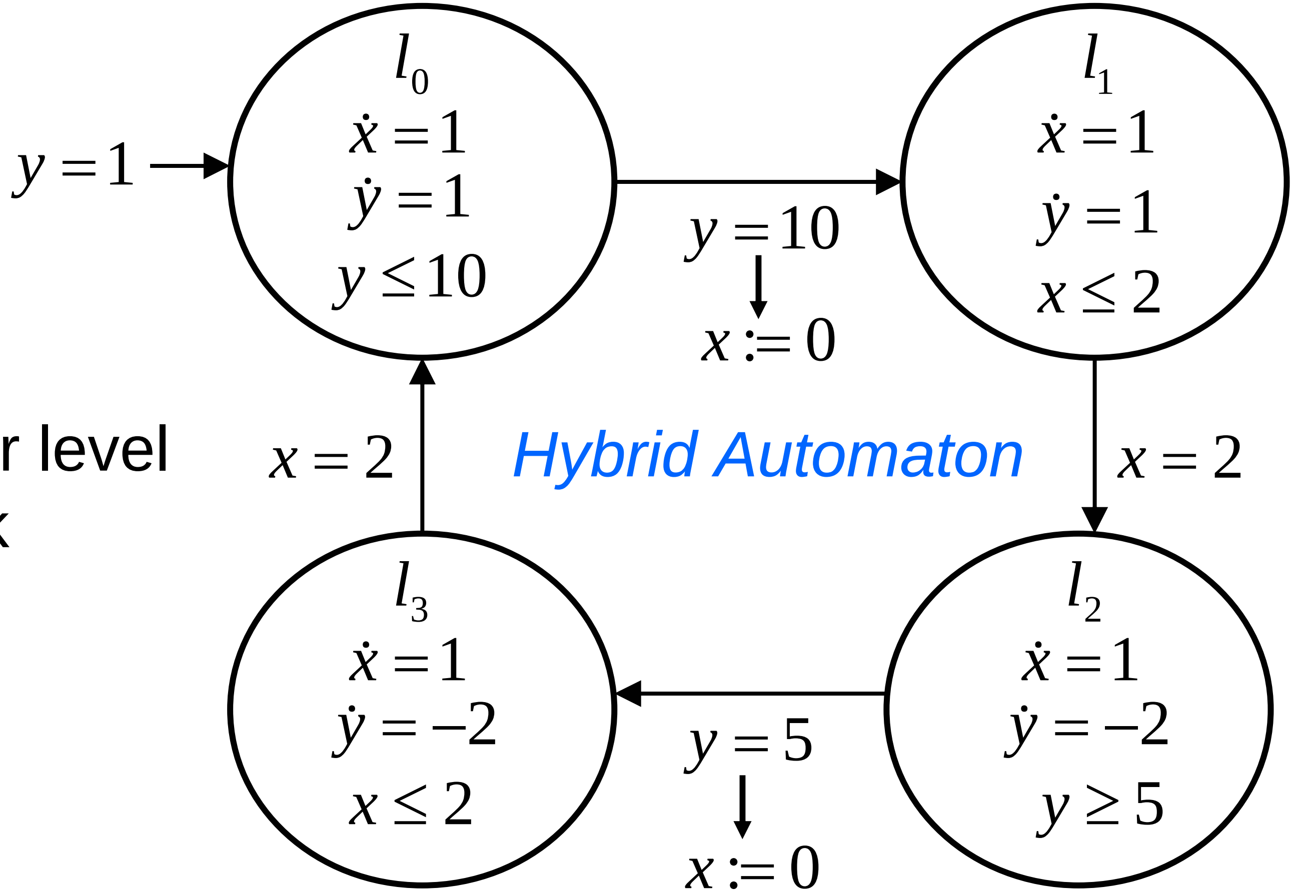
School of Information Science,

Japan Advanced Institute of Science and Technology

Example: Water-level monitor



Example: Water-level monitor



y: water level
x: clock

Computer tools for hybrid systems

- Real-time systems: KRONOS,UPPAAL, ...
- Hybrid systems: HyTech, CheckMate, d/dt, MLD (Mixed Logical Dynamical) systems with MIQP(Mixted Integer Quadratic Programming), ...

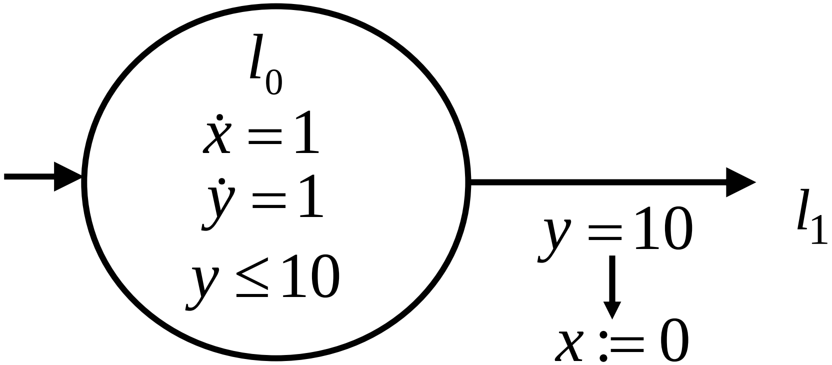
Aim and Method

- We would like to develop new computer tools for
  - optimization (including optimal control problems),
  - parameter design,
  - and also for verification and reachability analysis.
- We use symbolic computation techniques:
  - CLP (Constraint Logic Programming) → KCLP-HS
  - QE (Quantifier Elimination) → SyNRAC

KCLP-HS

- CLP = Prolog + Constraint Solver.
- Keyed CLP for HS =
  - Prolog Interpreter +
  - Linear Constraint Solver (simplex method) +
  - Linear/Quadratic Optimizer +
  - Global variables (keyed predicates) +
  - Convex Polyhedral Library (POLKA).

Modeling HA by KCLP-HS




```
l0(X, Y, Time):-
        X1 = X + D, Y1 = Y + D, D >= 0,
        Y1 = 10,
        l1(0, Y1, Time + D).
```

Modeling by CLP

```
l0(X, Y, Time):-
    X1 = X + D, Y1 = Y + D, D >= 0,
    Y1 = 10,
    l1(0, Y1, Time + D).
l1(X, Y, Time):-
    X1 = X + D, Y1 = Y + D, D >= 0,
    X1 = 2,
    l2(X1, Y1, Time + D).
l2(X, Y, Time):-
    X1 = X + D, Y1 = Y -2 * D, D >= 0,
    Y1 = 5,
    l3(0, Y1, Time + D).
l3(X, Y, Time):-
    X1 = X + D, Y1 = Y -2 * D, D >= 0,
    X1 = 2,
    l0(X, Y, Time + D).
```

Execution trace

```
(trace)| ?- l0(X, 1, 0).
1-0) CALL : l0(X, 1, 0) ?
1-0) TRY  : l0(X, 1, 0) :- _5 = X + _6, _8 = 1 + _6, _6 >= 0, _8 = 10, l1(0, _8, 0 + _6)
1-0) SUC  : l0(X, 1, 0) :- _5 = X + _6, _8 = 1 + _6, _6 >= 0, _8 = 10, l1(0, _8, 0 + _6)
1-1) CONSTRAINT : _5 = X + _6
1-1) SUC  : _5 = X + _6
1-1) CONSTRAINT : _8 = 1 + _6
1-1) SUC  : _8 = 1 + _6
1-1) CONSTRAINT : _6 >= 0
1-1) SUC  : _6 >= 0
1-1) CONSTRAINT : _8 = 10
1-1) SUC  : 10 = 10
1-1) CALL : l1(0, 10, 0 + _6) ?
1-1) TRY  : l1(0, 10, 9) :- _15 = 0 + _16, _18 = 10 + _16, _16 >= 0, _15 = 2, l2(_15, _18, 9 + _16)
1-1) SUC  : l1(0, 10, 9) :- _15 = 0 + _16, _18 = 10 + _16, _16 >= 0, _15 = 2, l2(_15, _18, 9 + _16)
1-2) CONSTRAINT : _15 = 0 + _16
1-2) SUC  : _15 = 0 + _16
1-2) CONSTRAINT : _18 = 10 + _16
1-2) SUC  : _18 = 10 + _16
1-2) CONSTRAINT : _16 >= 0
1-2) SUC  : _16 >= 0
```

Computation process

Goal

l0(X, 1, 0)

Computation process

l0(X, 1, 0)

⇕ unification

```
l0(X, Y, Time):-
        X1 = X + D, Y1 = Y + D, D >= 0,
        Y1 = 10,
        l1(0, Y1, Time + D).
```

Computation process

l0(X, 1, 0)

⇕ unification

l0(X, 1, 0):-

X1 = X + D, Y1 = 1 + D, D >= 0,

Y1 = 10,

l1(0, Y1, 0 + D).

