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Multi-Attribute Target-Oriented Decision Analysis and Its Application to Kansei Evaluation of Japanese Traditional Crafts

by

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Abstract

Multi-attribute decision analysis (MADA) problems involve the task of ranking a finite number of decision alternatives, each of which is explicitly described in terms of different characteristics (also, often called attributes, decision criteria, or objectives), which have to be taken into account simultaneously. Among various MADA methods, multi-attribute utility theory (MAUT) is one widely used method to solve MADA problems. However, substantial empirical evidence and recent research have shown that it is usually difficult to build mathematically rigorous utility functions based on attributes and the conventional attribute utility function often does not provide a good description of individual behavioral/psychological preferences.

As a substitute for utility theory, in 1979 Kahneman and Tversky proposed the S-shaped value function in the Prospect Theory to better represent decision makers'(DMs) behavioral/psychological preferences, and in 1999 Heath et al. suggested that the inflection point in the S-shaped value function can be interpreted as a target. To develop this concept further, target-oriented decision analysis involves interpreting an increasing, bounded function, properly scaled, as a cumulative distribution function (cdf) and relating it to the probability of meeting or exceeding a target value, i.e. it argues that target serves as reference point and alters outcomes in a manner consistent with the value function of *Prospect Theory*. As an emerging area considering the behavioral aspects of decision analysis, target-oriented decision analysis lies in the philosophical root of Simon's *bounded rationality* as well as represents the *S-shaped value function of Prospect Theory.*

In fact, decision analysis with targets/goals has a long history in the literature. Distance-based approach is one widely used method in decision analysis problems. However, different distances should have different impacts on DMs' preferences, which is missed in the distance-based approach. In this sense, revisiting the targets/goals in decision analysis problems is essential to many decision problems.

This research builds upon past research work and makes an intensive/in-depth study on target-oriented decision analysis from the following three aspects:

1. *Target-oriented decision analysis with different types of target preferences and hybrid uncertain targets: We propose two methods to target-oriented decision model with different target preferences and extend those two methods to target-oriented decision analysis with fuzzy targets*

   (a) Target-oriented decision analysis with different types of target preferences
   Original target-oriented decision model presumes that the DM has a monotonically increasing target preference, e.g., the attribute/criterion wealth. However, there are three types of target preferences: "the more the better" (corresponding to benefit target preference), "the less the better" (corresponding to cost target preference), and range targets (too much or too little is not acceptable).

   The key ideas of our methods are to use the cdf and level set of the probability distribution function (pdf) in the target-oriented decision model. Compared

with previous work, our methods can model different types of target preferences and induce four shaped value functions: S-shaped, inverse S-shaped, convex, and concave.

(b) Target-oriented decision analysis with fuzzy targets
In addition, target-oriented decision model assumes that target has a random pdf. It is well known that all facets of uncertainty cannot be captured by a single probability distribution. Fuzzy uncertainty is considered by DMs to linguistically specify their uncertain targets. In our research, we extend those two methods to decision analysis with fuzzy targets. Compared with the pioneering work on fuzzy decision analysis by Bellman and Zadeh, our research outperforms in terms of three aspects.

2. *Multi-attribute target-oriented decision analysis: We develop a non-additive multi-attribute target-oriented decision model based on fuzzy measure and fuzzy integral, and develop a prioritized aggregation operator to model the prioritization between targets/attributes.*

   (a) Non-additive multi-attribute target-oriented decision analysis
   In many situations, multiple attributes are of interest. Several researches have extended the target-oriented decision model into multi-attribute case. In their model, multi-additive value function is used to aggregate partial target achievements while assuming the mutual independence between different targets. However, it is recognized that in many decision problems attributes are interdependent. On the other hand, even if, in an objective sense the targets are mutually independent (probabilistically mutually independent), they are not necessary considered to be independent from the DM's subjective viewpoint. Thus traditional approaches are not adequate for such complex situations.

   The key idea of our work to model the interdependence between different targets is to use the fuzzy measure and fuzzy integral. In our research, several similarities between multi-attribute target-oriented decision model and non-additive fuzzy integral have been discovered. Hence, the λ-fuzzy measure is used as a technique to induce the possible combinations of indices of meeting targets and fuzzy integral is used to model the non-additive multi-attribute model. Compared with previous research, our method can model the interdependence from DM's subjective viewpoint as well as be of simple use in real applications.

   (b) Prioritized multi-attribute target-oriented decision analysis
   Furthermore, the importance information associated with different targets plays a fundamental role in the comparison between alternatives by overseeing tradeoffs between respective satisfactions of different targets. A concept closely related to the importance information is the priority, which does not allow the tradeoffs between different targets. In some cases, the DM may have a prioritization between different targets.

   In our research, a prioritized OWA aggregation operator has been proposed to model the prioritization between different targets based on the Ordered Weighted Averaging (OWA) operator and Hamacher t-norms.

3. *Application to Kansei evaluation problems: We extend the proposed decision models into Kansei evaluation context and propose a Kansei evaluation model based on prioritized multi-attribute fuzzy target-oriented decision analysis. A case study for Kansei evaluation of Japanese traditional crafts is also conducted to illustrate the proposed Kansei evaluation model.*

   Differed from existing work on Kansei evaluation, our proposed Kansei evaluation model can

   (a) solve the inconsistent preference order relations on Kansei attributes,
   (b) integrate the psychological preferences in satisfaction degree of Kansei attributes,
   (c) and consider the prioritization between different Kansei attributes.

   By using our model, consumers can choose their preferred products according to their Kansei preferences. The consumer-oriented Kansei evaluations for traditional crafts in Japan provides possible solutions for both consumer-oriented product design and recommendation strategy for traditional crafts in Japan. Thus we believe that the proposed Kansei evaluation model would be of great help for marketing or recommendation purposes.

In conclusion, our efforts in studying the target-oriented decision model are to solve decision analysis with hybrid uncertain targets and different target preferences, non-additive and prioritized multi-attribute target-oriented decision analysis, and then apply the decision models in Kansei evaluation problems.

Key word: S-shaped function; Bounded rationality; Target-Oriented decision model; Different target preferences; Possibilistic/Probabilistic uncertainty; Fuzzy measure and fuzzy integral; Prioritized aggregation; Kansei evaluation; Japanese traditional crafts.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

The study of decision analysis is part of many disciplines, including psychology, business, engineering, operations research, systems engineering, and management science. It is a scientistic discipline comprising a collection of principles and methods aiming to help individuals, groups of individuals, or organization in the performance of difficult decisions. In many decision problems, multiple attributes are of interest. A typical problem in multi-attribute decision analysis (MADA) is concerned with the task of ranking a finite number of decision alternatives, each of which is explicitly described in terms of different characteristics (also, often called attributes, decision criteria, or objectives), which have to be taken into account simultaneously [127].

Among various MADA methods, multi-attribute utility theory (MAUT) [75] is one widely used method to solve MADA problems. Other methods involving attributes, utility and relative measurement, include the analytical hierarchy process (AHP) [116] and the simple multi-attribute rating technique (SMART), which are simple versions of MAUT [6, 140]. In MAUT, by assuming the existence of a utility function, decision makers (DMs) try to maximize the utilities.

However, substantial empirical evidence has shown that it is difficult to build mathematically rigorous utility functions based on attributes and the conventional attribute utility function often does not provide a good description of individual behavioral/psychological preferences [17, 70]. Recent research has shown that the behavioral aspects of decision analysis has grown, and this was recognized by the award of the 2002 Nobel Prize in Economics to Daniel Kahneman [135].

There are several decision theories focusing on DM's psychological/behavioral preferences. In the next section, we will introduce some notable behavioral decision theories and the development of target-oriented decision analysis, which will be studied in this thesis.

1.1 Development of Target-Oriented Decision Model

1.1.1 Expected utility and prospect theory

In 1947, Von Neumann and Morgenstern [134] enunciated various axioms of rationality which implied that

1. For any rational individual, it was always possible to define the utility of a consequence as that probability p making the individual indifferent between receiving

that consequence and receiving a lottery with a probability p chance of leading to the best possible consequence and a $(1 - p)$ chance of leading to the worst possible consequence.

2. The rational individual, when choosing among several possible decisions, would always choose that decision whose possible consequences have the maximum expected utility.

Prospect theory by Kahneman and Tversky [70] has attracted a good deal of attention as an alternative to the well-established utility/value theory. The appeal of prospect theory stems from its descriptive power. Experiments support that prospect theory is consistent with the behavior of DMs [133]. Prospect theory also permits prediction and description of behavior that violates axioms of rationality, e.g., the transitivity axiom.

There are some fundamental differences between prospect theory and utility/value theory. In the original prospect theory by Kahneman and Tversky [70], outcomes are expressed and evaluated as positive or negative deviations (gains and losses) from a reference alternative. Coding the outcomes as gains and losses is the most important part of the editing phase that consists of a preliminary analysis of the offered prospects. The other major operations (combination, segregation and cancelation) of the editing phase are of less importance for our purpose and are discussed by Kahneman and Tversky [70]. S-shaped marginal value functions (difference functions) are applied on the gains and losses. Another difference between the prospect theory and utility theory is that when evaluating gambles to form the functions, prospect theory uses weighting functions rather than probabilities, which may, but do not have to, coincide. The weighting function does not obey the axioms of probability theory and it measures the impact of probabilities on choices rather than the likelihood of the underlying events [70]. In addition to the original risky choice problem, Tversky and Kahneman [132] have also applied the concepts of reference alternatives and loss aversion to riskless choice. The idea is that a certain magnitude of loss is valued more than the same magnitude of gain. The marginal value of both gains and losses decreases by their size. These properties give again rise to an asymmetric S-shaped value function, concave above the reference point and convex below it.

1.1.2 Optimizing and satisficing

Two of the most important approaches to decision analysis are optimizing and satisficing. Rational decision making is based on the optimizing principle. As Simon argued in [120], the traditional utility theory presumes that a rational DM was assumed to have "a well-organized and stable system of preferences, and a skill in computation". However, there were serious costs associated with the memory and computations required to calculate the utility of various outcomes and choose the outcome of highest utility. Most decisions don't seem to be worth the time and effort required to make such computations. Even for decisions which are worth such time and effort, few, if any, individuals seem oriented toward making such computations.

For this reason, Simon [120] enunciated his famous *behavioral* model for rational choice, by enunciating the so-called theory of *bounded rationality*. In this theory, an individual has certain pre-specified requirements. If those requirements are met, the individual continues with his current decisions. When those requirements are not met, the individual actively searches for alternative decisions. Instead of looking for the 'optimum' decision, the individual adopts the first alternative he discovers which satisfies the requirements. Simon's work establishes that DMs consciously make decisions by satisficing targets and not by optimizing utility functions. Although simple and appealing from this target-oriented point of view, its resulted model is still not complete because there may be uncertainty about the target itself.

In fact, decision analysis with targets/goals has a long history in the literature. Distance-based approach is one widely used method in decision analysis problems. However, different distances should have different impacts on DMs' preferences, which is missed in the distance-based approach. In this sense, revisiting the targets/goals in decision analysis problems is essential to many decision problems.

1.1.3 Target-oriented decision analysis: A behavioral decision model

Castagnoli and LiCalzi [25] show that for any utility function, there exists a random target with the utility being the cumulative distribution function (cdf) of the random target, see also Bordley [18]. Heath *et al.* [54] suggest that the inflection point in this S-shaped value function can be interpreted as a target. To develop this concept further, target-oriented decision analysis involves interpreting an increasing, bounded function, properly scaled, as a cdf and relating it to the probability of meeting or exceeding a target value [130]. As an emerging area considering the behavioral aspects of decision analysis, target-oriented decision analysis lies in the philosophical root of Simon's bounded rationality as well as represents the S-shaped value function of Prospect Theory, i.e. it argues that target serves as reference point and alters outcomes in a manner consistent with the value function of Prospect Theory.

In fact, there are three approaches to decision analysis in the literature,

1. Normative decision theory: Axioms of rationality.
2. Prescriptive approach: What people should do?
3. Descriptive approach: What people actually do?

The first theory develops theories of coherent or rational behavior of decision analysis. Based on an axiomatic footing, certain principles of rationality are developed to which a rational DM has to adhere if he or she wants to reach the "best" decision. The second one, prescriptive approach, focuses on which principle people should follow. The last one, so-called psychological/behavioral decision theory, empirically investigates how the (naïve) DMs really make their decisions and develops descriptive theories of decision behavior based on empirical findings.

In fact, utility-maximization is only one way of modeling normatively rational behavior. Alternatively, there exists another valid way, so-called *target-oriented approach*. Consider the subjective expected utility theory perfected in Savage [117]. It provides an axiomatic foundation which implies that the DM should choose an action d which maximizes his expected utility $\mathrm{Val}(d) = \mathrm{EU}(X_d)$ with respect to a subjective probability distribution, where X_d denotes the random consequence associated with the decision d. As proved by Bordley and LiCalzi [18] and LiCalzi [88], *the target-based model satisfies* *the Savage axioms*, thus there is no way to tell if an individual follows Savage's axioms by maximizing his expected utility or is maximizing the probability of meeting his uncertain target. Due to the equivalence of utility-based decision theory and target-oriented decision theory, the target-oriented decision theory is a normative decision theory. Furthermore, in maximizing expected utility, a DM behaves as if maximizing the probability that performance is greater than or equal to a target, whether the target is real or just a convenient interpretation. In this sense, the target-oriented decision model is also a prescriptive approach to decision analysis.

Although the Prospect Theory can better represent DMs' behavioral preferences, the weighting function in Prospect Theory does not obey the axioms of probability theory and it measures the impact of probabilities on choices rather than the likelihood of the underlying events [84]. Therefore, prospect theory postulates a model which in general is not linear in the known probabilities. Whereas, the target-oriented decision model is equivalent to the expected utility theory. The target-oriented decision model argues that target serves as reference point and alters outcomes in a manner consistent with the value function of Prospect Theory. In this sense, the target-oriented decision model is a behavioral model for decision analysis.

Due to the appealing features of target-oriented decision analysis, since its formulation by Bordley and LiCalzi [15, 18], it has received a lot of attention in the past nine years. Abbas and Howard [2] model target setting in organizations. They define "aspiration equivalents" for the alternatives under consideration based on the organization's utility function, drawing an analogy with the notion of *satisficing* by seeking an alternative that meets or exceeds an aspiration level [120], and show that these aspiration equivalents can be used as targets. LiCalzi and Sorato [89] use the Pearson systems probability distributions to model the uncertain targets. Taking a different tack, instead of random uncertainty, Huynh et al. [61, 62] propose a target-oriented approach to decision analysis under uncertainty with fuzzy targets. [1]

In many decision analysis situations, multiple attributes are of interest [75], thus it is important to extend basic target-oriented model to the multi-attribute case. Bordley and Kirkwood [17] consider situations in which a target-oriented approach is natural and define a target-oriented DM for a single attribute as one with a utility that depends only on whether a target for that attribute is achieved. They extend this definition to targets for multiple attributes, requiring that the DM's utility for a multidimensional outcome depend only on the subset of attributes for which targets are met, and they develop a target-oriented approach to assess a multi-attribute preference function. Abbas and Howard [2] introduce a class of multi-attribute utility functions called attribute dominance utility functions that can be manipulated like joint probability distributions and allow the use of probability assessment methods in utility elicitation. Taking a different tack, Tsetlin and Winkler [131] consider decision analysis in a multi-attribute target-oriented setting and study the impact on changes of expected utility in a parameters of performance and target distributions via statistics technique. In addition, Tsetlin and Winkler [130] point out that a multi-attribute utility function cannot always be expressed in the form of the

[1] It should be noted that target-oriented decision model is used in decision analysis under uncertainty (DAUU). In the literature, there two types of DAUU problems: no probability information is given and probability information is available (also called as decision analysis under risk) [20]. The probability information can be subjective or objective. In fact, when there is no probability information, we can use some kind of subjective probability, see [117].

cdf.