Computation process

Goal

X1 = X + D, Y1 = 1 + D, D >= 0, Y1 = 10, l1(0, Y1, D)

Computation process

Goal

X1 = X + D, Y1 = 1 + D, D >= 0, Y1 = 10, l1(0, Y1, D)

Computation process

Constraints

l1(0, Y1, D)

$X1 = X + D,$
$Y1 = 1 + D,$
$D \geq 0,$
$Y1 = 10$

Computation process

l1(0, Y1, D)

Constraints

X1 = X + D,
Y1 = 1 + D,
$D \geq 0$,
Y1 = 10

Constraint Solver

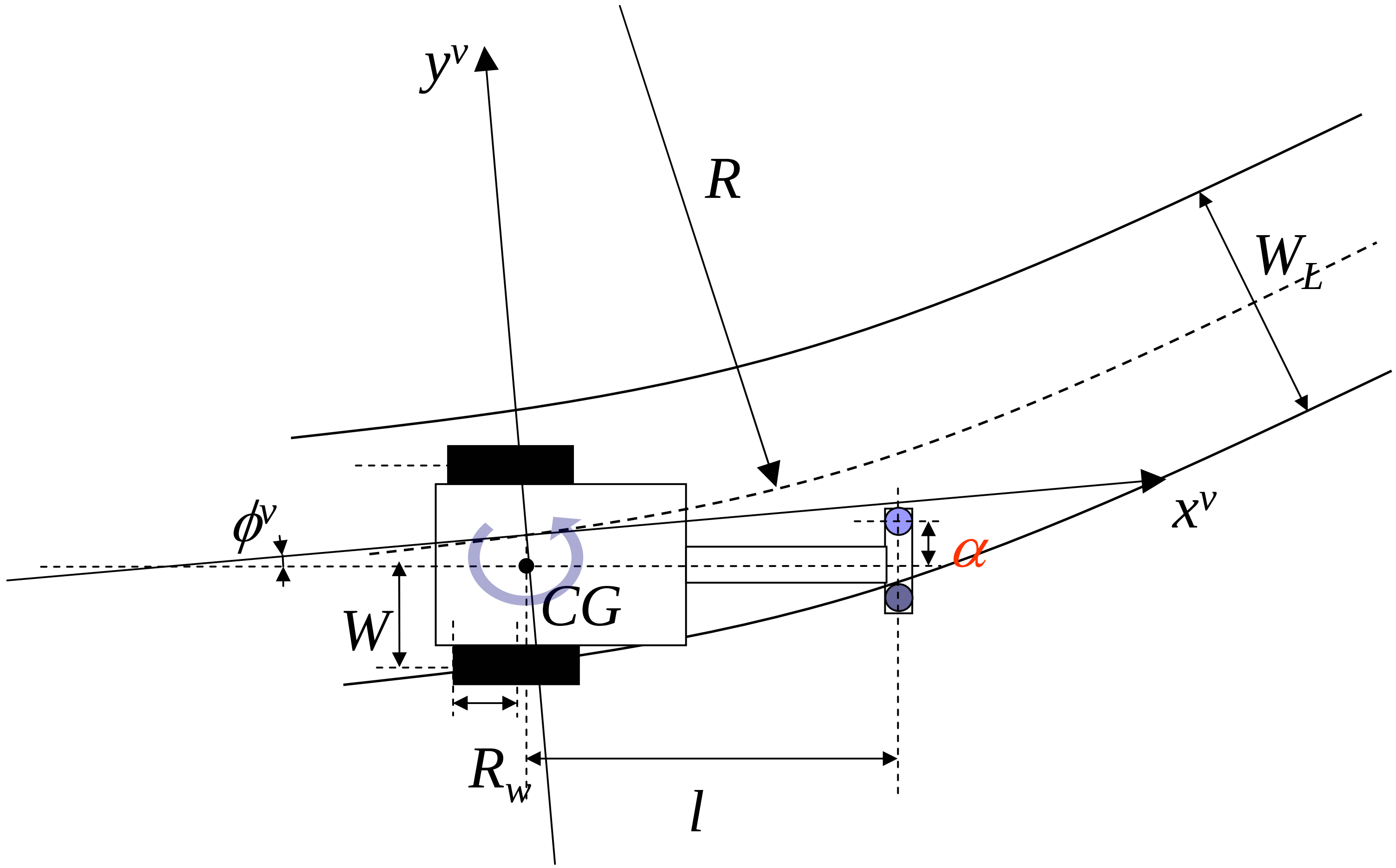
TRUE

Computation process

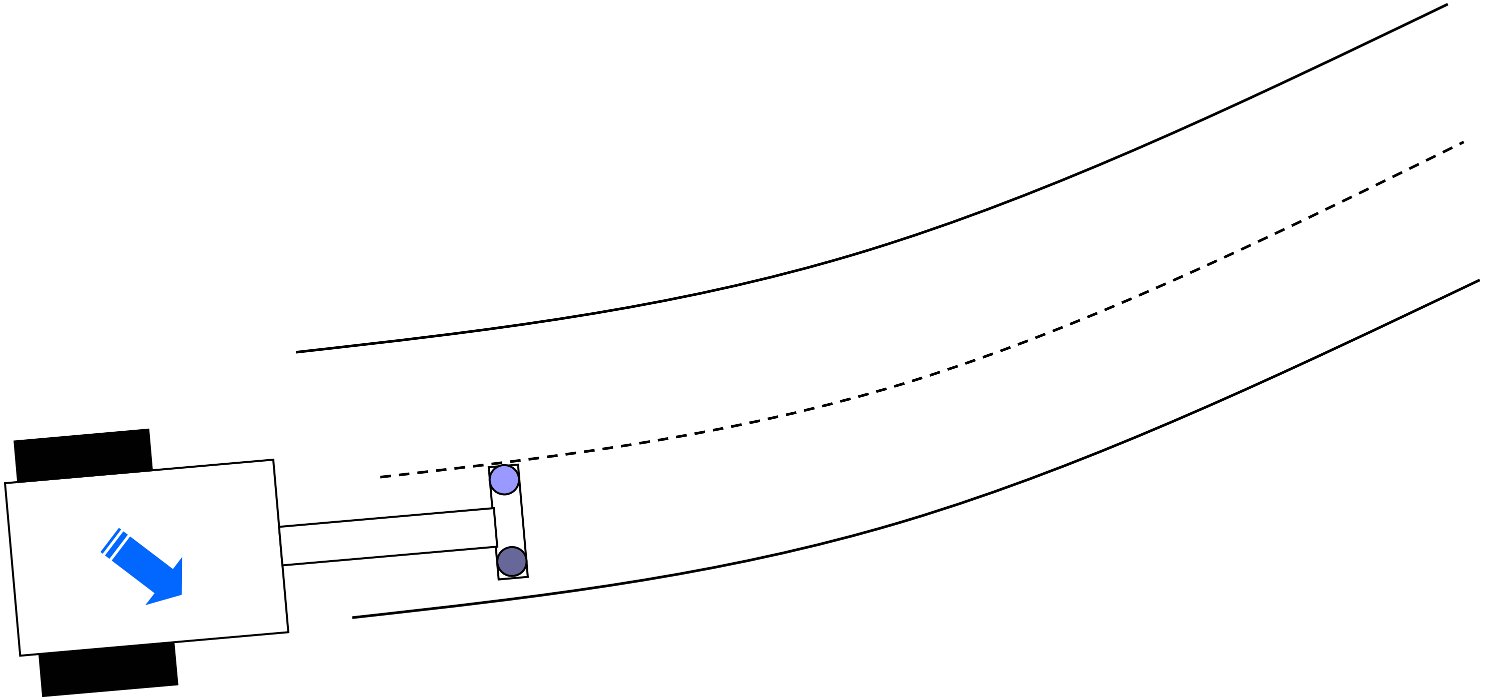
Goal

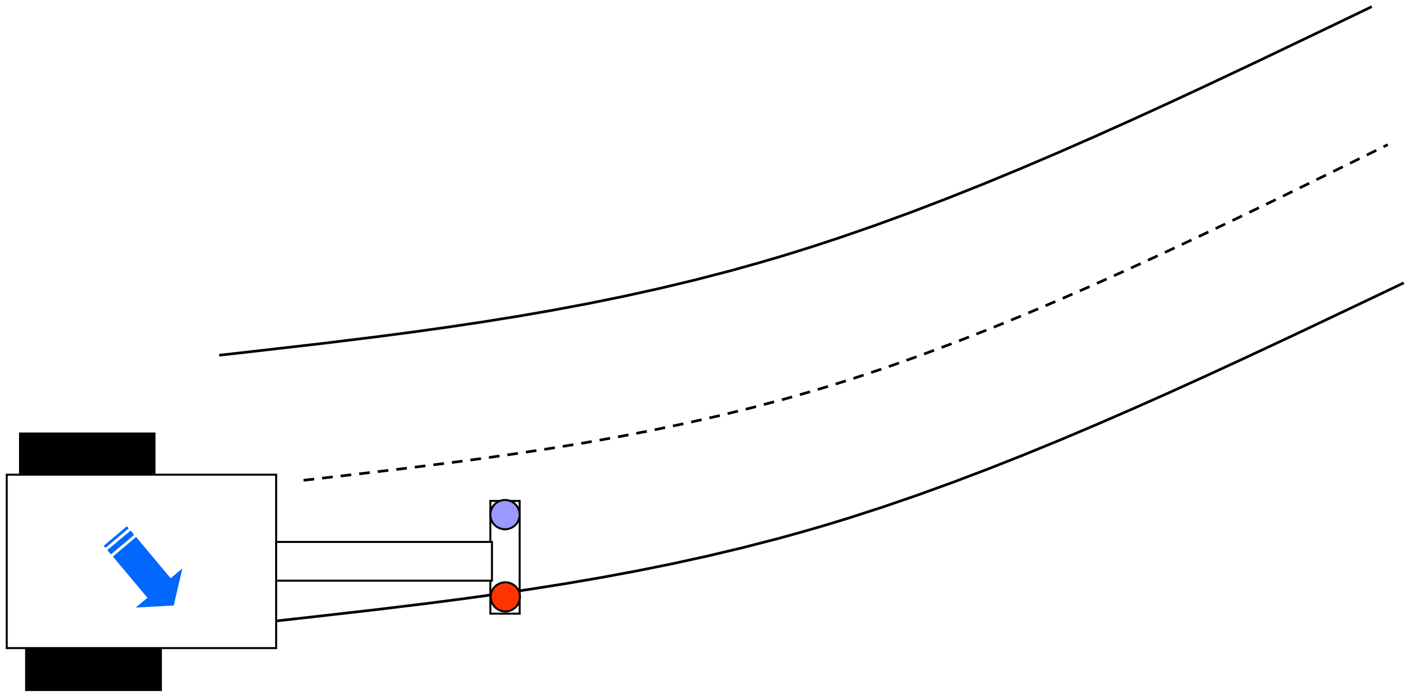
l1(0, 10, D) with Constraints $X1 = X + D, Y1 = 1 + D, D \geq 0$