Fig. 1.1 shows the historical formulation and main development of target-oriented decision model.

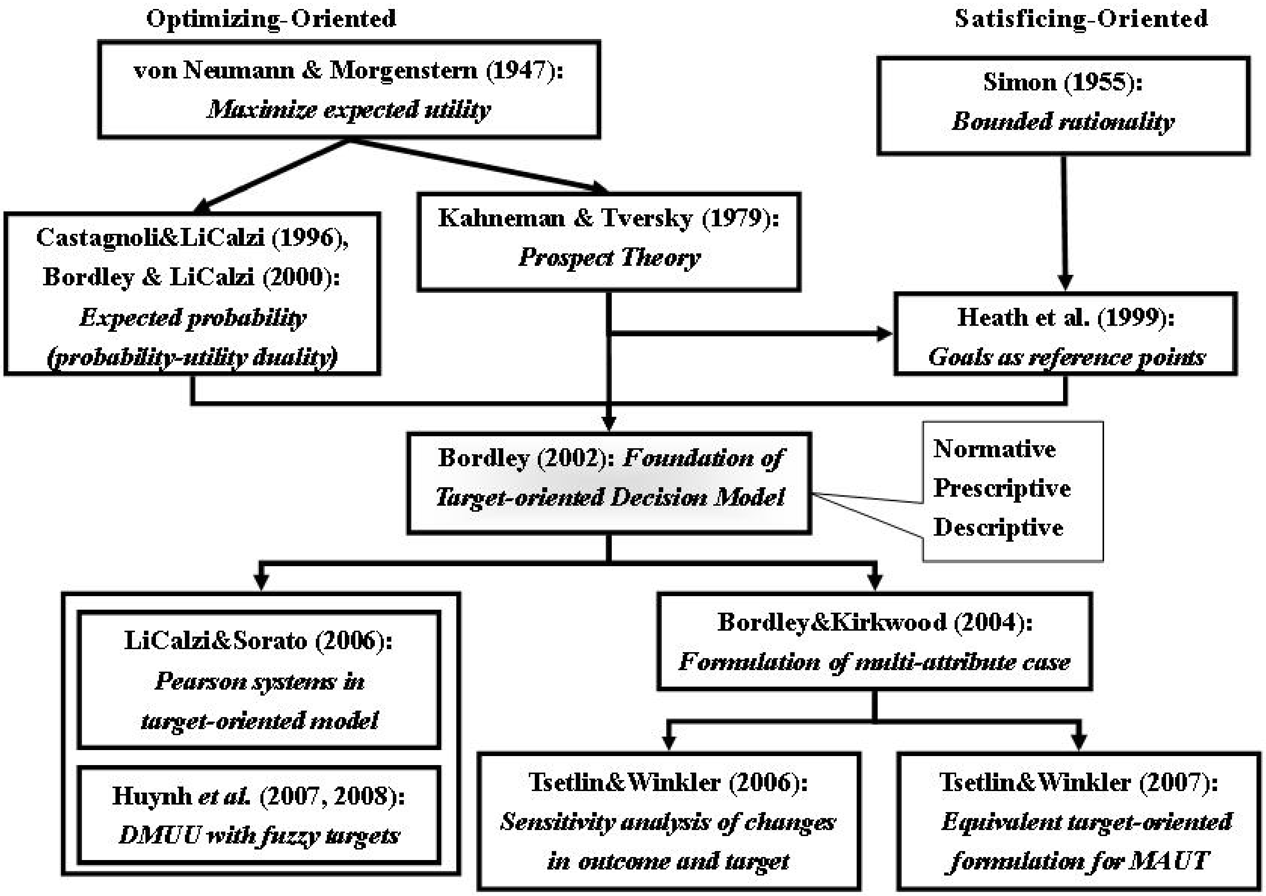

Figure 1.1: History formulation and development of target-oriented decision model

1.2 Problem Statement

We are highly motivated by the great appealing features of target-oriented decision model. After investigating and analyzing the current research on target-oriented decision model, our study focuses on three main problems: (1) single attribute target-oriented decision analysis; (2) multi-attribute target-oriented decision analysis; and (3) applications of target-oriented decision analysis. We will state our research problems in a great detail.

1. The first main problem investigated here is *Target-oriented decision analysis with different types of target preferences and hybrid uncertain targets*.
   The original target-oriented decision model presumes that the DM has a monotonically increasing target preference, e.g., the attribute/criterion wealth. However, there are three types of target preferences in most situations: "the more the better" (corresponding to benefit target preference), "the less the better" (corresponding to cost target preference), and range targets (too much or too little is not acceptable). These three target preferences are missed in the literature. Moreover,

target-oriented decision model views the cdf as the probability of meeting some target. Can the cdf be also used to model the other two types of target preferences?

In addition, target-oriented decision model assumes that target has a random probability distribution function (pdf). Some researchers try to use different pdfs to model the uncertain target. For example, LiCalzi and Sorato [89] use the Pearson system probability distributions to model the uncertain target. Bordely and Kirkwood [18] and Tsetlin and Winkler [131] use the normal distribution to model the uncertain target. However, it is not so easy to define the suitable pdf for the uncertain target. It is also well known that all facets of uncertainty cannot be captured by a single probability distribution. In decision analysis, fuzzy set is often used by DMs to linguistically specify their uncertain requirements. Thus is is necessary to consider fuzzy target-oriented decision analysis. Although Huynh et al. [61, 62] consider fuzzy uncertainty in target-oriented decision model, they only consider the payoff variables. In fact, fuzzy decision analysis has received a lot of attention since their pioneering work on fuzzy decision analysis by Bellman and Zadeh [12], and the Bellman-Zadeh paradigm is still widely used in many studies and applications. Thus it is necessary to do a comparative analysis with Bellman-Zadeh paradigm.

2. The second main problem is to consider *multiple attribute target-oriented decision analysis*.

   In many situations, multiple attributes are of interest. Some researches have extended the target-oriented decision model into multi-attribute case. In their model, multi-additive value function (MAVF) is used to aggregate partial target achievements by presuming the independence between different targets, e.g. [17]. Although independence assumption leads to convenient and simple use in real applications, interdependence/interaction phenomena among the targets are quite natural. Toward this end, Tsetlin and Winkler [131] consider decision analysis in a multi-attribute target-oriented setting and study the impacts on changes of expected utility in a parameters of performance and target distributions via simple statistics techniques. They firstly assume targets have some predefined probability distribution (norma distribution), and then model the interaction phenomena between different targets by using a function of correlations. However, as targets may have different probability distributions, their approach is limited and too complex in real applications. Furthermore, even if, in an objective sense the targets are mutually independent (probabilistically mutually independent), they are not necessary considered to be independent from the DM's subjective viewpoint. If the DM specifies fuzzy targets, Tsetlin and Winkler's approach will not be suitable. In this regard, traditional analytic methods are inadequate and not applicable for modeling such complex situations.

   Another subproblem in this part is the prioritization of different targets. Consideration of different relative importance information of different targets is important as some targets are more important than others. In this case, the DM associates different importance weights with different targets. The importance information associated with different targets plays a fundamental role in the comparison between alternatives by overseeing tradeoffs between respective satisfactions of different targets. A concept closely related to the importance information is the priority, which does not allow the tradeoffs between different targets. In some cases, the DM may have a prioritization between different targets. This type of multi-attribute target-oriented decision analysis is also missed in the literature.

3. Finally yet importantly, the third main problem is the *Application of target-oriented decision model.*
   Since its formulation, the research of target-oriented decision model mostly focuses on the theoretical part. In the literature, only one new product development example is studied. Due to the research context in Japan Advanced Institute of Science and Technology (JAIST), the Kansei evaluation problem will be studied as an application of target-oriented decision model. Many studies have attempted to solve Kansei evaluation in the literature. Generally speaking, there are two types of approaches to Kansei evaluation.

   Statistical methods: Statistical analysis plays an important role and is widely accepted as the most systematic tool for Kansei evaluation.

   Decision analysis methods: In addition to these methods, in closely similar and related studies on sensory evaluation or subjective evaluation, decision analysis has also been utilized in the evaluation problems.

   Previous studies have significantly advanced the issue of Kansei and Kansei-related evaluations. However, there are still two problems we need to solve. Firstly, consumers' preferences on Kansei attributes vary from person to person according to character, feeling, aesthetic and so on. For example, a Kansei attribute *fun* having left and right Kansei words as <solemn, funny>. Some consumers may prefer *solemn*, others may prefer *funny*, and there are also some consumers preferring *neither solemn nor funny*. In this regard, we will have *inconsistent order relations* on Kansei attributes. Furthermore, a consumer usually may have a priority order of the Kansei attributes, i.e., some Kansei attributes may be necessary to be satisfied.

   The objective of this part is to consider Kansei evaluation based on target-oriented decision model.

1.3 Main Contributions

All in all, the objective of this study is to include DMs' behavioral/psychological preferences into MADA problems, and then apply the proposed models into Kansei evaluation problems. Although not all multi-attribute problems deal with risk, the shape of the value function of the target-oriented decision model is the same as the gain/loss function of Prospect Theory, which represents DMs' behavioral preferences.

Our research strategy is threefold. Firstly, single target-oriented decision model will be studied to discuss different target preferences and hybrid uncertain targets. Secondly, multi-attribute target-oriented decision analysis will be studied to model the non-additive representation and prioritized representation. Thirdly, we will develop a Kansei evaluation model based on prioritized multi-attribute target-oriented decision analysis and conduct a case study for Kansei evaluation of Japanese traditional crafts to illustrate the proposed model. The main contributions of this thesis are summarized as follows.

1. **The first main contribution is that *we propose two methods to target-oriented decision model with different types of target preferences and extend those two methods to fuzzy target-oriented decision analysis.***

   (a) *The first sub-contribution in this part is that we develop two methods for target-oriented decision analysis with different target preferences.*
   In most studies on target-oriented decision analysis, monotonic assumptions are given in advance to simplify the problems, e.g., the attribute/criteria wealth. However, there are three types of target preferences. Thus two methods have been proposed to model the different target preference types: cdf based method and level set based method. No matter which method is selected, these two methods can both induce four shaped value functions: S-shaped, inverse S-shaped, convex shaped, and concave shaped, which represent DM's psychological preference depending different target preferences. The main difference between these two methods is that the level set based model induces a stricter value function than the cdf based model.

   (b) *The second sub-contribution in this part is that we extend those two random target-oriented decision analysis to fuzzy uncertain targets.*
   Target-oriented decision model assumes that target has a random probability distribution. Fuzzy numbers are usually used by DMs to linguistically specify their uncertain targets. In this thesis, two methods of fuzzy target-oriented decision analysis with respect to different target preferences have been proposed. To do so, firstly, a thorough analysis of possibility-probability transformations is given, and then the proportional approach is properly used to transform a possibility distribution into its associated probability distribution. Secondly, two methods of fuzzy target-oriented decision analysis have been obtained based on the random target-oriented decision model. Finally, some widely used fuzzy targets used in the pioneering work on fuzzy decision analysis by Bellman and Zadeh [12] are selected to illustrate the fuzzy target-oriented model. Our research outperforms better in terms of three aspects.

   The publications related to this part are [153, 155, 157, 158, 159, 160].

2. **The second main contribution is that *we develop a non-additive multi-attribute target-oriented decision model based on fuzzy measure and fuzzy integral, and put forward a prioritized OWA aggregation operator to model the prioritization between targets/attributes.***

   (a) *The first sub-contribution in this part is that we model the interdependence between different targets based on λ-fuzzy measure and Choquet fuzzy integral.*
   The use of fuzzy measures and fuzzy integral in MADA enables us to model some interaction phenomena existing among different attributes. As we shall see, multi-attribute target-oriented function has a similar structure with fuzzy measure, and fuzzy integral does not assume the independence. The fact that fuzzy integral model does not need to assume the independence of each target, means it can be used in non-linear situations. Thus we use fuzzy measure and fuzzy integral to model the interaction among targets. Since the specification

for fuzzy measures requires the values of a fuzzy measure for all subsets, the λ-fuzzy measure is used in order to reduce the difficulty of collecting information and the Choquet fuzzy integral is used to model the dependence in multi-attribute target-oriented decision analysis. A bisection search algorithm is also designed to identify the fuzzy measures of individual attributes group with a given λ value.

(b) *The second sub-contribution in this part is that we put forward a prioritized OWA aggregation operator to model the prioritization between different targets.*
To consider the prioritization between different targets, firstly the OWA operator is used to obtain the satisfaction degree for each priority level. To preserve the tradeoffs among the attributes in the same priority level, the degree of satisfaction for each priority level is viewed as a pseudo attribute. Secondly, we suggest that roughly speaking any t-norm can be used to model the priority relationships between the attributes in different priority levels. To keep the slight change of priority weight, strict Archimedean t-norms perform better in inducing priority weight. As Hamacher family of t-norms provide a wide class of strict Archimedean t-norms ranging from the product to weakest t-norm, Hamacher t-norms are selected to induce the priority weight for each priority level. Thirdly, considering DM's requirement toward the higher priority levels, a benchmark based approach is proposed to induce priority weight for each priority level, i.e. "the satisfactions of the higher priority targets are larger than or equal to the DM's requirements". We suggest that the weights of lower priority level should depend on the benchmark achievement of all the higher priority levels. To illustrate the effectiveness and advantages of the prioritized OWA operator mentioned above, we conduct several comparative analysis with previous work on prioritized aggregation.

The publications related to this part are [154, 159, 161].

3. **The third contribution is that *we develop a Kansei evaluation model based on prioritized multi-attribute fuzzy target-oriented decision analysis. A case study is also conducted to illustrate the proposed Kansei evaluation model.***
To overcome the those two above-mentioned problems in current research on Kansei evaluation, we put forward a Kansei evaluation model based on fuzzy target-oriented decision analysis and prioritized OWA aggregation operator. Firstly, like the traditional Kansei evaluation method, a preparatory experiment study is conducted in advance to select Kansei attributes by means of semantic differential (SD) method. In order to obtain Kansei data of products, a number of people are selected to assess products regarding these Kansei attributes. Secondly, these Kansei data are used to generate Kansei profiles for evaluated products by making use of the voting statistics. Thirdly, according to consumer-specified preferences on Kansei attributes, three main types of fuzzy targets are defined, to represent the consumers' preferences. Based on the principle of target-oriented decision analysis, we can obtain the satisfaction degrees (probabilities of meeting targets) regarding the Kansei attributes selected by consumers for all the evaluated products. Finally, considering

prioritization of the Kansei attributes, the prioritized OWA aggregation operator is used to aggregate the partial satisfaction degrees for the evaluated products.

Kansei evaluation has been applied to consumer products with successful results, e.g., table glasses, housing assessment, telephones, cars, and mobile phones. However, Kansei evaluation of traditional crafts has not been addressed yet. In Japan, there are many traditional crafts such as fittings, textile, etc. These beautiful, elegant and delicate products are closely related to and have played an important role in Japanese culture and life. Evaluations of these traditional crafts would be of great help for marketing or recommendation purposes. Thus the Japanese traditional crafts are used as a case study to illustrate the proposed Kansei evaluation model. By using our model, consumers can choose their preferred crafts according to their preferences.

The publications related to this part are [63, 152, 155, 156].

1.4 Overview of the Thesis

As we shall see in the next chapter, MADA problems can be categorized into two main steps: (1) to transform the consequence data into values according to DM's preferences; (2) to aggregate multiple scores into a global score. Thus, in this thesis the research topic, multi-attribute target-oriented decision analysis, is divided into two parts: (1) single attribute target-oriented decision analysis, which focuses on the transformations from the consequence data into target achievements/satisfaction degrees; (2) multi-attribute target-oriented decision analysis, which focuses on the aggregation of partial target achievements according to the principle of target-oriented decision analysis.

Fig. 1.2 summaries of the organization of the thesis. A detailed explanation is as follows:

- **Chapter 1** describes the motivations and objectives of this thesis, including the development history of target-oriented decision model, problems statement, and the organization of the thesis.

- **Chapter 2** is a background and literature review of MADA problems, including the following aspects: structure of MADA, a summary of MADA methods, and the inclusion of DMs' behavioral preferences into MADA.

- **Chapter 3** & **Chapter 4** address decision analysis under hybrid uncertain performance targets with different target preferences. **Chapter 3** deals with decision analysis under random uncertain target with different target preferences, where two approaches have been proposed. In **Chapter 4**, we also consider decision analysis under uncertainty with fuzzy targets.

- **Chapter 5** & **Chapter 6** consider multi-attribute target-oriented decision model. Particularly, due to the similarity of structure between multi-attribute target-oriented model and non-additive fuzzy measure and fuzzy integral, λ-fuzzy integral is used to model the interdependence among different targets in **Chapter 5**. Furthermore, in most cases importance information and priority information play different role in aggregation step. To consider the prioritization among different targets, a prioritized Ordered Weighted Averaging (OWA) aggregation operator based on OWA operator and triangular norms (T-norms) has been proposed in **Chapter 6**.

- As an application of the proposed prioritized multi-attribute target-oriented decision model, Kansei evaluation has been studied in **Chapter 7**. A case study of Japanese traditional crafts has also been conducted in **Chapter 8**
- **Chapter 9** contains a summary of the main contributions of the research and suggestions for future work.

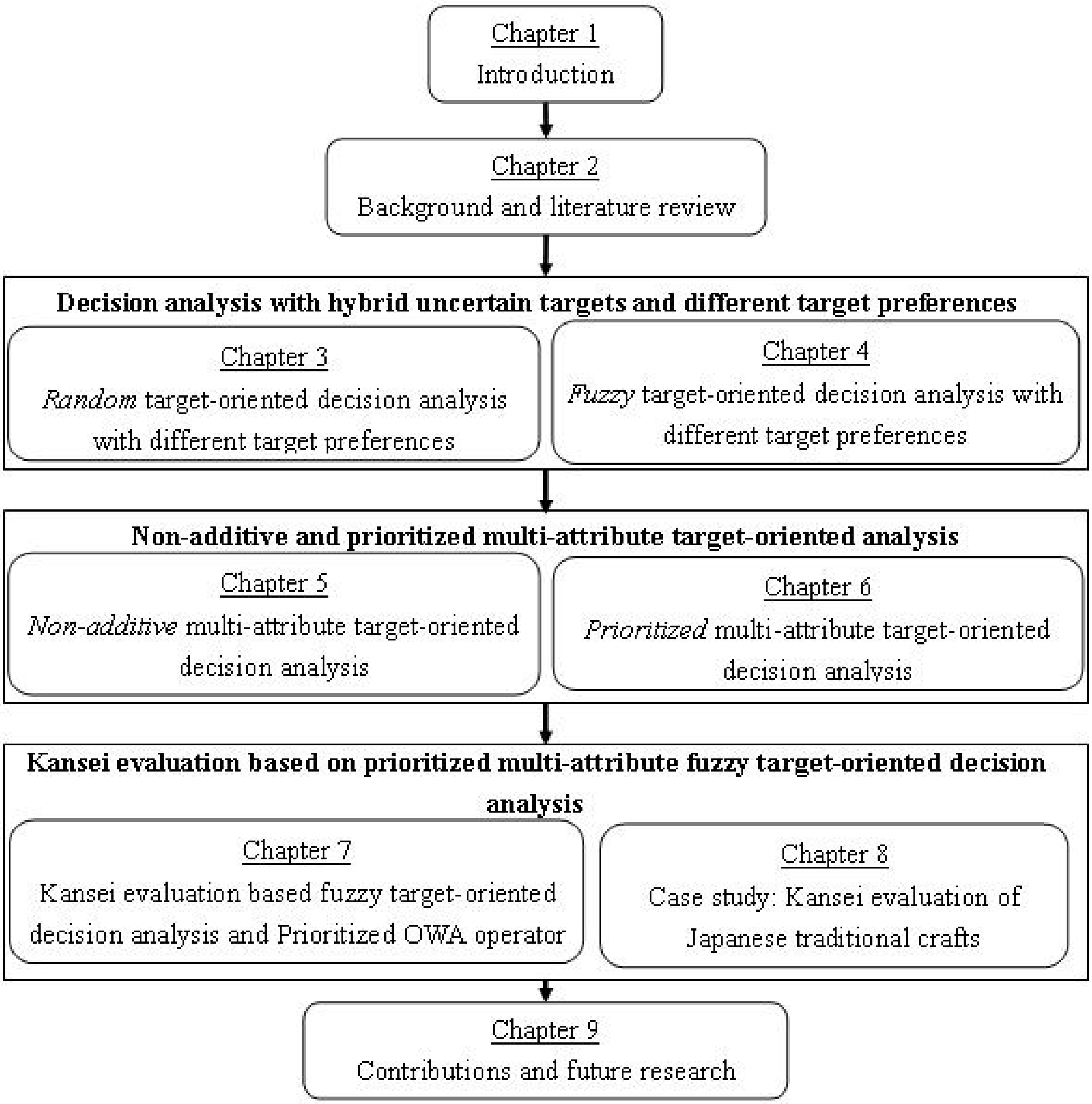

Figure 1.2: Content of this thesis

Chapter 2

Background and Literature Review of Multi-Attribute Decision Analysis

Abstract: In this chapter, a background and literature review of multi-attribute decision analysis (MADA) are presented to provide a foundation for the research in this thesis. We first provide some basic information about MADA and its related research. Secondly, the structure of MADA is presented. Thirdly, we classify different MADA methods from three aspects. Finally, two behavioral MADA methods are introduced and discussed.

2.1 Relationships Between MCDA, MADA, and MODA

Decision analysis is characterized by its involvement with information, value assessments, and optimization. Thus, whereas inventiveness seeks many possible answers and analysis seeks one actual answer, decision making seeks to choose the one best answer [38]. But the "one best answer" can be difficult to obtain, particularly when the decision is based on several objectives. Multi-criteria decision analysis (MCDA), sometimes called multi-criteria decision making (MCDM), is a discipline aimed at supporting decision makers (DMs) who are faced with making numerous and conflicting evaluations.

MCDA is one of the most widely used decision methodologies in the sciences, business, and engineering worlds. Some applications of MCDA in engineering include the use on flexible manufacturing systems, layout design, integrated manufacturing systems. A typical problem in MCDA is concerned with the task of ranking a finite number of decision alternatives, each of which is explicitly described in terms of different characteristics (also, often called attributes, decision criteria, or objectives), which have to be taken into account simultaneously [128]. *Attributes* are generally defined as characteristics that describe in part the state of a product or system, while *objectives* are attributes with a goal and a direction "to do better" as perceived by the DM [64]. Specifically, *goals* are things desired by the DM expressed in terms of a specific state in space and time. Thus, while objectives give the desired direction, goals give a *desired* (or *target*) level to achieve [64]. In many cases, however, the terms "objective" and "goal" are used interchangeably.

Ching-Lai Hwang has been on the forefront of the development of new techniques and the enhancement of existing ones that aid the DM. His two references [64, 65] list a multitude of techniques, grouped in two classes: the Multi-Attribute Decision Analysis (MADA) and Multi-Objective Decision Analysis (MODA) techniques. With the above-mentioned definition for objectives in mind, MODA problem involve the design of alternatives which optimize or "best satisfy" the objectives of the DM [64]. MADA problems involve the selection of the "best" alternative from a pool of preselected alternatives described in terms of their attributes" [64]. In other words, MODA problems are optimization problems (continuous MCDA), whereas MADA problems are product selection problems (discrete MCDA). Together all techniques for solving both problems can be classified as MCDA techniques. While criteria typically describe the standards of judgment or rules to test acceptability, here they simply indicate attributes and/or objectives.

In this thesis, we use MADA to represent the discrete MCDA problems (product selection problems), MADA and MCDA are used interchangeably. Whereas MODA is used to denote the continuous MCDA problems.

2.2 Structure of Multi-Criteria Decision Analysis

MCDA begins with a serial process of defining objectives, arranging them into criteria (attributes), identifying all possible alternatives, and then measuring consequences. Note that a consequence is a direct measurement of the succuss of an alternative according to a criterion, and it does not include preferential information. The process of structuring MCDA problems has received a great deal of attention. We follow the three steps of Roy's general modeling methodology for decision analysis problems [114]:

1. **Object of decision**. That is, defining the object upon which the decision has to be made and the rationale of the decision.

2. **Family of criteria**. That is, the identification and modeling of a set of criteria that affect the decision, and which are exhaustive and non-redundant.

3. **Global preference model**. That is, the definition of the function that aggregates the marginal preferences upon each criterion into the global preference of the DM about each alternative.

We will explain these three aspects in a great detail.

1. **Object of decision**
   The first and the most important step for studying a multi-attribute decision problem is the identification of decision object. Roy [114] refers to the notion of the decision "problematic". The four types of common decision problematics identified in MCDA literature are as follows:

   (a) ***Choice***, which involves choosing one alternative from a set of alternatives.

   (b) ***Sorting***, which involves classifying alternatives in predefined homogenous groups in a given preference order.

   (c) ***Ranking***, which involves ranking alternatives from best to worst.

   (d) ***Description***, which involves describing all the alternatives in terms of their major distinguishing features.

2. **Family of criteria/attributes**
   The set of all alternatives is analyzed in terms of multiple attributes, in order to model all possible impacts, consequences, or attributes. In MCDA, there types of criteria are formally used [66]:

   (a) ***Measurable***, is a criterion that allows quantified measurement upon an evaluation scale.

   (b) ***Ordinal***, is a criterion that defines an ordered set in the form of a qualitative or a descriptive scale.

   (c) ***Interval, probabilistic, fuzzy data***. Sometimes uncertainty must be considered. The data may be expressed as interval data, probabilistic data, and fuzzy data. Probabilistic is a criterion that uses probability distributions to cover uncertainty in the evaluation of alternatives. Fuzzy is a criterion where evaluation of alternatives is represented in relationship to its possibility to belong in one of the intervals of a qualitative or descriptive evaluation scale.

   Generally speaking, the data used in MCDA can be divided into two large categories, numerical data and non-numerical data. With numerical data, information is conveyed using the known properties of numbers. Non-numerical data may use numbers, but only nominally, or may apply non-numerical structures. In this thesis, only the numerical data are considered.

3. **Global preference model**
Throughout this step, the development of a global preference model provides a way to aggregate the values of each criterion in order to express the preferences between the different alternatives. MCDA literature identifies the following categories of preference modeling approaches:

   (a) **Value-Focused models [73]**, where a value system for aggregating the user preferences on the different criteria is constructed. In such approaches, marginal preferences upon each criterion are synthesized into a total value using a synthesizing utility function.

   (b) **Outranking Relations models [113]**, where preferences are expressed as a system of outranking relations between the alternatives, thus allowing the expression of incomparability. In such approaches, all alternatives are one-to-one compared between them, and preference relations are provided as relations "a is preferred to b", "a is equally preferred to b", and "a is incomparable to b".

   (c) **Multi-Objective Optimization models [168]**, where criteria are expressed in the form of multiple constraints of a multi-objective optimization problem. In such approaches, usually the goal is to find a Pareto optimal solution for the original optimization problem.

   (d) **Preference Disaggregation models [66]**, where the preference model is derived by analyzing past decisions. Such approaches build on the models proposed by the previous ones (thus they are sometimes considered as a sub-category of other modeling approaches' categories), since they try to infer a preference model of a given form (e.g. value function) from some given preferential structures that have led to particular decisions in the past. Inferred preference models aim at producing decisions that are at least identical to the examined past ones.

2.3 Multi-Attribute Decision Analysis Methods Based on Decision Makers' Preference Expressions

During the last thirty years, a multitude of models has been developed to solve MADA problems. The **value-focused thinking** [73] method provides a systematics analysis method, which will be studied in this thesis. To better review different models of MADA, we shall discuss MADA from DMs' preference information based on the work by Chen [32]. Generally speaking, there are three kinds of preference expressions: value functions (preferences on consequences), weights (preferences on criteria), and aggregation operators (preferences on aggregation modes).

Common to all MADA techniques is the concept of a decision matrix. The basic structure of a decision matrix is an M-by-N matrix, as shown in Table 2.1. In this table, $\mathcal{A} = \{A^1, A^2, \cdots, A^m, \cdots, A^M\}$ is the set of alternatives, and $\mathcal{X} = \{X_1, X_2, \cdots, X_n, \cdots, X_N\}$ is the set of attributes/criteria. The consequence on attribute X_n of alternative A^m is expressed as $X_n(A^m)$, which can be shortened to X_n^m when there is no possibility of confusion. Note that there are M alternatives and N attributes altogether.

Table 2.1: Multi-attribute decision matrix

Alternatives	Attributes				
	X_1	$\cdots$	X_n	$\cdots$	X_N
A^1	X_1^1	$\cdots$	X_n^1	$\cdots$	X_N^1
A^2	X_1^2	$\cdots$	X_n^2	$\cdots$	X_N^2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A^m	X_1^m	$\cdots$	X_n^m	$\cdots$	X_N^m
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A^M	X_1^M	$\cdots$	X_n^M	$\cdots$	X_N^M

2.3.1 Preferences on consequence data

There are several ways for a DM to express preferences based directly on consequences. Among them, the best known are utility theory-based definitions [75] and outranking based definitions [114]. Some normalization methods can be regarded as transformations from consequences to preferences.

Definition The DM's preference on consequence for attribute X_n of alternative A^m is a value $c_n(X_n^m) = c_n^m$. The DM's preference on consequences over all attributes for alternative A^m is the value vector

$$\mathsf{c}^m = (c_1^m, \cdots, c_n^m, \cdots, c_N^m). \tag{2.1}$$

Values are refined data obtained by processing consequences according to the needs and objectives of the DM. The relationship between consequences and values can be expressed as a value function as

$$c_n^m = f_n(X_n^m) \tag{2.2}$$

where c_n^m and X_n^m are a value and a consequence, respectively, and $f_n(X_n^m)$ is a mapping function to realize the transformation from consequences to preferences. A commonly used mapping function is to as follows

$$f_n(X_n^m) : X_n^m \to [0, 1] \tag{2.3}$$

There are many approaches to generating values based on consequences. In this thesis, we consider two types of approaches based on Chen [32]: single alternative-based methods and binary alternative-based methods.