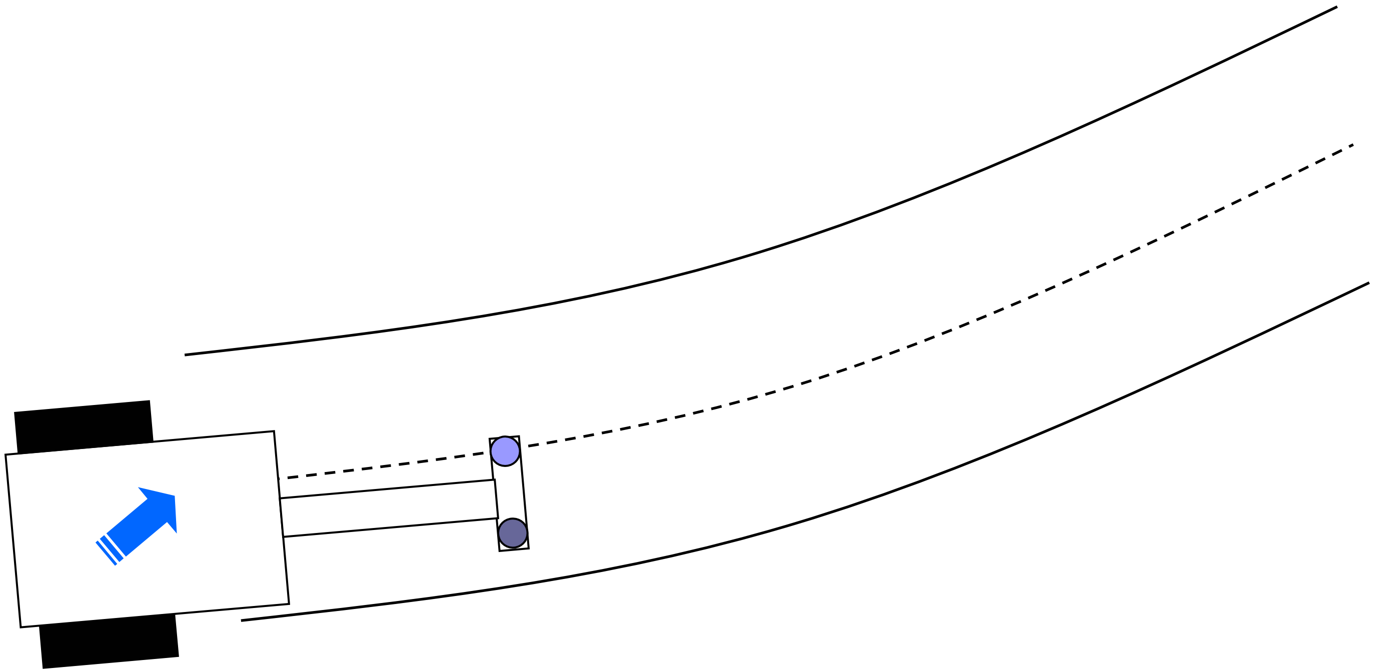
Example: Two-wheeled vehicle

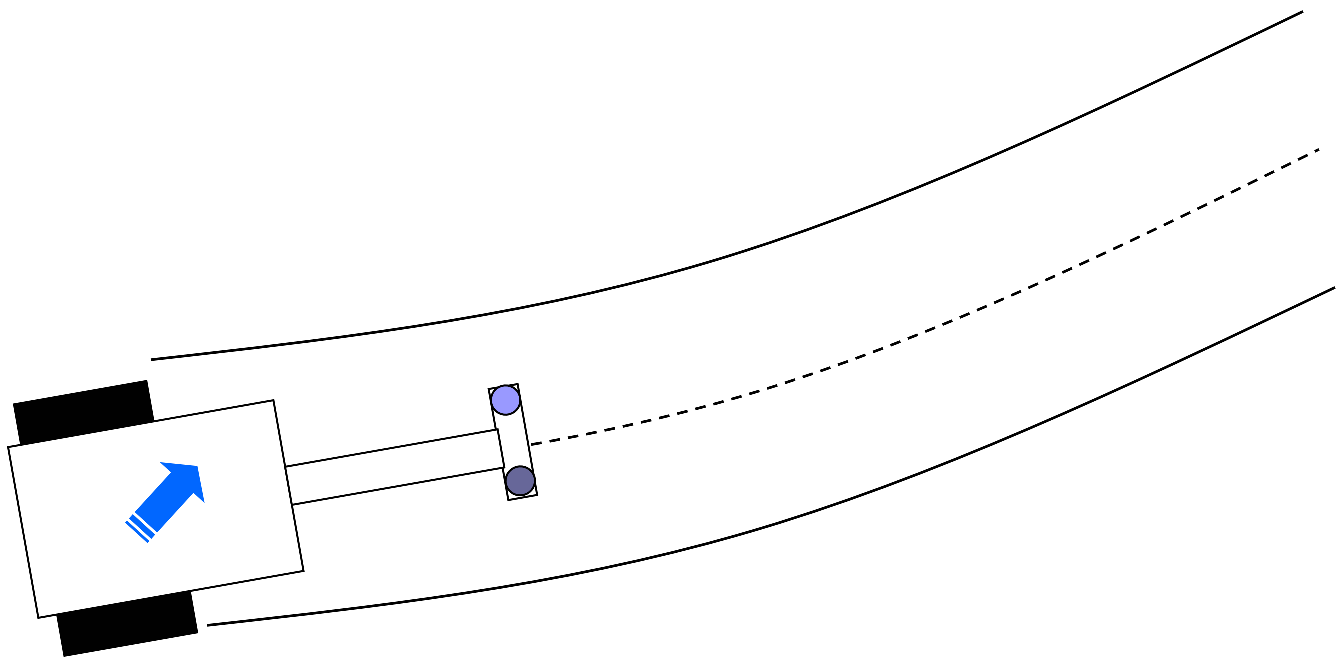


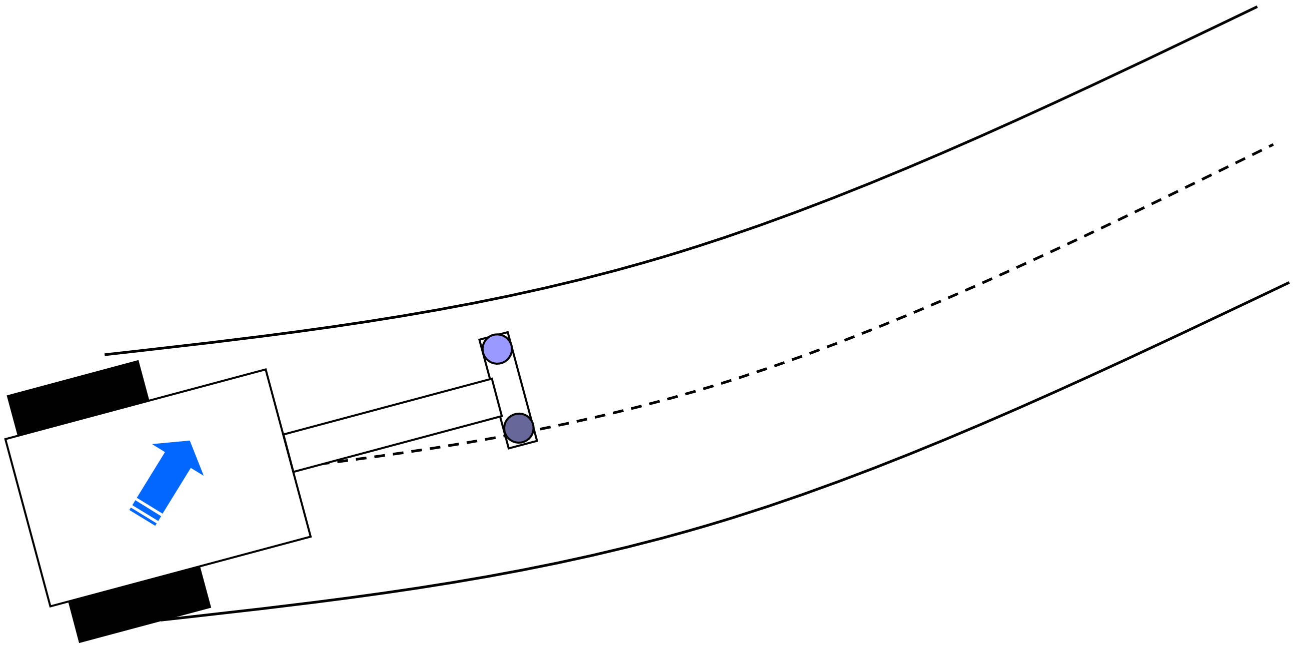
[Konaka 2004]