1. **Single alternative-based methods**
   Single alternative-based methods focus on the expressions of values according to one alternative, such methods are as follows.

   (a) **Utility functions [75]**
   In this method, a subset of alternatives are selected and ranked subjectively.

This subset of alternatives is further used to determine a utility function for all alternatives based on a monotonic piecewise linear utility function for the attributes and their subjective preferences.

(b) **Normalization functions [65]**
There are two types normalization functions: linear and non-linear. In both these two methods, the first step is to identify the maximum and minimum values for each attribute, denoted as

$$X_n^{\min} = \min_{m=1,\cdots,M} [X_n^m] \tag{2.4}$$

and

$$X_n^{\max} = \max_{m=1,\cdots,M} [X_n^m] \, . \tag{2.5}$$

Two simple but frequently used linear transformation functions are shown in Table 2.2

Table 2.2: Two linear transformation functions

Methods	Attribute type	
	Benefit attribute	Cost attribute
Method 1	$c_n^m = \frac{X_n^m - X_n^{\min}}{X_n^{\max} - X_n^{\min}}$	$c_n^m = \frac{X_n^{\max} - X_n^m}{X_n^{\max} - X_n^{\min}}$
Method 2	$c_n^m = \frac{X_n^m}{X_n^{\max}}$	$c_n^m = \frac{X_n^{\min}}{X_n^m}$

These two transformation methods assume that all consequences are real numbers. There are some drawbacks in method 2. When the consequence data is non-positive (negative and zero), the transformation method will not be suitable. However, in many MADA problems, the consequence data are given in positive real numbers. In addition, due to the simplicity of these two methods, they are are widely used in the literature.

(c) **Fuzzy set based approach [12]**
In their pioneering work on fuzzy decision making, Bellman and Zadeh [12] suggested that an attribute can be represented as a fuzzy subset over the alternatives. In particular, they modeled objectives and attributes together to form the decision space which is represented by a fuzzy set whose membership function is the degree to which each alternative is a solution. This method is widely used in fuzzy MADA problems.

(d) **Aspiration-level functions [93]**
The approach involves the user choosing levels of the objectives that he desires to achieve (levels of aspiration), and provides him with various kinds of feedback.

2. **Binary alternative-based methods**
Binary relation-based models focus on expressions of values via comparisons of two

alternatives. The following binary relation-based methods employ numerical data to represent values.

(a) **Analytic Hierarchy Process Method [116]**
The Analytic Hierarchy Process (AHP) method was originally introduced by Saaty and is intended to solve such product selection problems that have a hierarchical structure of attributes. Attributes in one level are compared in terms of relative importance with respect to an element in the immediate higher level, treating the pairwise comparison with the eigenvector method as outlined in [119]. This process is executed from the top down starting with the overall goal as the single top element of the hierarchy and closing with the alternatives at the very bottom, ranking the attributes/alternatives at each level with respect to the overall goal.

While AHP method is well known, it has several disadvantages as outlined in [119]. First, it requires attributes to be independent with respect to their preferences, which is rarely the case in product selection cases. Second, all attributes and alternatives are compared with each other (at a given level), which may cause a logical conflict of the kind: $A > B$ and $B > C$ but $C > A$. The likelihood of such conflicts occurring in the hierarchy tree increases dramatically with the number of alternatives and attributes. Last but not least, AHP has the potential of introducing a rank reversal of alternatives, depending on the number of alternatives assessed, which is particularly troublesome for normative decision making environments [119].

(b) **ELECTRE [114]**
ELECTRE [114] is a family of MADA methods that originated in Europe in the mid-1960's. The acronym ELECTRE stands for: ELimination Et Choix Traduisant la REalité (ELimination and Choice Expressing REality). The method was first proposed by Bernard Roy and his colleagues at SEMA consultancy company. A team at SEMA was working on the concrete, multiple criteria, real-world problem of how firms could decide on new activities and had encountered problems using a weighted sum technique. Bernard Roy was called in as a consultant and the group devised the ELECTRE method. As it was first applied in 1965, the ELECTRE method was to choose the best action(s) from a given set of actions, but it was soon applied to three main problems: choosing, ranking and sorting. ELECTRE employs concordance and discordance matrices to transform consequences to values.

(c) **The PROMETHEE method [21]**
The PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluation) is a MADA method developed by Brans and Vincke [21]. It is a quite simple ranking method in conception and application compared with other methods used for multi-attribute analysis. It is well adapted to problems where a finite number of alternatives are to be ranked according to several, sometimes conflicting criteria/attributes. The evaluation table is the starting point of the PROMETHEE method. In this table, the alternatives are evaluated on the different criteria.

The implementation of PROMETHEE requires two additional types of information, namely: (1) Information on the relative importance that is the weights of the criteria/attributes considered. (2) Information on the DM's preference function, which he/she uses when comparing the contribution of the alternatives in terms of each separate criterion.

2.3.2 Preferences on attributes

Preferences on criteria/attributes refer to expressions of the relative importance of criteria/attributes. They are generally called **weights**; the weight for attribute X_n is w_n. It is usually assumed that $w_n \geq 0$ for all criteria, and $\sum_{n=1}^{N} w_n = 1$. A weight vector is denoted $\mathrm{W} = (w_1, \cdots, w_n, \cdots, w_N)$.

Belton and Stewart [13] summarize two kinds of weights: tradeoff-based weights and non-tradeoff based weights. Tradeoff-based weights emphasize the "compensation" of values across attributes, which permits preference data to be compared as they are aggregated into a single representative evaluation. Non-tradeoff based weights do not permit direct tradeoffs across criteria; they are usually associated with outranking methods. Several weights determination approaches are as follows:

- AHP and geometric ratio weighting. These two methods are *integrated* methods, which means they proceed from values and weight assessments to aggregated preferences to final results.

- Swing weights apply ratio data to represent weights. This direct estimation method is preferred by von Winterfeldt and Edwards [13].

- Ordered weighted averaging (OWA) weights. Many techniques are available to calculate the OWA weights [44]. We could resolve a mathematical programming problem [44, 138, 139], associate it with a linguistic quantifier [44, 141], or obtain OWA weights via an analytic method [43].

- Data envelopment analysis (DEA) employs ratio data to represent weights. This method, proposed by Cook et al. [35], has the unique feature that the values of weights are determined by optimizing the measure of each alternative.

2.3.3 Preferences on aggregation modes

After the basic construction of an MADA problem and acquisition of preferences from the DMs, a global model to aggregate preferences and solve a specified problem (choose, rank or sort) may be constructed. By using this, an aggregation function F is used to aggregate each c_n^m into an overall degree of satisfaction $\mathrm{Val}(A^m)$ with respect to the set of attributes such that

$$\mathrm{Val}(A^m) = F\left(c_1^m, \cdots, c_n^m, \cdots, c_N^m\right) \tag{2.6}$$

The choice of the form for F models the DM's desired imperative and individual preference for combining the criteria [94, 148].

Consideration of different relative importances of different criteria is important as some criteria are more important than others. In this case, the DM associates different importance weights with different attributes [23, 79, 94, 101, 125, 126, 141, 144]. There are several approaches to incorporating and/or assigning weights to different criteria. Typical is some form of weighted arithmetic mean, such as quasi-arithmetic means, weighted arithmetic means, weighted quasi-arithmetic means [23]. These aggregation operations work well in situations in which any differences are viewed as being in conflict because the operator reflects a form of compromise behavior among the various criteria [94, 137]. In general, the importance information associated with different criteria plays a fundamental role in the comparison between alternatives by overseeing tradeoffs between respective satisfactions of different criteria [148, 150].

Many studies have attempted to include different priorities of criteria into MADA problems in the literature. Generally speaking, approaches to prioritized MADA can be classified into two categories according to our knowledge. Approaches belonging to the first class aim to use non-monotonic intersection operator [58, 142] and triangular norms (t-norms) to model the priority relationships among criteria, for more detail, see [30, 39, 95, 143, 145, 150, 31].

Furthermore, in real applications, for purposes of simplicity, independence among different attributes are given in advance. Although independence assumption leads to convenient and simple use in real applications, interdependence/interaction phenomena among the attributes is very natural. Considering the interdependence among different attributes, fuzzy measure and fuzzy integral are widely used in MADA [47, 49, 48, 50, 51].

Fig. 2.1 shows the relationships among the main different aggregation methods. It should be noted that, in the literature of MADA, aggregation operators have been widely studied. For more details and properties of aggregations operators, see [23, 22, 36, 50, 126].

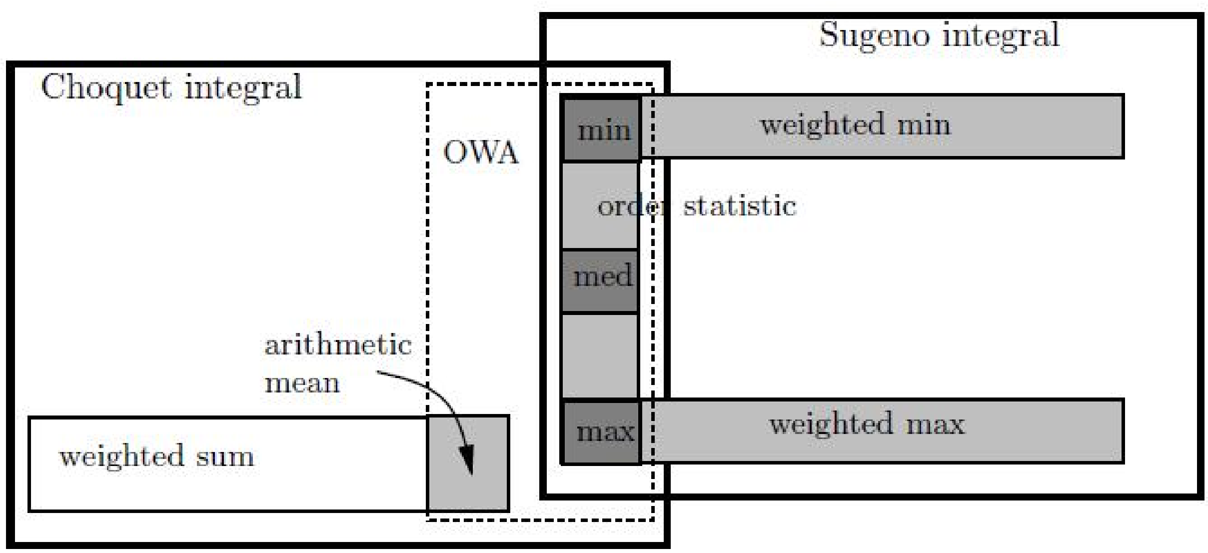

Figure 2.1: Set relations between various aggregation operators, adapted from Detyniecki [36]

2.4 Inclusion of Decision Makers' Behavioral Preferences into Multi-Attribute Decision Analysis

Recent research shows that DMs' behavioral aspects play an important role in human decision making, and this was recognized by the award of the 2002 Nobel Prize in Economics to Daniel Kahneman. In the following, we will introduce two behavioral MADA methods based on the S-shaped value function in Prospect Theory [70].

2.4.1 The TODIM method

The TODIM method (an acronym in Portuguese of Interactive and Multi-Criteria Decision Making), is a discrete multi-criteria method based on **Prospect Theory** [70]. The TODIM method makes use of a global measurement of value calculable by the application of the paradigm of Prospect Theory. In this way, the method is based on a description, proved by empirical evidence, of how people effectively make decisions under risk. The use of TODIM relies on a global multi-attribute value function, which is built in parts, with their mathematical descriptions reproducing the gain/loss function of Prospect Theory. The global multi-attribute value function of TODIM then aggregates all measures of gains and losses over all attributes.

From the construction of the aforementioned TODIM additive difference function, which functions as a multi-attribute value function and, as such, must also have its use validated by the verification of the condition of mutual preferential independence [75], the method leads to a global ordering of the alternatives. It can be observed that the construction of the multi-attribute value function, or additive difference function, of the TODIM method is based on a projection of the differences between the values of any two alternatives to a referential attribute or reference attribute.

The TODIM method makes use of pair comparisons between the decision attributes, using technically simple resources to eliminate occasional inconsistencies arising from these comparisons. It also allows value judgments to be carried out in a verbal scale, using a attributes hierarchy, fuzzy value judgments and making use of interdependence relationships among the alternatives. Roy and Bouyssou [113] state that it is: "...a method based on the French School and the American School. It combines aspects of the Multi-Attribute Utility Theory, of the AHP method and the ELECTRE methods" (p. 638). The concept of introducing expressions of losses and gains in the same multi-attribute function, present in the formulation of the TODIM method, gives this method some similarity to the PROMETHEE methods, which make use of the notion of net outranking flow, in other words, it is based on a notion extremely similar to a net flow, in the PROMETHEE sense.

Let us consider the multi-attribute decision matrix, as showed in Table 2.1. The TODIM method consists of the following four steps [46]:

1. Obtain the consequence data and the weights of attributes
   The consequence on attribute X_n of alternative A^m is expressed as X_n^m. The weights of attributes are expressed as $\mathrm{W} = (w_1, \cdots, w_n, \cdots, w_N)$, where for all attributes $w_n \geq 0$ and $\sum_{n=1}^{N} w_n = 1$.

2. Normalize the consequence data
   The value transformed from the consequence data over all attributes for alternative A^m is a value vector, denoted as

$$\mathsf{c}^m = (c_1^m, \cdots, c_n^m, \cdots, c_N^m)$$

In TODIM method, the normalization method is as follows

$$c_n^m = \frac{X_n^m}{\sum_{m=1}^{M} X_n^m} \tag{2.7}$$

3. Obtain the partial matrices of dominance and the final matrix of dominance

   In this step, the DM must indicate which attribute is to be chosen as the reference attribute for the calculations according to the importance weight assigned to each attribute. In this way, the attributes with the highest value accorded to its importance will usually be chosen as the reference attribute. The reference attribute is denoted as X_r. We also denote $w_{n/r}$ is the weight of attribute X_n divided by the weight of the reference attribute X_r. Using $w_{n/r}$ allows all pairs of differences between performance measurements to be translated into the same dimension, i.e. that of the reference attribute. The measurement of dominance of each alternative A^m over each alternative A^l, now incorporated to Prospect Theory, is given by the mathematical expression

$$\delta(A^m, A^l) = \sum_{n=1}^{N} \Phi_n(A^m, A^l),\ \forall (m,l) \tag{2.8}$$

   where

$$\Phi_n(A^m, A^l) = \begin{cases} \sqrt{\frac{w_{n/r}(c_n^m - c_n^l)}{\sum_{n=1}^{N} w_{n/r}}}, & \text{if } c_n^m - c_n^l \geq 0; \\ \frac{-1}{\theta}\sqrt{\frac{\left(\sum_{n=1}^{N} w_{n/r}\right)(c_n^l - c_n^m)}{w_{n/r}}}, & \text{if } c_n^m - c_n^l < 0. \end{cases} \tag{2.9}$$

   Here $\delta(A^m, A^l)$ represents the measurement of dominance of alternative A^m over alternative A^l; N is the number of attributes; X_n is any attribute, for $n = 1, \cdots, N$; $w_{n/r}$ is equal to w_n divided by w_r, where r is the reference attribute; c_n^m and c_n^l are the values of the alternatives A^m and A^l with respect to attribute X_n, respectively; θ is the attenuation factor of the losses, different choices of θ lead to different shapes of the prospect theoretical value function in the negative quadrant.

   The expression $\Phi_n(A^m, A^l)$ represents the parcel of the contribution of attribute X_n to function $\delta(A^m, A^l)$, when comparing alternative A^m with alternative A^l.

   - If the value of $c_n^m - c_n^l$ is positive, it will represent a gain for the function $\delta(A^m, A^l)$ and, therefore the expression $\Phi_n(A^m, A^l)$ will be used.
   - If $c_n^m - c_n^l$ is nil, the value zero will be assigned to $\Phi_n(A^m, A^l)$.
   - If $c_n^m - c_n^l$ is negative, $\Phi_n(A^m, A^l)$ will be negative.

   The construction of function $\Phi_n(A^m, A^l)$ in fact permits an adjustment of the data of the problem to the value function of Prospect Theory, thus explaining the aversion and the propensity to risk.

4. Obtain the global dominance matrices
   The overall values of the various alternatives are combined to produce a rank ordering by computing the following values:

$$\xi_m = \frac{\sum_{n=1}^{N} \delta(A^m, A^l) - \min \sum_{n=1}^{N} \delta(A^m, A^l)}{\max \sum_{n=1}^{N} \delta(A^m, A^l) - \min \sum_{n=1}^{N} \delta(A^m, A^l)} \tag{2.10}$$

where ξ_m is the overall value of alternative A^m.

The TODIM method assumes that the consequence data in certain. However, in most cases, the consequence data may be uncertain, e.g. fuzzy interval, probabilistic. In addition, the TODIM method assumes the mutual preference independence among different attributes, this is unrealistic in many situations. Furthermore, the TODIM method needs a normalization of consequence data. As we introduced before, the normalization is a kind of preference expression. Finally, the TODIM method tries to part away from the utility axiomatization.

2.4.2 The SMAA-P method

Stochastic multicriteria acceptability analysis (SMAA) methods handle imprecise, partly missing, or conflicting weight information by exploring the weight space in order to describe what kind of weights, if any, make an alternative most preferred. During the analysis, both criteria measurements and weights are constrained by their distributions. Several versions of SMAA have been developed. The original SMAA method [81] considers alternatives' acceptability for the first rank through additive value functions. SMAA-2 [83] extends the analysis to consider all ranks thereby improving the possibilities of finding good compromise solutions in group decision making problems. SMAA-O [82] is developed for problems with mixed ordinal and cardinal criteria.

The SMAA-P method combines features from prospect theory and the SMAA-2 method. Similar to SMAA-2, SMAA-P has been developed for multi-attribute group decision problems where neither criteria measurements nor weights are precisely known. The descriptive measures that SMAA-P computes for the alternatives are also similar to the rank acceptability indices, central weight vectors and confidence factors of SMAA-2. The main difference between SMAA-P and SMAA-2 is that SMAA-P is not based on a utility or value function model; instead the DMs preferences are represented in the spirit of prospect theory for riskless choice.

The SMAA-P method considers a special case, where the weights of attributes and consequence data are constrained by probability distributions. In addition, it focuses on group decision analysis problems.

2.5 Conclusions

In this chapter, we give a background and literature review of (MADA) problems to provide a foundation for the research in this thesis. The basic context of MADA is explained as follows:

1. Firstly, MADA, MCDA, and MODA problems are introduced and classified.
2. Secondly, the structure of MADA problems is explained in detail. The value-focused model will be used in this thesis.
3. Thirdly, a summary of MADA techniques based on DMs' preference expression is given: (1) preferences on consequence data; (2) preferences on attributes; and (3) preferences on aggregation modes.

4. Finally, two main techniques for including behavioral preferences into MADA are discussed: the TODIM method and SMAA-P method.

Based on the background information and literature review, from the next chapter, we shall begin our topic: multi-attribute target-oriented decision analysis and its applications. In the literature, the MADA problems can be categorized into two main steps: (1) transformation from consequence data into values according to DM's preferences, and (2) aggregation of partial values into a global value. Thus, we shall study the multi-attribute target-oriented decision analysis from two aspects: single attribute case (focusing on how to obtain values) and multi-attribute case (focusing on how to model the aggregation under the target-oriented decision principle).

Chapter 3

Random Target-Oriented Decision Analysis with Different Target Preferences

Abstract: In most studies on target-oriented decision analysis, monotonically increasing assumption is given in advance to simplify the decision problems, e.g., the attribute wealth. In this case, the decision maker (DM) prefers "the more the better", and then target-oriented decision model views the cumulative distribution function (cdf) as the probability of meeting targets. However, there are another two types of target preferences: "the less the better" (corresponding to cost target preference), and range targets (too much or too little is not acceptable). The main focus of this chapter is to model the three types of target preferences in target-oriented decision model. Toward this end, two methods have been developed to model the different target preference types: cdf based method and level set based method. The results show that no matter which method is selected, these two methods can both induce four shaped value functions: S-shaped, inverse S-shaped, convex, and concave. These four shaped value functions can represent DMs' psychological preferences. The main difference between these two methods is that the level set based method induces a steeper value function than the cdf based method.

3.1 Introduction

Traditionally, when modeling a decision maker (DM)'s rational choice between acts with uncertainty, it is assumed that the uncertainty is described by a probability distribution on the space of states, and the ranking of acts is based on the expected utilities of the consequences of these acts. This utility maximization principle was justified axiomatically in von Neumann and Morgenstern [134] and Savage [117]. As Simon [120] argued, the traditional utility theory presumes that a rational DM was assumed to have "a well-organized and stable system of preferences, and a skill in computation" that was unrealistic in many decision contexts [16]. At the same time, Simon proposed his famous behavioral model for rational choice, so-called *bounded rationality*, which implies that due to the cost or the practical impossibility of searching among all possible acts for the optimal, the DM simply looks for the first "satisfactory" act that meets some predefined targets. It is also concluded that human behavior should be modeled as *satisficing* instead of optimizing. Intuitively, the satisficing approach has some appealing features because thinking of targets is quite natural in many situations.

Particularly, in an uncertain environment, each act a may lead to different outcomes usually resulting in a random consequence X_a. Then, given a target t, the agent can only assess the probability $\Pr(X_a \succeq t)$ of the act a's consequence meeting the target. In this case, according to the optimizing principle, the agent should choose an act a that maximizes the probability $\mathrm{Val}(a) = \Pr(X_a \succeq t)$ [96]. Although simple and appealing from Simon's satisficing-oriented point of view, its resulted model is still not complete because there may be uncertainty about the target itself. Therefore, Castagnoli and LiCalzi [25]and Bordley and LiCalzi [18] have relaxed the assumption of a known target by considering a random consequence T instead. Then the target-oriented decision model prescribes that the DM should choose an act a that maximizes the probability of meeting an uncertain target T, provided that the target T is stochastically independent of the random consequences to be evaluated. The satisficing approach is sufficient but not necessary to make target-oriented decisions.

On the other hand, traditional utility theory presumes that the DM has to define a utility function for an attribute. However, substantial empirical evidence has shown that it is difficult to build mathematically rigorous utility functions based on attributes [17] and the conventional attribute utility function often does not provide a good description of individual preferences [70, 132, 133]. As a substitute for the utility theory, Kahneman and Tversky [70] proposed an S-shaped value function, and Heath et al. [54] suggested that the inflection point in this S-shaped value function can be interpreted as a target. To develop this concept further, target-oriented decision analysis involves interpreting an increasing, bounded function, properly scaled, as a cumulative distribution function (cdf) and relating it to the probability of meeting or exceeding a target value. Note that if a target is fixed, its cdf simplifies to a step function with a single step at the target. Abbas and Matheson [3] model target setting in organizations. They define "aspiration equivalents" for the alternatives under consideration based on the organization's utility function, drawing an analogy with Simon's [120] notion of satisficing by seeking an alternative that meets or exceeds an aspiration level, and show that these aspiration equivalents can be used as targets.

Target-oriented decision analysis focuses on using a distribution to represent the utility function. In fact, Berhold [14] notes that "there are advantages to having the utility function represented by a distribution" (p. 825), arguing that it permits the use of the known properties of distribution functions to find analytic results. Manski [96] calls this the "utility mass model". Castagnoli and LiCalzi [25] provided a formal equivalence of von Neumann and Morgenstern's expected utility model and the target-based model with reference to preferences over lotteries and lately, Bordley and LiCalzi [18] showed a similar result for Savage's expected utility model with reference to preferences over acts. Thus, despite the differences in approach and interpretation, both target-oriented decision procedure and utility-based decision procedure essentially lead to only one basic model for decision analysis. In maximizing expected utility, a DM behaves as if maximizing the probability that performance is greater than or equal to a target, whether the target is real or just a convenient interpretation.

In general, target-oriented decision model lies in the philosophical root of *bounded rationality* [120] as well as represents the S-shaped value function in *prospect theory* [70]. Although previous research greatly advanced target-oriented decision analysis, in most studies on target-oriented decision analysis, monotonically increasing assumption of attribute is given in advance to simplify the decision problems, e.g. the attribute wealth. In decision analysis under uncertainty based on target-oriented decision model, the payoff variable is also the monotonically increasing preference. In this case, the DM prefers "the more the better". However, as well-known, in the context of decision analysis involving targets, usually there are three types of targets: "the more the better" (corresponding to benefit target), "the less the better" (corresponding to cost target), and target values are fairly fixed and not subject to much change, i.e., too much or too little is not acceptable (we shall call this type of targets as range level type). Thus it is important to consider these three types of targets. The target-oriented decision model views the cdf as the probability of meeting the uncertain target T. In case of benefit target, the probability of meeting target is indeed the cdf. Can the cdf also be used in other types of target preferences? Furthermore, in the probability theory, the level set of probability density function (pdf) also provides a convenient way to represent the probability distribution. Can the the level set of pdf also be used in the target-oriented decision model?

Due to the above-mentioned two observations, the main focus of this chapter is to consider the target-oriented decision model with different types of target preferences by making use of the cdf and the level set of pdf. The key idea of our work is to add a target achievement level u. The rest of this chapter is organized as follows. In Section 3.2, we present the cdf based method for target-oriented decision analysis with different target preferences. The level set based method for target-oriented decision analysis with different target preferences is showed in Section 3.3. In Section 3.4 we use two examples to illustrate the proposed two methods. A comparative analysis with related research is also given in Section 3.5. Section 3.6 gives some discussions of the proposed model. Section 3.7 gives some concluding remarks.

3.2 Cumulative Distributive Function based Method for Different Types of Target Preferences

For notational convenience, let us designate an evaluation attribute by X, and an arbitrary specific level of that evaluation attribute by x. We also restrict the variable

x to a bounded domain $D = [X_{\min}, X_{\max}]$. [1] Suppose that a DM has to rank several possible decisions $\mathcal{A} = \{A^1, \cdots, A^m, \cdots, A^M\}$, where A^m represent the alternatives (or acts) available to a DM, one of which must be selected. Assume for simplicity that the set $\mathcal{A}$ of consequences is finite and completely ordered by a preference relation $\succeq$. Denote by p_{A^m} his probability distribution for the random consequence X^m associated with an act A^m. Let p be the DM's subjective probability distribution on the state space $\mathcal{S}$. The probability distribution p_{A^m} is induced by the alternative $A^m : \mathcal{S} \to \mathcal{A}$ through the equality $p_{A^m}(X^m = x) = p(A^m(s) = x)$.

The expected utility model suggests that the ranking be obtained by using the following value function[2]

$$\mathrm{Val}(A^m) = \mathrm{EU}(X^m) = \sum_x U(x) \cdot p_{A^m}(x) \tag{3.1}$$

where $U(x)$ is a von Neumann and Morgenstern (NM-)utility function over consequences. Most often, it is assumed that the probability distribution satisfies that $\sum_x p_{A^m}(x) = 1$.

As pointed out by Bordley and Kirkwood [17], an expected utility DM is defined to be target oriented for a single attribute decision if the DM's utility for an outcome depends only on whether a target is achieved with respect to x. Thus a target-oriented DM has only two different utility levels, and because a utility function is only specified to within a positive affine transformation, these two utility levels can be set to one (if the target is achieved) and zero (if the target is not achieved). Then a target-oriented DM's expected utility for alternative A^m is

$$\begin{aligned}
\mathrm{Val}(A^m) &= \Pr(X^m \succeq T) \\
&= \sum_x \left[\Pr(x \succeq T) * 1 + (1 - \Pr(x \succeq T)) * 0\right] p_{A^m}(x) \\
&= \sum_x \Pr(x \succeq T) p_{A^m}(x)
\end{aligned} \tag{3.2}$$

where T is an uncertain target having a random distribution on D, $\Pr(x \succeq T)$ is the probability of meeting the uncertain target T and T is stochastically independent of X^m. The idea that the NM-utility function U should be interpreted as a probability distribution may appear unusual but, in fact, NM-utilities are probabilities [1, 18]. With the assumption that the attribute is monotonically increasing, x and t are mutually independent, Bordley and Kirkwood [17] suggest the following function

$$\Pr(x \succeq T) = \int_{X_{\min}}^{x} p(t)dt, \tag{3.3}$$

where t is a random level of the uncertain target T and $p(t)$ is the probability density function of uncertain target T.

In most studies on target-oriented decision making, monotonic assumptions of attributes (e.g., wealth) are given to simplify the problems. In many decision problems involving goals/targets, usually there are three types of goal preferences [71].

[1] Without loss of generality, we can set $X_{\min} = -\infty$ and $X_{\max} = +\infty$. However, to clearly show our work and better compare with other research, we shall use $D = [X_{\min}, X_{\max}]$ instead.

[2] It should be noted that when there is only one state of nature in $\mathcal{S}$, then the problem reduces to single attribute decision problem under certainty. Although this thesis focuses on multi-attribute decision analysis problem, here, without loss of generality, we shall use the general representation.

- Goal values are adjustable: "more is better" (we shall call benefit targets);
- Goal values are adjustable: or "less is better" (with respect to cost targets);
- Goal values are fairly fixed and not subject to much change, i.e. too much or too little is not acceptable (we shall call this type of target as equal or range level targets). Examples where this might hold include manufacturing processes where there is an "ideal" level for some characteristic of the product, materials management with a target inventory level, or medical conditions with an ideal level for a medical indicator, such as blood pressure.

Now let us consider these three target preferences via the cdf. The target-oriented decision model assumes that the pdf of the uncertain target is unimodal as well as views the mode value of the pdf of the uncertain target as the reference point, denoted as T_m [18]. To model the three types of target preferences, we define

$$\Pr(x \succeq T) = \int_{X_{\min}}^{X_{\max}} u(x,t)p(t)dt. \tag{3.4}$$

where $u(x,t)$ is used to denote the target achievement levels.

1. Benefit target
   In case of benefit target, the DM has a monotonically increasing preference, i.e. "the more the better". As target-oriented model assumes that there are only two levels of utility (1 or 0), thus, we define as follows:

   $$u(x,t) = \begin{cases} 1, & x \geq t; \\ 0, & \text{otherwise.} \end{cases} \tag{3.5}$$

   Then we can obtain the probability of meeting uncertain target as the following function

   $$\Pr(x \succeq T) = \Pr(x \geq T) = \int_{X_{\min}}^{x} p(t)dt. \tag{3.6}$$

   This is consistent with the target-oriented model in the literature [18, 25], i.e. the target-oriented model views the cdf as the probability of meeting the uncertain target T.