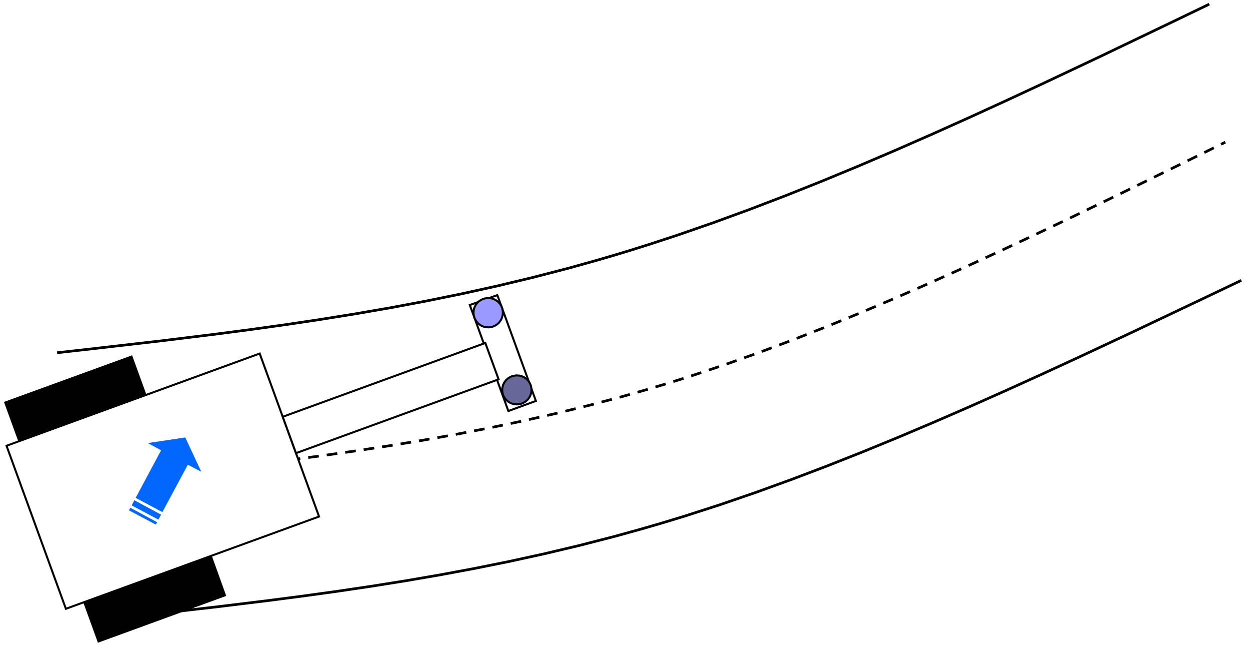


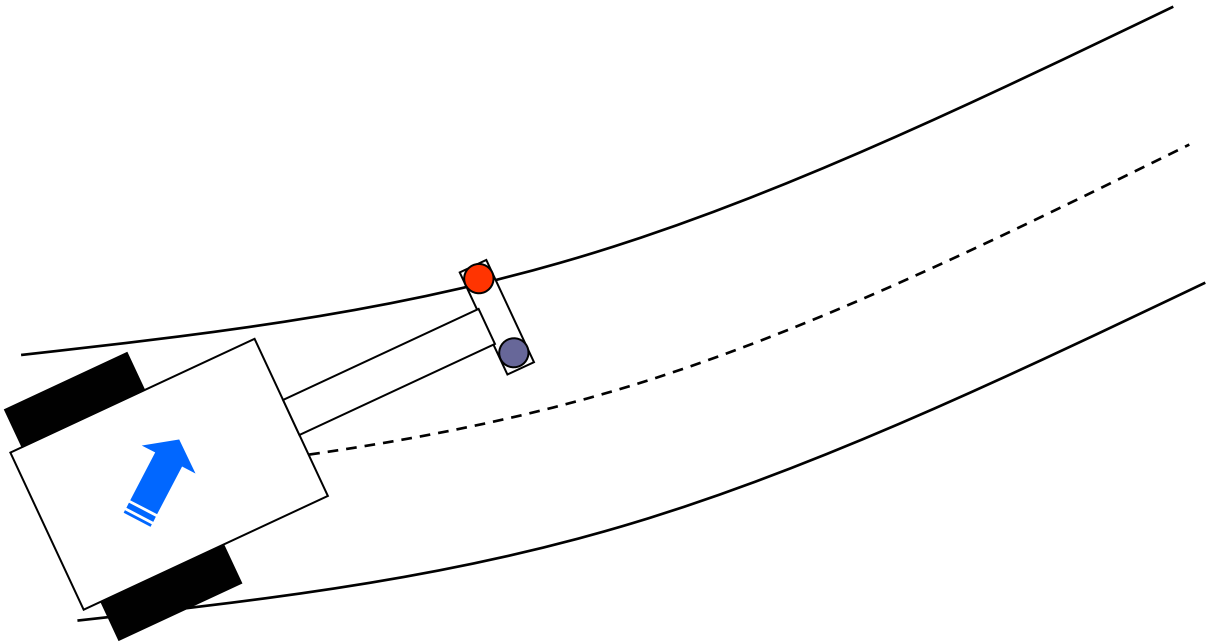


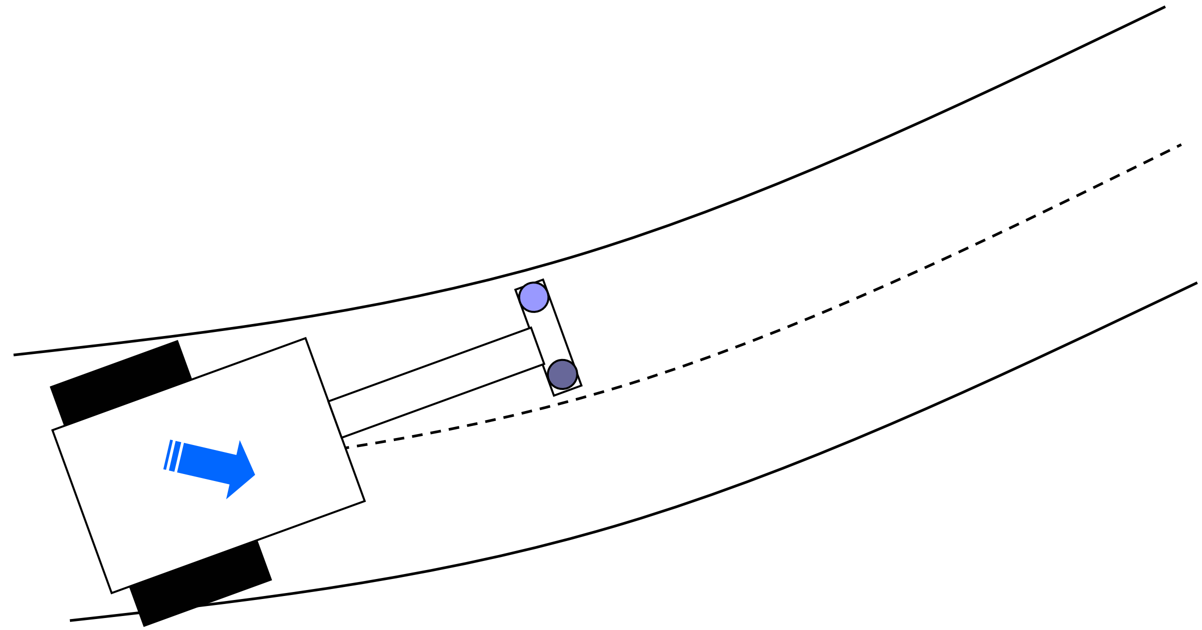


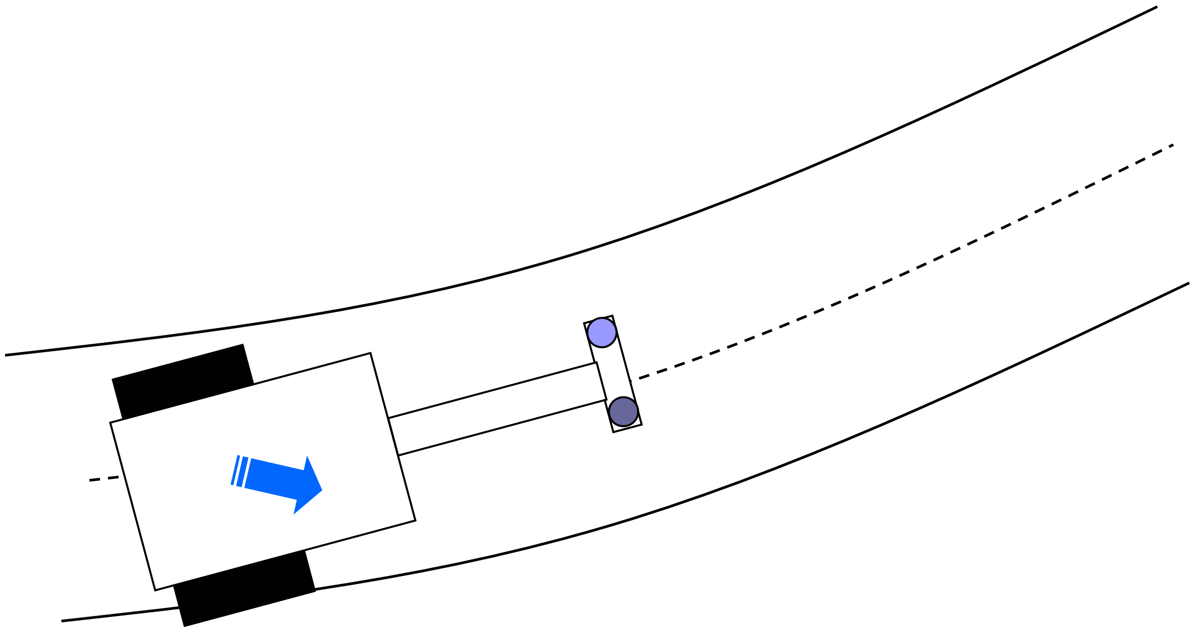


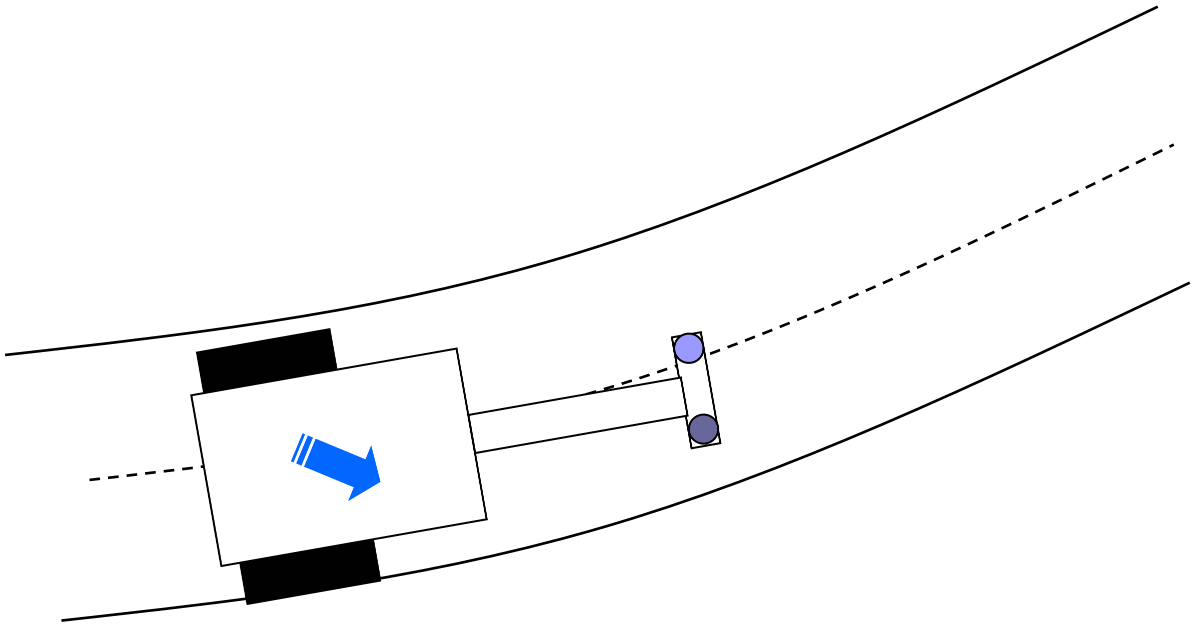


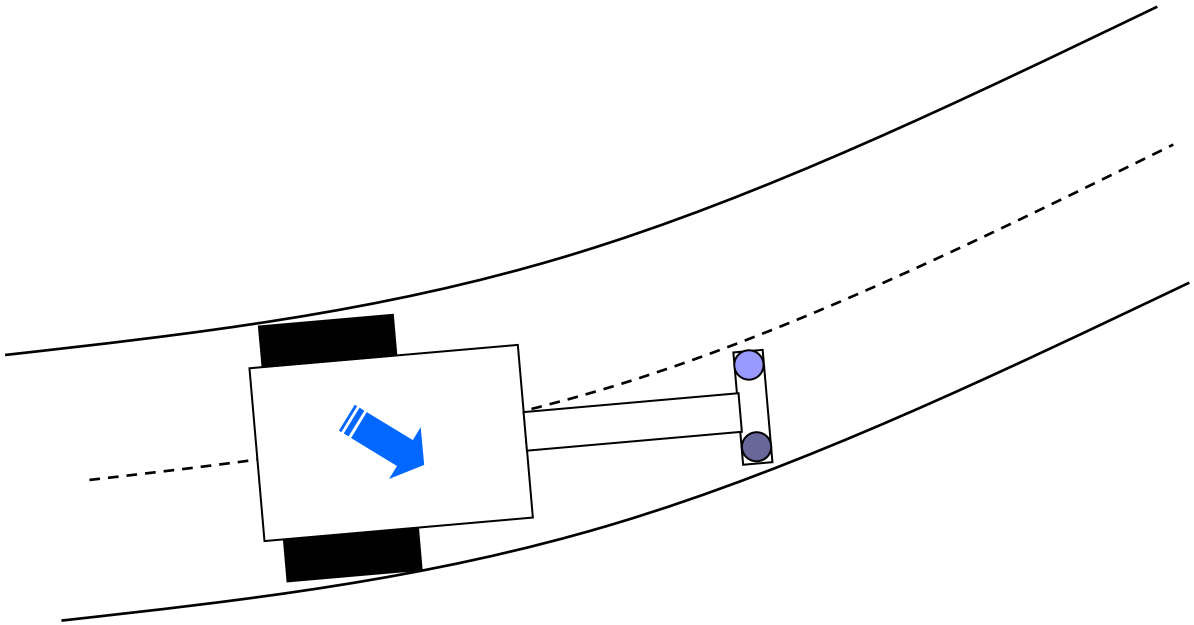


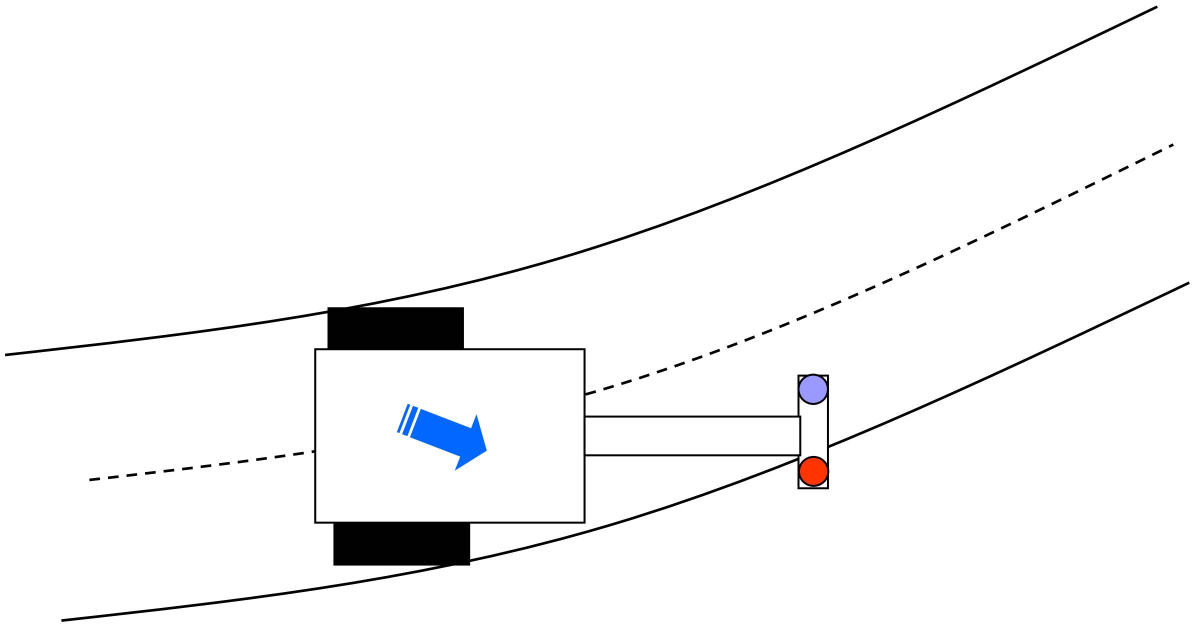


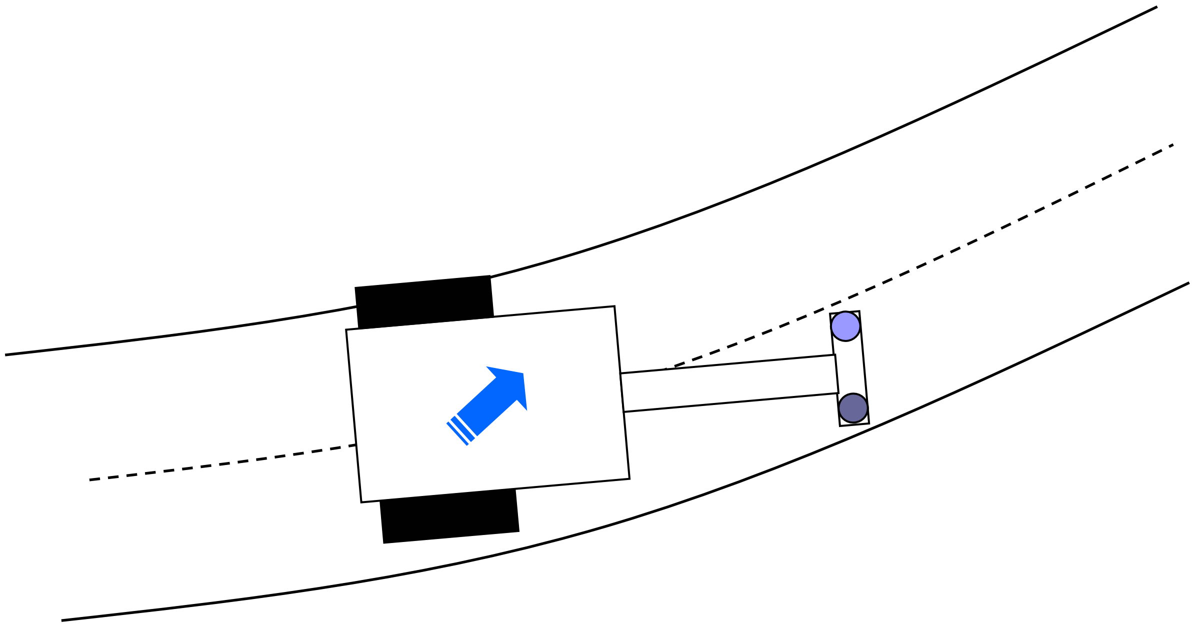




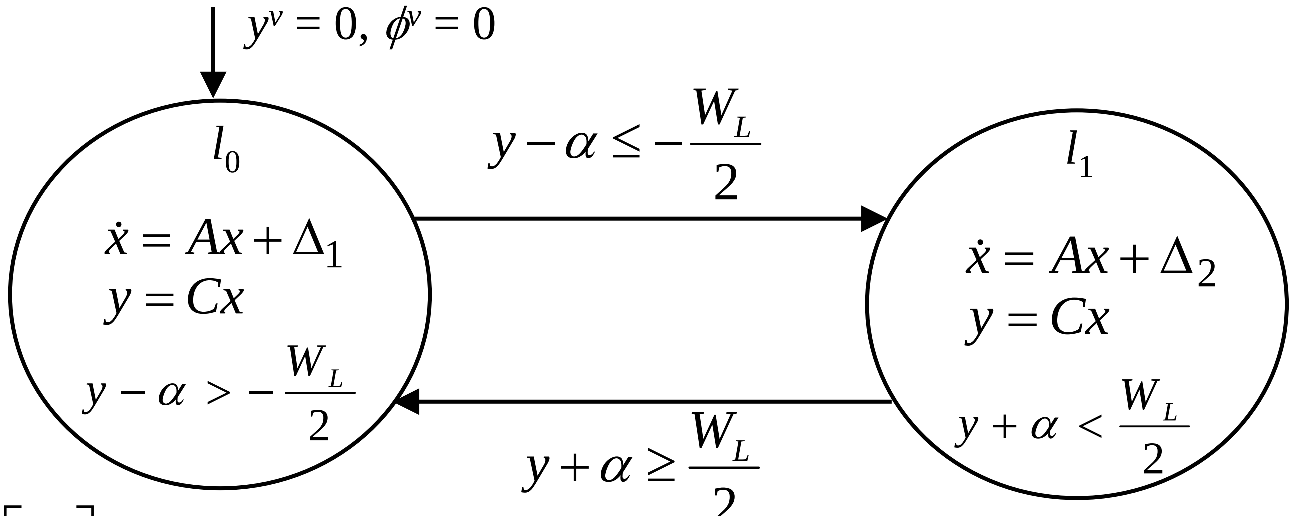








Hybrid automaton



$$x = \begin{bmatrix} y^v \\ \phi^v \end{bmatrix}$$