2. Cost target
   Similar with the benefit target, for cost target we define

   $$u(x,t) = \begin{cases} 1, & x \leq t; \\ 0, & \text{otherwise.} \end{cases} \tag{3.7}$$

   Then we can induce the probability of meeting a cost target T as follows

   $$\begin{aligned} \Pr(x \succeq T) &= \Pr(x \leq T) \\ &= \int_{x}^{X_{\max}} p(t)dt \\ &= 1 - \int_{X_{\min}}^{x} p(t)dt \end{aligned} \tag{3.8}$$

3. Equal/range target
   In this case, the mode value T_m is the reference point. There will be added loss of value for missing the reference point on the low side, or added loss for exceeding the reference point. When $x = T_m$ the probability of meeting target should be equivalent to one. Based on this observation, we define the probability of meeting uncertain target T as follows:

   (a) When $x < T_m$

$$\Pr(x \succeq T) = \Pr(x \cong T) = \frac{\int_{X_{\min}}^{x} p(t)dt}{\int_{X_{\min}}^{T_m} p(t)dt}. \tag{3.9}$$

   (b) When $x = T_m$

$$\Pr(x \succeq T) = \Pr(x \cong T) = \frac{\int_{T_m}^{T_m} p(t)dt}{\int_{T_m}^{T_m} p(t)dt} = 1. \tag{3.10}$$

   (c) When $x > T_m$

$$\Pr(x \succeq T) = \Pr(x \cong T) = \frac{\int_{x}^{X_{\max}} p(t)dt}{\int_{T_m}^{X_{\max}} p(t)dt}. \tag{3.11}$$

   The main idea behind this definition is that we use a relative probability of meeting targets. As target-oriented decision model views the mode T_m as the reference point. If the arbitrary specific level x of attribute X is less than the reference point, it can be viewed as as pseudo benefit attribute. When the arbitrary specific level x of attribute X is greater than the reference point, it can be viewed as as pseudo cost attribute. Otherwise, we can define $u(x,t)$ as follows

$$u(x,t) = \begin{cases} 1, & x = t; \\ 0, & \text{otherwise.} \end{cases}$$

   It should be note that in case of benefit and cost targets, we can also use this relative probability of meeting uncertain targets. When the DM prefers monotonically increasing preference, then we can define

$$\Pr(x \geq T) = \frac{\int_{X_{\min}}^{x} p(t)dt}{\int_{X_{\min}}^{X_{\max}} p(t)dt} = \int_{X_{\min}}^{x} p(t)dt.$$

   When the DM prefers monotonically decreasing preference, then we can define

$$\Pr(x \leq T) = \frac{\int_{x}^{X_{\max}} p(t)dt}{\int_{X_{\min}}^{X_{\max}} p(t)dt} = \int_{x}^{X_{\max}} p(t)dt.$$

   Generally speaking, when the DM has an range level target preference, the reference point T_m may have a interval range, such that $T_m \equiv [T_{ml}, T_{mu}]$. In this case, the probability of meeting target T becomes

(a) When $x < T_{ml}$

$$\Pr(x \cong T) = \frac{\int_{X_{\min}}^{x} p(t)dt}{\int_{X_{\min}}^{T_{ml}} p(t)dt}. \tag{3.12}$$

(b) When $x \in [T_{ml}, T_{mu}]$

$$\Pr(x \cong T) = \frac{\int_{T_{ml}}^{T_{mu}} p(t)dt}{\int_{T_{ml}}^{T_{mu}} p(t)dt} = 1. \tag{3.13}$$

(c) When $x > T_{mu}$

$$\Pr(x \cong T) = \frac{\int_{x}^{X_{\max}} p(t)dt}{\int_{T_{mu}}^{X_{\max}} p(t)dt}. \tag{3.14}$$

3.3 Target-Oriented Decision Analysis Based on the Level Set of Probability Density Function

The level set of the pdf provides a convenient way to represent the probability distribution. Dubois et al. [40] call this level set "confidence interval" which is different from the confidence interval in measurement theory. In this section, we shall consider the target-oriented decision model with different target preferences by means of the level set of the pdf.

Let T be an uncertain target having a random pdf over the bounded domain $D = [X_{\min}, X_{\max}]$, $p(t)$ be the pdf of the random target T. Let σ be any given probability level, where $0 \leq \sigma \leq \sup T$ ($\sup T$ denotes the support of the pdf of uncertain target), T_σ consists of all the elements whose probabilities are greater than or equal to σ such that

$$T_\sigma = \{t \in D | p(t) \geq \sigma\}, \tag{3.15}$$

T_σ is called the σ-level set of random target T. It should be noted that target-oriented decision analysis assumes that the uncertain target has a unimodal pdf, thus we can express as

$$T_\sigma = [T_\sigma^l, T_\sigma^r], \tag{3.16}$$

where T_σ^l and T_σ^r are the left and right bound of level cut, respectively.

Based on the distribution function of level sets of pdf provided before, similar but different from Garcia et al. [45], we define the following function:

$$\Pr(x \succeq T) = \int_0^{\sup T} u(x, T_\sigma) T_\sigma d\sigma, \tag{3.17}$$

where $u(x, T_\sigma)$ indicates the degree that the target achievement in the level set T_σ, $\sup T$ denotes the support of the pdf of uncertain target, and $u(x, T_\sigma) \in [0, 1]$, $\sup u(x, T_\sigma) = 1$.

Considering different target preferences, we further define

$$\Pr(x \succeq T) = \begin{cases} \Pr(x \geq T) = \int_0^{\sup T} u(x \geq T_\sigma) T_\sigma d\sigma, & \text{benefit target preference;} \\ \Pr(x \leq T) = \int_0^{\sup T} u(x \leq T_\sigma) T_\sigma d\sigma, & \text{cost target preference;} \\ \Pr(x \cong T) = \int_0^{\sup T} u(x \cong T_\sigma) T_\sigma d\sigma, & \text{equal/range target preference.} \end{cases} \tag{3.18}$$

Now let us consider these three cases in more detail.

Benefit target preference

If the DM has a monotonically increasing target preference, for an interval $T_\sigma = [T_\sigma^l, T_\sigma^r]$, to ensure that $u(x, T_\sigma) \in [0, 1]$ and $\sup u(x, T_\sigma) = 1$, we define

$$u(x \geq T_\sigma) = \frac{\int_{T_\sigma^l}^{T_\sigma^r} u(x,t)p(t)dt}{\int_{T_\sigma^l}^{T_\sigma^r} p(t)dt} \tag{3.19}$$

As target-oriented model assumes that there are only two levels of utility (1 or 0), thus we define

$$u(x,t) = \begin{cases} 1, & \text{if } x \geq t; \\ 0, & \text{otherwise.} \end{cases}$$

where $u(x,t)$ denotes whether the attribute level achieves target level or not. Then we can obtain $u(x \geq T_\sigma)$ as follows:

$$u(x \geq T_\sigma) = \begin{cases} 0, & \text{if } x < T_\sigma^l; \\ \frac{\int_{T_\sigma^l}^{x} p(t)dt}{\int_{T_\sigma^l}^{T_\sigma^r} p(t)dt}, & \text{if } T_\sigma^l \leq x \leq T_\sigma^r; \\ 1, & \text{if } x > T_\sigma^r. \end{cases} \tag{3.20}$$

By substituting Eq. (3.20) into the general representation of level set based target-oriented decision model Eq. (3.18), we can obtain the probability of meeting uncertain target T.

Cost target preference

In case of cost target preference, similarly we define

$$u(x,t) = \begin{cases} 1, & \text{if } x \leq t; \\ 0, & \text{otherwise.} \end{cases}$$

and then we can obtain $u(x \leq T_\sigma)$ as follows:

$$u(x \leq T_\sigma) = \begin{cases} 1, & \text{if } x < T_\sigma^l; \\ \frac{\int_{x}^{T_\sigma^r} p(t)dt}{\int_{T_\sigma^l}^{T_\sigma^r} p(t)dt}, & \text{if } T_\sigma^l \leq x \leq T_\sigma^r; \\ 0, & \text{if } x > T_\sigma^r. \end{cases} \tag{3.21}$$

It is clear that $u(x \leq T_\sigma) = 1 - u(x \geq T_\sigma)$, thus we obtain

$$\begin{aligned} \Pr(x \leq T) &= \int_0^{\sup T} u(x \leq T_\sigma) T_\sigma d\sigma \\ &= \int_0^{\sup T} (1 - u(x \geq T_\sigma))\, T_\sigma d\sigma \\ &= \int_0^{\sup T} T_\sigma d\sigma - \int_0^{\sup T} u(x \geq T_\sigma) T_\sigma d\sigma \\ &= 1 - \Pr(x \geq T) \end{aligned} \tag{3.22}$$

Equal/range target preference

In case of non-monotonic target preference, there exists an "ideal" level. Recall that target-oriented decision analysis views the modal value T_m of the pdf as reference point (reflection point), then there will be added loss of value for missing the reference point on the low side, or added loss for exceeding the reference point. In other words, when $x = T_m$ the probability of meeting target should be equivalent to one; when $x < T_m$ it can be viewed as pseudo benefit attribute; and when $x > T_m$ it can be viewed as pseudo cost attribute. Due to this observation, we can define the following function:

1. When $x < T_m$,

$$u(x \cong T_\sigma) = \begin{cases} 0, & \text{if } x < T_\sigma^l; \\ \frac{\int_{T_\sigma^l}^{x} p(t)dt}{\int_{T_\sigma^l}^{T_m} p(t)dt}, & \text{otherwise.} \end{cases} \tag{3.23}$$

2. When $x = T_m$,

$$u(x \cong T_\sigma) = \frac{\int_{T_m}^{x} p(t)dt}{\int_{T_m}^{T_m} p(t)dt} = 1 \tag{3.24}$$

3. When $x > T_m$,

$$u(x \cong T_\sigma) = \begin{cases} 0, & \text{if } x > T_\sigma^r; \\ \frac{\int_{x}^{T_\sigma^r} p(t)dt}{\int_{T_m}^{T_\sigma^r} p(t)dt}, & \text{otherwise.} \end{cases} \tag{3.25}$$

It should be noted that if the mode value T_m is an interval range, such that $T_m \equiv [T_{ml}, T_{mu}]$, then we can define $u(x \cong T_\sigma) = 1$ if $T_{ml} \leq x \leq T_{mu}$. Typical examples of this case are the trapezoidal distributions.

3.4 Illustrative Examples

In this section, we shall consider two special cases to illustrate the proposed two methods.

3.4.1 Normally distributed targets

In real applications, the uncertain targets may have different probability distributions. For example, Tsetlin and Winkler [131] used the normal probability distribution, LiCalzi and Sorato [89] used the Pearson system probability distributions to represent the uncertainty of the target. Choosing a suitable probability distribution for uncertain target T is due to specific problems. As the normal distribution is widely used as a model of quantitative phenomena in the natural and behavioral sciences, we shall assume that the uncertain target is normally distributed over the bounded domain D and with mode value T_m. We assume a DM has three types of monotonic preferences: benefit, cost, and equal/range target. According to the two target-oriented decision methods proposed in previous sections, we can obtain the probability of meeting the normal target with respect to these three target preference types. We will discuss these three cases in great detail.

- Firstly, let us consider the benefit case. Fig. 3.1 graphically depicts the pdf, induced probability of meeting the normal target. To distinguish these two methods, $\Pr_{\mathtt{I}}(x \geq T)$ is used to denote the value function induced by the cdf based method, whereas

$\Pr_{\mathtt{II}}(x \geq T)$ is used to denote the value function induced by the level set based method.

Looking at the induced value functions $\Pr_{\mathtt{I}}(x \geq T)$ and $\Pr_{\mathtt{II}}(x \geq T)$ with respect to benefit target preference, as shown in Fig. 3.1. It is clearly seen that no matter which method is chosen, the induced value function (utility function) corresponds to an S-shaped function, which is equivalent to the S-shaped utility function of Prospect Theory by Kahneman and Tversky [70] as well as is consistent with "Goals as reference point" by Heath et al. [54]. The induced value functions have the following two properties:

1. **Gain and loss**
   The target divides the space of outcomes into regions of gain and loss (or success and failure). Thus, the value function assumes that people evaluate outcomes as gains or losses relative to the reference point T_m.
2. **Diminishing sensitivity**
   The value function draws an analogy to psychophysical process and predicts that outcomes have a smaller marginal impact when they are more distant from the reference point T_m.

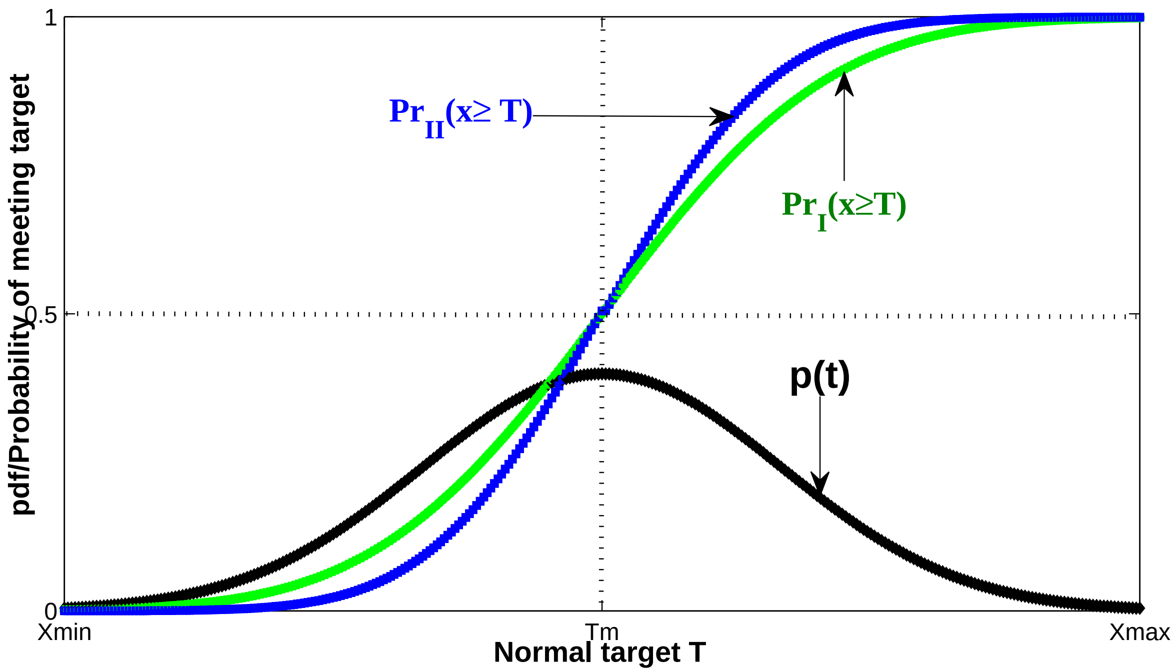

Figure 3.1: Induced value functions with a normally distributed target by means of cdf and level set based methods, with respect to *benefit* target preference

Remark It should be noted that in their Prospect Theory [70] Kahneman and Tversky assume another principle: *outcomes that are encoded as losses are more painful than the similar sized gains are pleasurable.* In their words, "losses loom larger than gains". From Fig. 3.1, the induced value function by target-oriented model does not entirely satisfy this principle. The main reasons for this observation are twofold. The first reason is the distribution type of target. The normal target is symmetrically distributed around the mode value T_m. Another reason is the bounded domain. In fact, when the attribute value has a bounded domain, and the reflection point in the Prospect Theory is the middle value of the domain, the value function induced by prospect theory will also not satisfy this principle.

In addition, from Fig. 3.1 it is clearly that although those two induced value functions have an S-shaped value function, the behaviors of value function are different. The value function $\Pr_{\mathtt{II}}(x \geq T)$ induced by the level set based method is stepper towards the mode value T_m of the corresponding target T than that $\Pr_{\mathtt{I}}(x \geq T)$ by the cdf based method. This practically implies that the level set based value function reflects a *stronger decision attitude* by the DM towards the target T than that defined by the cdf function. A similar result for this phenomenon is given in Huynh *et al.* [61].

- In case of cost target, the DM will have a monotonically decreasing preference. According to Eq. (3.8), we can obtain the value function induced by the cdf based method, denoted as $\Pr_{\mathtt{I}}(x \leq T)$. By means of Eqs. (3.18) and (3.21) we can obtain the probability of meeting target based on the level set of pdf, denoted as $\Pr_{\mathtt{II}}(x \leq T)$. Fig. 3.2 graphically depicts the pdf of the normal target T, its induced value functions by the cdf based method and level set based method.

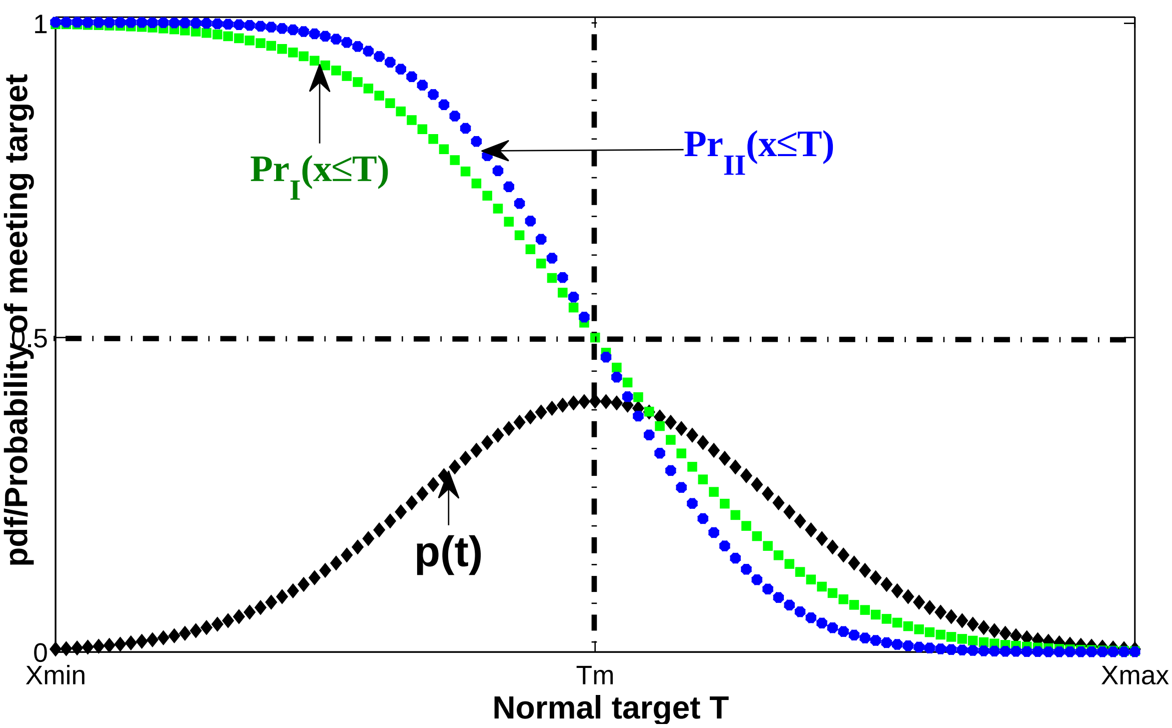

Figure 3.2: Induced value functions with a normally distributed target by means of cdf and level set based methods, with respect to *cost* target preference

From Fig. 3.2, it is clear that no matter which method is chosen, these two methods both induce an inverse S-shaped value function. The reference point T_m divides the value function into two parts: gains and losses (the value below T_m can be viewed as a kind of gains; the value upper than T_m can be viewed a kind of losses).

In addition, the value function draws an analogy to psychophysical process and predicts that outcomes have a smaller marginal impact when they are more distant from the reference point T_m. Finally, the behaviors of value function $\Pr_{\text{II}}(x \leq T)$ induced by the level set based method is also stepper towards the mode value of the corresponding target than that $\Pr_{\text{I}}(x \leq T)$ induced by the cdf based method.

- In case of equal/range target preference, according to Eqs. (3.9)-(3.11) we can obtain the value function induced by the cdf based method, denoted as $\Pr_{\text{I}}(x \cong T)$. By means of Eqs. (3.18) and (3.23)-(3.25), we can induce the value function via the level set based method, denoted as $\Pr_{\text{II}}(x \cong T)$. Fig. 3.3 graphically depicts the pdf of the normal target T, its induced value functions by the cdf based method and level set based method with respect to equal/range target preference.

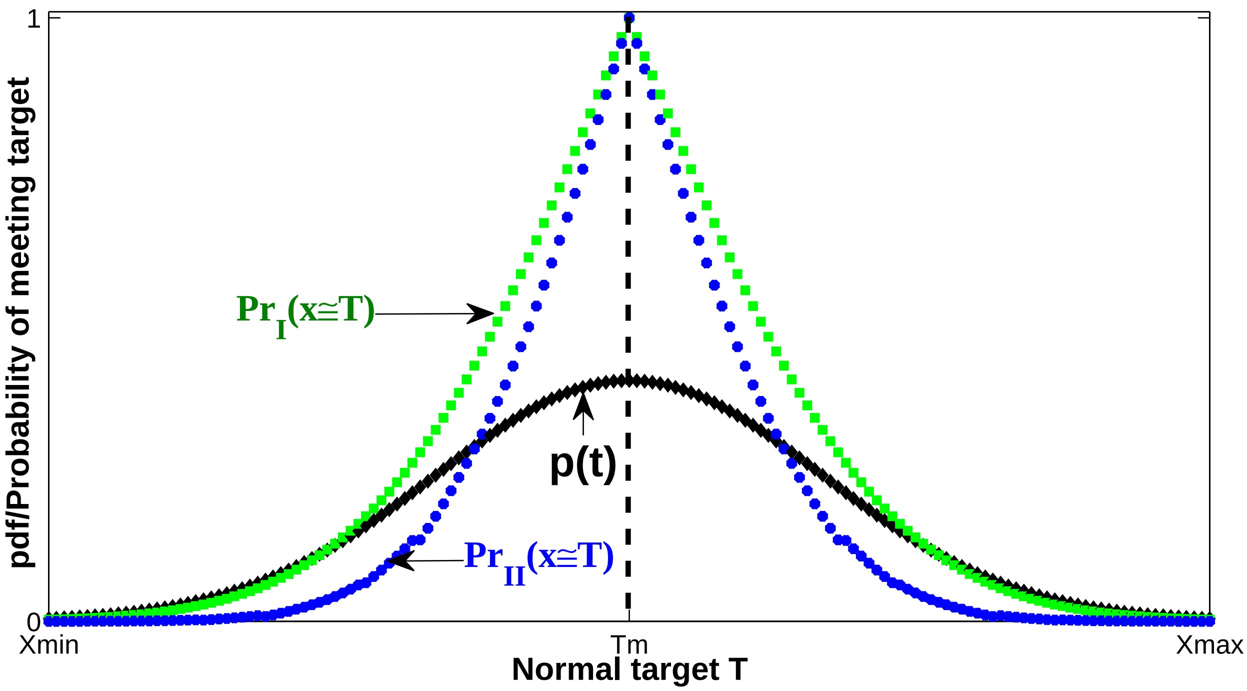

Figure 3.3: Induced value functions with a normally distributed target by means of cdf and level set based methods, with respect to *equal/range* target preference

As the DM assumes interval/range target preference, there will be added loss of value for missing the reference point on the low side, or added loss for exceeding the reference point T_m. Thus the reference point T_m is the reflection point. As illustrated in Fig. 3.3, the value functions induced by the cdf based method and level set based method have a convex shaped function, i.e. below or upper than the mode value T_m is viewed as loss. Furthermore, it is clear that the behavior of value function $\Pr_{\text{II}}(x \cong T)$ induced by the level set based method is stepper towards the mode value T_m of the corresponding target than that $\Pr_{\text{I}}(x \cong T)$ induced by the cdf based method.

3.4.2 Uniformly distributed target

Furthermore, let us consider a special case. Without additional information about the target distribution, we can assume that the random target T has a uniform distribution on D with the pdf $p(t)$ defined by

$$p(t) = \begin{cases} \frac{1}{X_{\max} - X_{\min}}, & X_{\min} \leq t \leq X_{\max}; \\ 0, & \text{otherwise.} \end{cases} \tag{3.26}$$

Under the assumption that the random target T is stochastically independent of any alternative, by means of the cdf based method and the level set based method we can obtain the same value function with respect to benefit and cost target preferences as follows

$$\Pr(x \succeq T) = \begin{cases} \Pr(x \geq T) = \frac{x - X_{\min}}{X_{\max} - X_{\min}}, & \text{for benifit target;} \\ \Pr(x \leq T) = \frac{X_{\max} - x}{X_{\max} - X_{\min}}, & \text{for cost target.} \end{cases} \tag{3.27}$$

In this case, the level set and cdf based methods are equivalent. From Eq. (3.27) it is easily seen that, for benefit and cost attribute there is no way to tell whether the DM selects an alternative by traditional normalization method or by target-oriented model. In other words, in this case the target-based decision model with the decision function is equivalent to the traditional normalization function. Fig. 3.4 graphically depicts the value function induced by target-oriented decision model under uniformly distributed target.

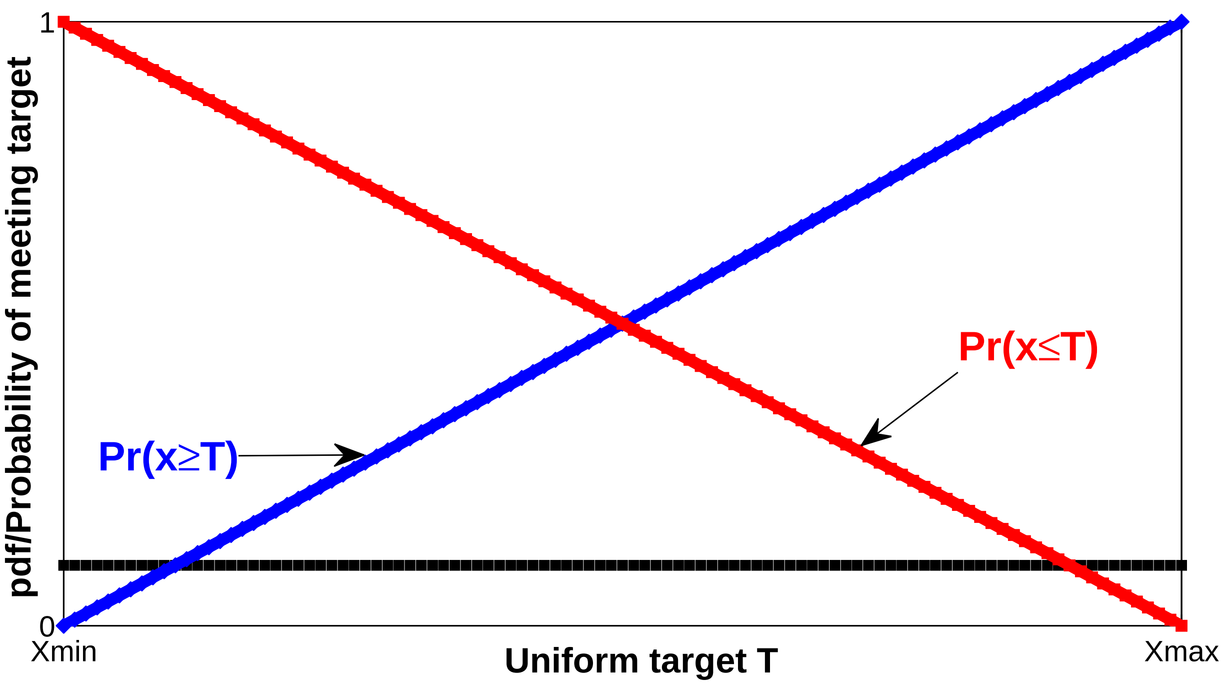

Figure 3.4: Uniformly distributed target with benefit and cost target preference

3.5 Comparison and Relationship with Related Research

In this section, we shall compare our research with related work.

3.5.1 Comparison with Bordley and Kirkwood's approach

To show that the goal programming method is just a special case of target-oriented decision model, Bordley and Kirkwood [17] define the following function to induce the goal programming problem. For a decision with a single evaluation attribute X and a target T, let t be the possibly uncertain target level for T and x be the possibly uncertain actual performance, and assume that utility is specified as a function $u(x,t)$

$$u(x,t) = \begin{cases} -a(t-x), & \text{if } x < t; \\ b - c(x-t), & \text{otherwise.} \end{cases} \tag{3.28}$$

where $a \geq 0$, $b \geq 0$, and $c \geq 0$. Then they define

1. If the DM prefers monotonically increasing preference (benefit target), then we can set $a = 0, b = 1, c = 0$;

2. if the DM prefers non-monotonic preference (equal/range target), then we can set $a > 0, b = 0, c > 0$.

Examples where this might hold include manufacturing processes where there is an "ideal" level for some characteristic of the product, materials management with a target inventory level, or medical conditions with an ideal level for a medical indicator, such as blood pressure.

However, this approach is debatable. As pointed out by Bordley and Kirkwood, an expected utility DM is defined to be target oriented for a single attribute decision if the DM's utility for an outcome depends only on whether a target is achieved with respect to X ([17], p. 824). Thus we shall have only two utility levels $u(x,t) = 1$ or $u(x,t) = 0$. The above functions allow more than two utility levels, thus there exists some inconsistency in Bordley and Kirkwood's approach. Furthermore, consider the uncertain target having a normal distribution, as shown in Fig. 3.3. If we assume the DM has an equal/range target preference, substituting Eq. (3.28) into Eq. (3.4) we will always obtain the non-positive (negative or zero) value function. In target-oriented decision model, the target achievement (probability of meeting target) belongs to $[0,1]$, thus there exists some conflict in Bordley and Kirkwood's approach. The main reason for this problem is that, target-oriented decision model uses the cdf or some function of the cdf to represent the degree of satisfaction, while goal programming approach focus on using distance-based function to represent the degree of satisfaction.

3.5.2 Relationship with Prospect Theory

Prospect theory [70] deals with decision making under risk, where probability distributions of the lotteries are known to agent. Prospect theory assumes that the ranking procedure is linear in the distorted probabilities. In other words, the ranking procedure is generated by the value function

$$\text{Val}(A^m) = \sum_x U(x) \cdot \phi\left[p_{A^m}(x)\right], \tag{3.29}$$

which is linear in ϕ but not in $p_{A^m}(x)$. The weighting function does not obey the axioms of probability theory and it measures the impact of probabilities on choices rather than the likelihood of the underlying events [84]. Therefore, prospect theory postulates a model which in general is not linear in the known probabilities. It is apparent how little prospect theory tries to part away from the expected utility model [88].