$$A = \begin{pmatrix} 0 & a_1 \\ 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 \\ a_2 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 \\ a_3 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & l \end{pmatrix}$$

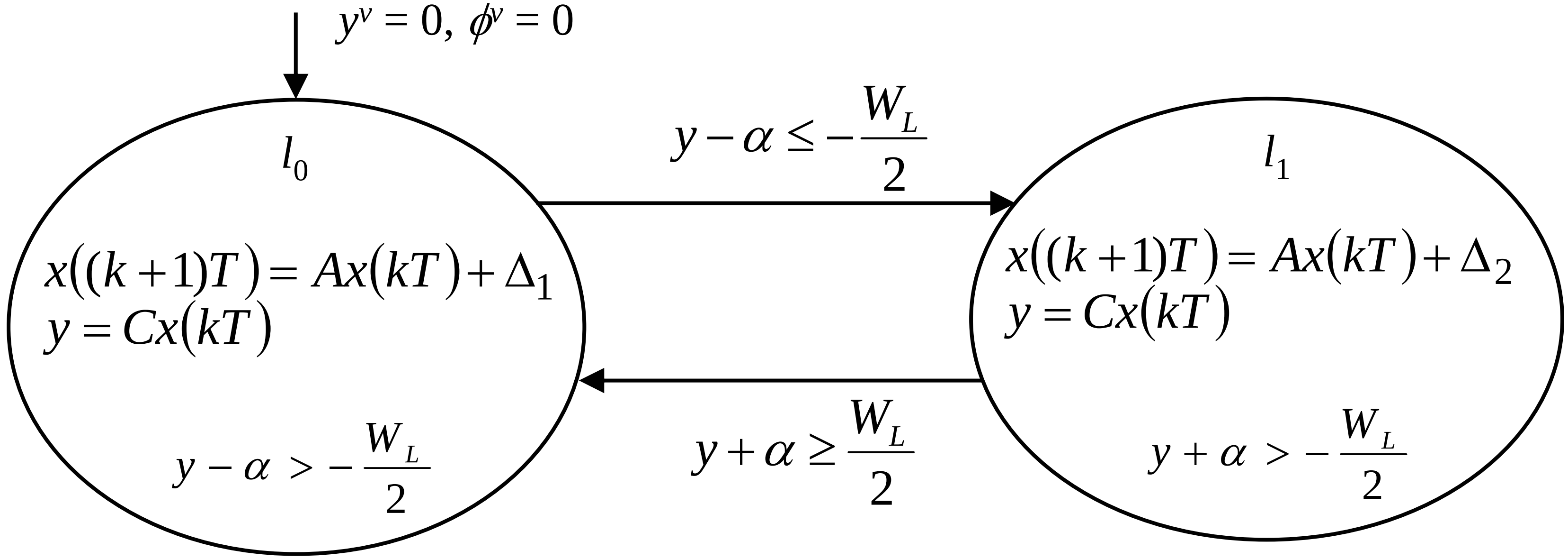
$$a_1 = \frac{R_w}{2}(w_H + w_L)$$

$$a_2 = -\frac{R_w}{2}\left(\frac{w_H + w_L}{R} + \frac{w_H - w_L}{W}\right)$$

$$a_3 = -\frac{R_w}{2}\left(\frac{w_H + w_L}{R} - \frac{w_H - w_L}{W}\right)$$

Continuous Time (differential equations)

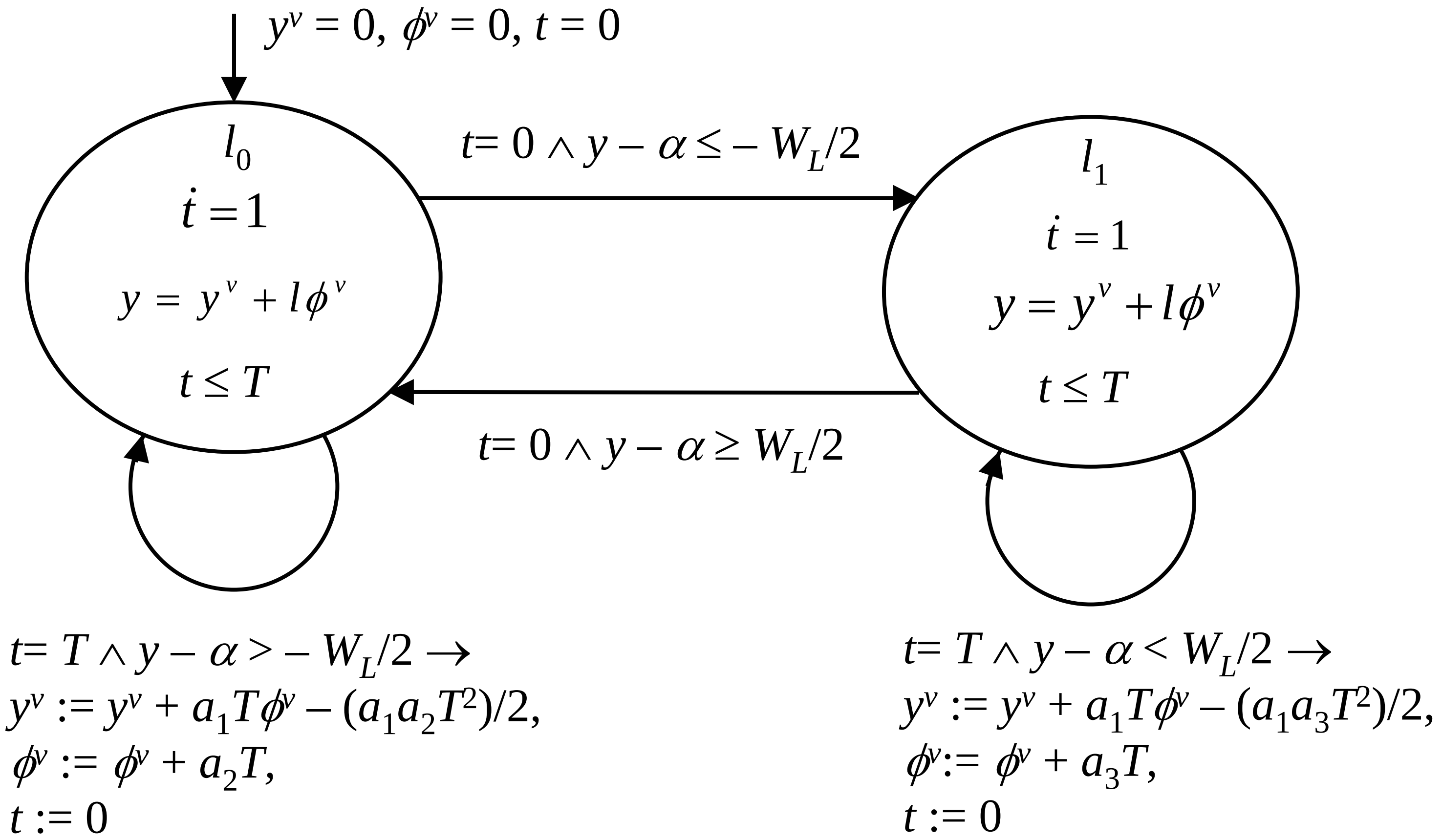
Hybrid automaton



$$x = \begin{bmatrix} y^v \\ \phi^v \end{bmatrix},\ A = \begin{pmatrix} 1 & a_1 T \\ 0 & 1 \end{pmatrix},\ C = \begin{pmatrix} 1 & l \end{pmatrix},\ \Delta_1 = \begin{pmatrix} -\frac{a_1 a_2}{2} T^2 \\ a_2 T \end{pmatrix},\ \Delta_2 = \begin{pmatrix} -\frac{a_1 a_3}{2} T^2 \\ a_3 T \end{pmatrix}$$

Discrete Time (difference equations)

Hybrid automaton



HA with one clock variable

Parameter design problem

Find the value of α that minimizes

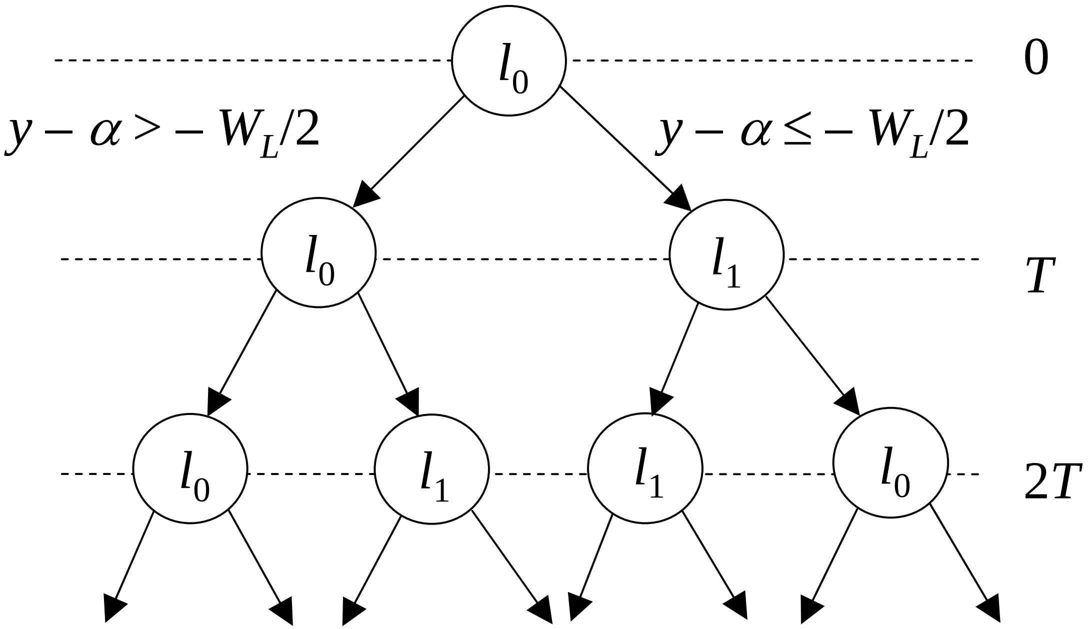
$$J = \sum_{k=1}^{k_f} \left(x(k)^T x(k) + \lambda \left| l(k) - l(k-1) \right| \right) \rightarrow Minimize$$

(Quadratic objective function)

CLP code

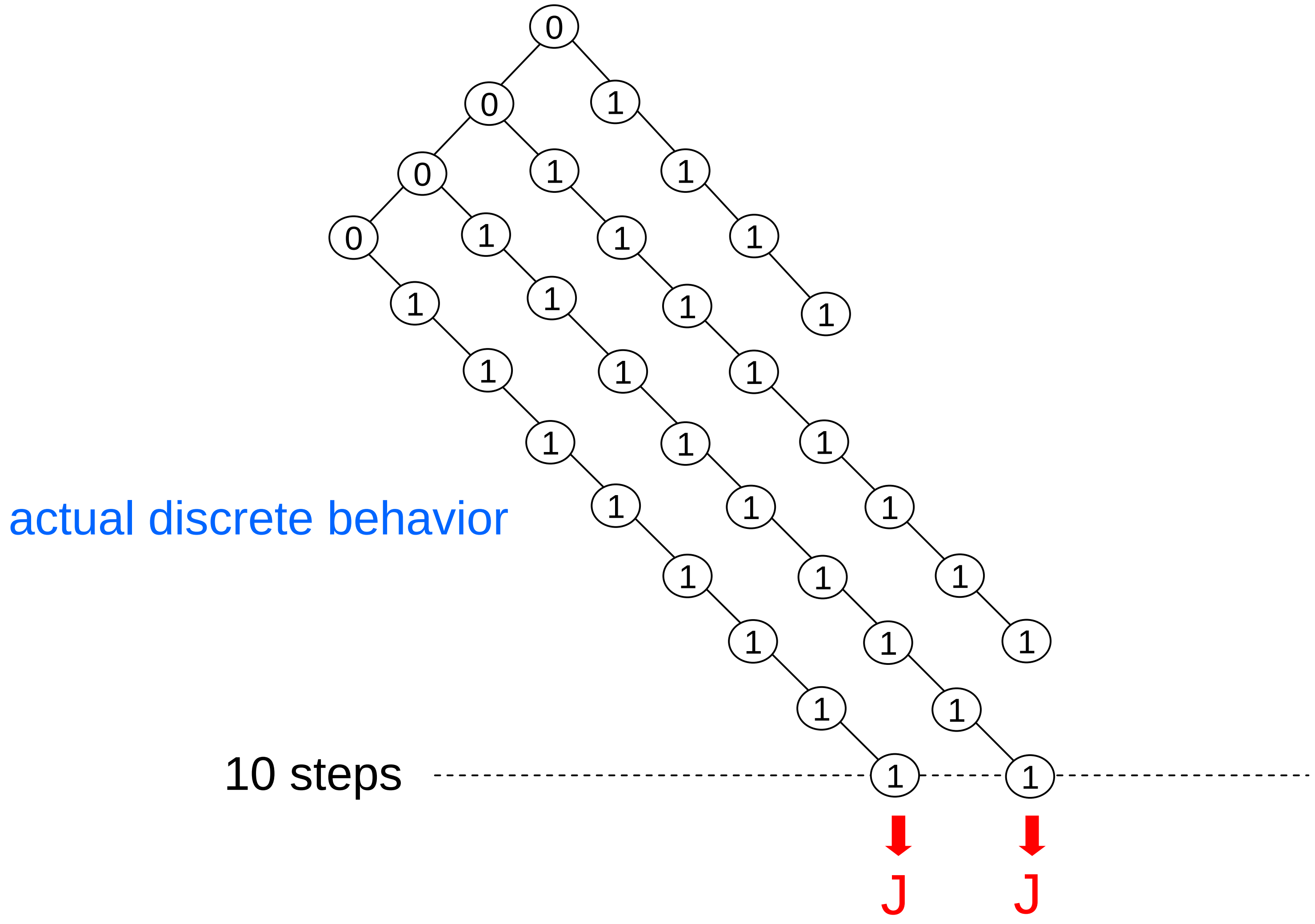
```
l1(X1, X2, T, J, J):- FinalTime(: FT), T >= FT, !.
l1(X1, X2, T, J, J1):- (optval, !, J <= @optval; true),
            Alpha(: Alpha), X1 + 0.215 * X2 - Alpha > - 0.05,
            l1(X1 + 0.022765 * X2 + 0.000781, X2 - 0.06864,
              T + 0.1, J + X1 * X1 + X2 * X2, J1).
l1(X1, X2, T, J, J1):- (optval, !, J <= @optval; true),
            Alpha(: Alpha), X1 + 0.215 * X2 - Alpha <= - 0.05,
            Lambda(: L), l2(X1, X2, T, J + L, J1).
l2(X1, X2, T, J, J):- FinalTime(: FT), T >= FT, !.
l2(X1, X2, T, J, J1):- (optval, !, J <= @optval; true),
            Alpha(: Alpha), X1 + 0.215 * X2 + Alpha < 0.05,
            l2(X1 + 0.022765 * X2 - 0.00061, X2 + 0.053464,
              T + 0.1, J + X1 * X1 + X2 * X2, J1).
l2(X1, X2, T, J, J1):- (optval, !, J <= @optval; true),
            Alpha(: Alpha), X1 + 0.215 * X2 + Alpha >= 0.05,
            Lambda(: L), l1(X1, X2, T, J + L, J1).
```

Search strategy



go(F, L, A, Z, J):-
FinalTime(: F), Lambda(: L), Alpha(: A),
min(J, l1(0, 0, 0, 0, J)),
fourier([A], Z). ← projecting constraints to those on variable A

Search strategy



Comparing with tentative optimal value

```
l1(X1, X2, T, J, J1):- (optval, !, J <= @optval; true),
           Alpha(: Alpha), X1 + 0.215 * X2 - Alpha > - 0.05,
           l1(X1 + 0.022765 * X2 + 0.000781, X2 - 0.06864,
              T + 0.1, J + X1 * X1 + X2 * X2, J1).
```

If the tentative optimal value is obtained,
then compare it with the current value of *J*.

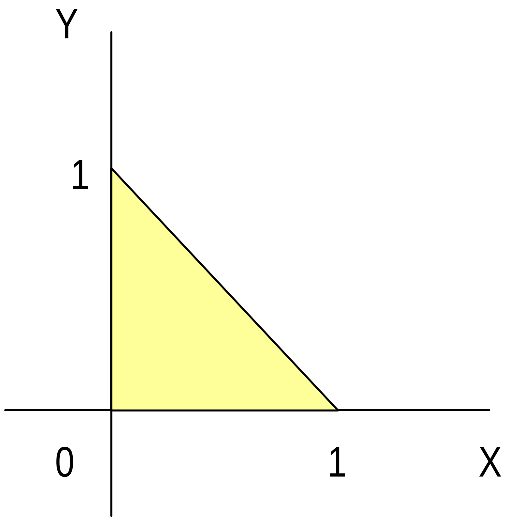
Operations on convex polyhedra

polka_constraints2poly()

[[X, Y], [X + Y <= 1, X >= 0, Y >= 0]]

CLP constraints

polka_poly2constraints()



convex polyhedron

POLKA Library
(http://www.irisa.fr/prive/Bertrand.Jeannet/newpolka.html)
supports intersection, emptyness test, change of dimension, convex hull etc of convex polyhedra.

CLP code using POLKA

```
I1(_, _, _, P, _, _, _):- polka_poly_is_empty(P), !, fail.
I1(T, _, _, P, P, J, J):- FinalTime(: FT), T >= FT, !.
I1(T, X1, X2, P, P1, J, J1):- (optval, !, entailed(J <= @optval); true),
        polka_constraints2poly([[A], [X1 + 0.215 * X2 - A > - 0.05]], PN),
        polka_poly_intersection(P, PN, PNN),
        I1(T + 0.1, X1 + 0.022765 * X2 + 0.000781, X2 - 0.06864,
           PNN, P1, J + X1 * X1 + X2 * X2, J1).
I1(T, X1, X2, P, P1, J, J1):- (optval, !, entailed(J <= @optval); true),
        polka_constraints2poly([[A], [X1 + 0.215 * X2 - A <= - 0.05]], PN),
        polka_poly_intersection(P, PN, PNN),
        Lambda(: L), I2(T, X1, X2, PNN, P1, J + L, J1).
I2(_, _, _, P, _, _, _):- polka_poly_is_empty(P), !, fail.
I2(T, _, _, P, P, J, J):- FinalTime(: FT), T >= FT, !.
I2(T, X1, X2, P, P1, J, J1):- (optval, !, entailed(J <= @optval); true),
        polka_constraints2poly([[A], [X1 + 0.215 * X2 + A < 0.05]], PN),
        polka_poly_intersection(P, PN, PNN),
        I2(T + 0.1, X1 + 0.022765 * X2 - 0.00061, X2 + 0.053464,
           PNN, P1, J + X1 * X1 + X2 * X2, J1).
I2(T, X1, X2, P, P1, J, J1):- (optval, !, entailed(J <= @optval); true),
        polka_constraints2poly([[A], [X1 + 0.215 * X2 + A >= 0.05]], PN),
        polka_poly_intersection(P, PN, PNN),
        Lambda(: L), I1(T, X1, X2, PNN, P1, J + L, J1).
```

Example: optimal control problem

$$x(t+1) = 0.8\begin{bmatrix} \cos\alpha(t) & -\sin\alpha(t) \\ \sin\alpha(t) & \cos\alpha(t) \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0] x(t)$$

$$\alpha(t) = \begin{cases} \pi/3 & ([1 \quad 0]x(t) \geq 0) \\ -\pi/3 & ([1 \quad 0]x(t) < 0) \end{cases}$$

SWITCH

$$x(t) \in [-10 \quad 10] \times [-10 \quad 10]$$

$$u(t) \in [-1 \quad 1]$$

$\leftarrow$ control input

$$J = \sum_{t=0}^{T-1} \left\| u(t) - u_f \right\|_{Q_1}^2 + \left\| x(t) - x_f \right\|_{Q_2}^2 \rightarrow Minimize$$

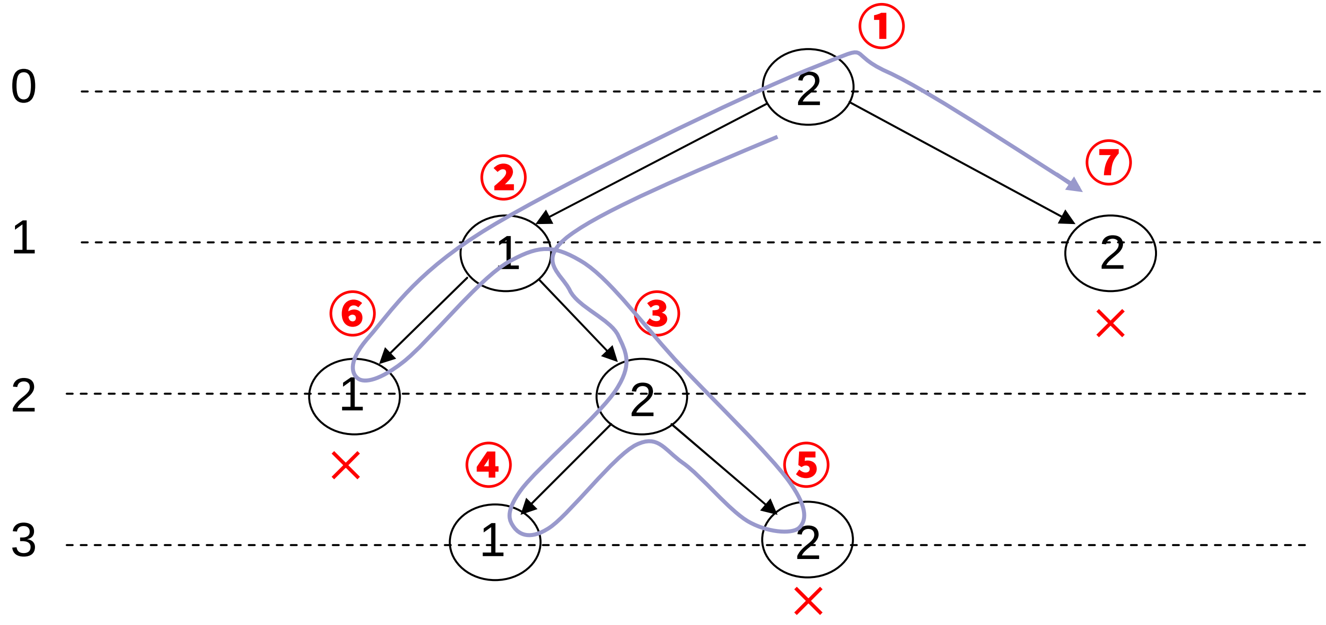
[Bemporad and Morari 1999]

CLP code

```
l1(T, L, J):- FinalTime(: F), T > F, !,qmin_list(L, [], J),  ← quadratic optimizer
     rec(0, RES), record(0, [J, RES]).
                                                ↓ checking the current value of J
l1(T, L, J):- X1(T : X1), X1 < 0, l2(T, L, J).
l1(T, L, J):- (optval, !, qmin_list(L, [], JTEMP, _), JTEMP <= @optval; true),
     X1(T : X1), X1 >= 0, X2(T : X2), U(T : U), LOC(T : 1),
     A11(1 : A11), A12(1 : A12), A21(1 : A21), A22(1 : A22),
     X1(T + 1 : A11 * X1 + A12 * X2),
     X2(T + 1 : A21 * X1 + A22 * X2 + U), QU(: QU), UU = U * QU,
     l1(T + 1, [X1, X2, UU | L], J).

l2(T, L, J):- FinalTime(: F), T > F, !, qmin_list(L, [], J),
     rec(0, RES), record(0, [J, RES]).
l2(T, L, J):- X1(T : X1), X1 >= 0, l1(T, L, J).
l2(T, L, J):- (optval, !, qmin_list(L, [], JTEMP, _), JTEMP <= @optval; true),
     X1(T : X1), X1 < 0, X2(T : X2), U(T : U), LOC(T : 2),
     A11(2 : A11), A12(2 : A12), A21(2 : A21), A22(2 : A22),
     X1(T + 1 : A11 * X1 + A12 * X2),
     X2(T + 1 : A21 * X1 + A22 * X2 + U), QU(: QU), UU = U * QU,
     l2(T + 1, [X1, X2, UU | L], J).
```

Search tree by KCLP-HS



Final Time = 3

Execution result

```
| ?- go(10).
JOPT = 2.837877
Time =  0: Loc = 2, X1 = -1.000000, X2 = 1.000000, U = -0.672764
Time =  1: Loc = 1, X1 = 0.292820, X2 = 0.420056, U = -0.220506
Time =  2: Loc = 2, X1 = -0.173895, X2 = 0.150389, U = -0.112616
Time =  3: Loc = 1, X1 = 0.034634, X2 = 0.068017, U = -0.029554
Time =  4: Loc = 2, X1 = -0.033270, X2 = 0.021649, U = -0.020139
Time =  5: Loc = 1, X1 = 0.001690, X2 = 0.011570, U = -0.003290
Time =  6: Loc = 2, X1 = -0.007340, X2 = 0.002509, U = -0.003876
Time =  7: Loc = 2, X1 = -0.001198, X2 = 0.002213, U = -0.000943
Time =  8: Loc = 1, X1 = 0.001054, X2 = 0.000772, U = -0.000974
Time =  9: Loc = 2, X1 = -0.000113, X2 = 0.000065, U = -0.000104
Time = 10: Loc = 1, X1 = 0.000000, X2 = 0.000000, U = 0.000000

*** yes ***

0.0300 sec.

| ?-
```

CPU Time (sec.) for $\lambda = 0.25$.

#Steps	10	20	30
MLD+MIQP	0.3	16	955.2
KCLP-HS	< 0.01	0.01	0.01
SyNRAC	0.36	1.41	2.77

[Konaka 2004]

CPU Time (sec.) for $\lambda = 0.5$.

#Steps	10	20	30
MLD+MIQP	0.34	4.25	53.98
KCLP-HS	< 0.01	< 0.01	0.01
SyNRAC	0.34	1.36	2.70

[Konaka 2004]

Conclusion

- We have shown several examples of design and verification problems on hybrid systems, and demonstrate how to solve them by using KCLP-HS and SyNRAC.
- The tools can be used not only for verification, but also for design and optimization. This is one of advantages of them over existing verification tools.
- There are instances of problems (particularly parameter design problems) in which computation by the tools is much faster than existing approaches such as mixed integer quadratic programming based on MLD system representation.