Target-oriented decision model focuses on whether the value function meets a random variable, T having a probability distribution. In addition, target-oriented model views the mode value of probability distribution as reference point, this point was illustrated by Heath et al. [54]. Finally, target-oriented decision model satisfies NM-utility axiomatization [18, 88].

3.6 Discussions

In fact, target-oriented decision analysis focuses on the decision analysis under risk (DAUR) problems. The most classical method for DAUR is to use the expected value. Our research focuses on using the expected probability of meeting uncertain targets. Thus our research can deal with DAUR problems. This thesis focuses on multi-attribute decision analysis (MADA), this chapter provides a way to induce the value functions for MADA problems. Although not all MADA problems deal with risk, the shapes of the value functions can better represent DM's behavioral/psychological preferences.

Here the behavioral/psychological preferences mean that the induced value functions are S-shaped, inverse S-shaped, convex, or concave. Kahneman and Tversky [70] have already done some empirical experiments to show DM's psychological preferences in decision analysis, our research is based on their work. In addition, the reflection point in the S-shaped value function of prospect theory can be viewed as a target, this was proved by Heath *et al.* [54]. Our research not only validates this point, but also other shaped value functions, e.g. inverse S-shaped, concave, and convex.

3.7 Summary

In this chapter, we proposed two methods to model the target-oriented decision analysis with different target preferences: cdf based method and level set based method. Both of these two methods can induce four shaped value functions: S-shaped, inverse S-shaped, convex, and concave, which represent DM's psychological preference. The main difference between these two methods is that the value function induced by the level set based method model is steeper than that induced by the cdf based method.

Target-oriented decision analysis presumes that target has a random probability distribution. In some cases, it is not so easy for the DM to specify a suitable pdf for the uncertain target. Furthermore, it is well known that all facets of uncertainty cannot be captured by a single probability distribution. In many applications, fuzzy subsets provide a very convenient object for the representation of uncertain information. In the next chapter, we shall discuss target-oriented decision analysis with fuzzy targets.

Chapter 4

Fuzzy Target-Oriented Decision Analysis with Different Target Preferences

Abstract: Simon proposed a behavioral model for rational choice, by enunciating the so-called theory of *bounded rationality* implying that the decision maker (DM) simply looks for the first "satisfactory" act that meets some predefined target. Target-oriented decision model has relaxed the assumption of a known target by considering a random consequence T instead. Then the target-oriented decision model prescribes that the DM should choose an act a that maximizes the probability of meeting the random target. However, in many situations, it is not so easy to specify a probability function for the uncertain target. Moreover, it is widely acknowledged that all facets of uncertainty cannot be captured by a single probability distribution. Fuzzy subset provides a very convenient object for representing uncertain information.

Toward this end, the main focus of this chapter is to discuss the issue of how to use fuzzy targets in the target-oriented decision model with different target preferences. To do so, we firstly analysis different fuzzy-probability transformation techniques, then the proportional transformation method is chosen to transform the fuzzy targets into probabilistic targets. Secondly, based on the probabilistic target-oriented decision model discussed in Chapter 3, we can finally obtain the fuzzy target-oriented decision model with different target preferences.

4.1 Introduction

Simon [120] proposed a behavioral model for rational choice, by enunciating the so-called theory of bounded rationality implying that due to the cost or the practical impossibility of searching among all possible acts for the optimal, the decision maker (DM) simply looks for the first "satisfactory" act that meets some predefined targets. Although simple and appealing from this satisficing-oriented point of view, its resulted model is still not complete because there may be uncertainty about the target itself. Target-oriented decision model has relaxed the assumption of a known target by considering a random consequence T instead. Then the target-based decision model prescribes that the agent should choose an act a that maximizes the probability of meeting an uncertain target T, assuming that the target T is stochastically independent of the random consequences to be evaluated.

However, it is now more and more widely acknowledged that all facets of uncertainty cannot be captured by a single probability distribution. Moreover, it is usually not easy for a DM to specify the probability distribution of the uncertain target. In many applications, fuzzy subsets provide a very convenient object for the representation of uncertain information. The subjective assessments provides by DMs are usually conceptually vague, with uncertainty that is frequently represented in linguistic forms. To help people easily express their subjective assessments, the linguistic variables are used to linguistically express requirements. Fuzzy numbers are usually used in decision analysis problems. Thus it is necessary to consider the fuzzy targets in target-oriented decision model. Toward this end, Huynh *et al.* [61, 62] have discussed the problem of decision analysis under uncertainty (DAUU) with a payoff variable. There are three drawbacks in their work. Firstly, only one target preference is considered. As we mentioned in Chapter 3, the DM can have three types of target preferences. Secondly, a thorough analysis of the possibility-probability conversion problems is ignored. In the literature, there are many techniques to transform a possibility into its associated probability. Thirdly, in fact, fuzzy decision analysis has received a lot of attraction since the pioneering work on fuzzy decision analysis by Bellman and Zadeh [12] in 1970. Bellman and Zadeh's paradigm is still widely used in most literature of fuzzy decision analysis, comparing fuzzy target-oriented decision model with Bellman and Zadeh paradigm will be of great help to fuzzy decision analysis. However, a complete analysis with Bellman and Zadeh is missed in their research.

Based on the above-mentioned observations, the main focus of this chapter is to revisit fuzzy targets in target-oriented decision model based on the probabilistic target-oriented decision model discussed in Chapter 3. To do so, firstly a through analysis of different possibility-probability transformation techniques is given and then the proportional transformation method is properly chosen. Secondly, we extend the probabilistic target-oriented decision model into fuzzy target case. Thirdly, we use several fuzzy targets, commonly used in Bellman and Zadeh paradigm, to illustrate the proposed fuzzy target-oriented decision model. Finally, we compare our work with Bellman and Zadeh in terms of three aspects.

The rest of this chapter is organized as follows. In Section 4.2 we introduce some concepts of possibility distribution and fuzzy subsets. Section 4.3 analyzes different possibility-probability conversion methods. Section 4.4 revisits fuzzy target-oriented decision model with different target preferences. In Section 4.5, four commonly used fuzzy targets in Bellman and Zadeh paradigm, are selected to illustrate our proposed models.

Section 4.6 gives a comparative analysis with Bellman and Zadeh paradigm from three aspects. Finally, some concluding remarks are given in Section 4.7.

4.2 Possibility Distributions and Fuzzy Subsets

Possibility measures are set functions similar to probability measures, but they rely on an axiom which only involves the operation "supremum". A possibility measure Π on a set X (e.g., the set of real numbers) is characterized by a possibility distribution $\pi : X \to [0, 1]$ and is defined by

$$\forall A \subseteq X, \Pi(A) = \sup\{\pi(x), x \in A\}. \tag{4.1}$$

On finite sets this definition reduces to

$$\forall A \subseteq X, \Pi(A) = \max\{\pi(x), x \in A\}. \tag{4.2}$$

To ensure $\Pi(x) = 1$, a normalization condition demands that $\pi(x) = 1$.

As pointed out by Zadeh [164], the membership function of a fuzzy set can be used for encoding a possibility distribution. Formally, the soft constraint imposed on a variable V in the statement "V is F", where F is a fuzzy set, can be considered as inducing a possibility distribution π on the domain of V such that $\mu(x) = \pi(x)$. In this thesis, we shall use membership function and possibility distribution interchangeably.

A fuzzy set F of $\mathbb{U}$ is a mapping from $\mathbb{U}$ into the unit interval: $\mu_F : \mathbb{U} \to [0, 1]$, where $\mu_F(x)$ is called the membership degree of x in F. A fuzzy number $\widetilde{A}$ is defined as a fuzzy set with the membership function $\mu_{\widetilde{A}}(x)$ of the set $\mathbb{R}$ of all real numbers that satisfies the following properties [76]:

- $\widetilde{A}$ is a normal fuzzy set, i.e., $\sup_{x\in\mathbb{R}} \mu_{\widetilde{A}}(x) = 1$;
- $\widetilde{A}$ is a convex fuzzy set, i.e.,

  $$\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2))$$

  for $\forall x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$;
- The support of $\widetilde{A}$, i.e., the set $\sup(\widetilde{A}) = \{x \in \mathbb{R} | \mu_{\widetilde{A}}(x) > 0\}$, is bounded.

A fuzzy number A can be conveniently represented by the canonical form [76]

$$\pi_{\widetilde{A}}(x) = \begin{cases} f_{\widetilde{A}}(x), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ g_{\widetilde{A}}(x), & c \leq x \leq d, \\ 0, & \text{otherwise.} \end{cases} \tag{4.3}$$

where $f_{\widetilde{A}}(x)$ is a real-valued function that is monotonically increasing, and $g_{\widetilde{A}}(x)$ is a real-valued function that is monotonically decreasing. In addition, as in most applications, we assume that functions $f_{\widetilde{A}}$ and $g_{\widetilde{A}}$ are continuous. If $f_{\widetilde{A}}(x)$ and $g_{\widetilde{A}}(x)$ are linear functions then $\widetilde{A}$ is called a trapezoidal fuzzy number and denoted by $[a, b, c, d]$. In particular, $[a, b, c, d]$ becomes a triangular fuzzy number if $b = c$.

For any fuzzy number $\widetilde{A}$ expressed in the canonical form, its α-cuts are expressed for all $\alpha \in [0,1]$ by the formula

$$\widetilde{A}_\alpha = \begin{cases} [f_{\widetilde{A}}^{-1}(\alpha), g_{\widetilde{A}}^{-1}(\alpha)], & \texttt{when } \alpha \in (0,1) \\ [\text{b,c}], & \texttt{when } \alpha = 1. \end{cases} \tag{4.4}$$

where $f_{\widetilde{A}}^{-1}$ and $g_{\widetilde{A}}^{-1}$ are the inverse functions of $f_{\widetilde{A}}$ and $g_{\widetilde{A}}$, respectively. In the case that $\tilde{A}$ degenerates into a crisp interval, i.e., $A = [a,b]$, we define $A_\alpha = A$ for all $\alpha \in (0,1)$.

4.3 Transformations from Possibility to Probability

Since possibility and probability represents different types uncertainties, there exists a transformation between them. The conversion problem between possibility and probability has its roots in the possibility/probability consistency principle of Zadeh [162], that he propose in the paper founding possibility theory in 1978 [164]. The possibility-probability consistency principle is a heuristic relationship between possibilities and probabilities. This principle can be summarized as: "the possibility of an event is always greater than or equal to the probability of the event". This is based on the consideration that possibility representation and probability representation are not just two equivalent representations of uncertainty, but the representation is weaker because it explicitly handles imprecision. Generally speaking, there are four possibility-probability transformation methods:

(1) Uniform distribution transformation [72]

$$p(x) = \pi(x) + (h_u - 1) \tag{4.5}$$

where h_u is the conversion constants which ensure that the area under the continuous probability function is equal to one.

This conversion of a fuzzy number into its corresponding probability distribution is computationally straightforward. However, when this transformation method is used, both the domain and the range of the resulting distribution may be reduced (or increased). The reduced (or increased) domain indicates the partial rejection (or addition) of some members from (or to) the set. Hence, this particular conversion method is not entirely suitable.

(2) Entropy-based transformation [7, 26]

Entropy is a measure of the uncertainty of a variable, it can also be looked upon as a measure of imprecision for a fuzzy variable [72]. Using the basic concept of entropy, the fuzzy imprecision can be transformed to random uncertainty. The basis of this transformation is that the measurement is invariant under transformation.

This transformation method has two drawbacks. Firstly, as pointed out by [40], the probabilistic representation and the possibilistic one are not just two equivalent representations of uncertainty, hence there should be no symmetry between the two mutual conversion procedures. The possibilistic representation is weaker than probabilistic representation. Secondly, even if the possibilistic representation and probabilistic one have the equivalent uncertainty, one precondition in this transformation from possibility to probability is to specify a probability distribution. It is usually difficult and inconvenient for the DM to specify the known probability distribution. Based on these two observations, the entropy-based transformation method will also be ill-suitable.

(3) α-cut based transformation [40, 42]

Dubois et al. [42] propose a conversion method based on the alpha-cut of fuzzy numbers

$$p(x) = \int_0^{\pi(x)} \frac{1}{|A_\alpha|} d\alpha \tag{4.6}$$

where π is the possibility distribution of the fuzzy number A and $|A_\alpha|$ is the width of the α-cut of π. One main problem of this approach is that for a unimodal fuzzy number, we can not obtain the probability of the mode value. For example, for a triangular fuzzy number (a, b, c) its associated probability density at point b will be $+\infty$.

Taking a different view, Dubois et al. [40] propose a method to convert symmetric triangular fuzzy number into its associated probability distribution based on the concept of “confidence interval”. The ”confidence interval” used by Dubois et al. [40] is different from the traditional confidence interval in measurement theory. They view the ”confidence interval” as the level sets of probability density function. As fuzzy numbers can have a variety of shapes as well as may be asymmetric, this method is also inappropriate.

(4) Proportional probability density function [26, 72, 146, 148]

Yager [146] investigates the problem of instantiating a possibility variable over a discrete domain by converting its possibility distribution into a probability distribution, via a simple normalization. This conversion has been extended into a continuous domain [148] as follows:

$$p(x) = \frac{\pi(x)}{\int_x \pi(x) dx}. \tag{4.7}$$

When applying this proportional probability density distribution to convert the fuzzy number, it is noted that the range of the membership grade of the resultant proportional distribution is greatly reduced when the fuzzy number has a wide domain. Consequently, the ability of the membership function to discriminate precisely among the members of the fuzzy set is impaired. Fortunately however, the domain of the fuzzy number is always sufficiently narrow to avoid this becoming a problem.

Based on the above analysis and comparisons, from the analytical point of view, the proportional proportional transformation approach can deal with different types of possibility distributions while following the possibility/probability consistency principle of Zadeh [164]. From the computational point of view, the proportional approach is convenient and simple in real applications. Thus, in this study, the proportional conversion method will be used to transform the possibility distribution to probability distribution.

4.4 Fuzzy Target-Oriented Decision Analysis

In many applications, the subjective assessments provided by DM(s) are usually conceptually vague, with uncertainty that is frequently represented in linguistic forms. To help people easily express their subjective assessments, the linguistic variables [163] are used to linguistically express requirements. Assume that the fuzzy targets linguistically specified by the DM have the canonical form, and $\pi(t)$ is the membership degree/possibility distribution of fuzzy target $\widetilde{T}$.

For notational convenience, we also designate an evaluation attribute by X, and an arbitrary specific level of that evaluation attribute by x. We also restrict the variable x to a bounded domain $D = [X_{\min}, X_{\max}]$. Suppose that a DM has to rank several possible decisions $\mathcal{A} = \{A^1, \cdots, A^m, \cdots, A^M\}$, where A^m represent the alternatives (or acts) available to a DM, one of which must be selected. Assume for simplicity that the set $\mathcal{A}$ of consequences is finite and completely ordered by a preference relation $\succeq$. By a fuzzy target we mean a possibility variable $\widetilde{T}$ over the attribute domain D, by a possibility distribution $\widetilde{T} : D \to [0, 1]$. We also assume further that $\widetilde{T}$ is a piecewise continuous function having a bounded support and $\int_D \widetilde{T}(t)dt > 0$.

Given a fuzzy target $\widetilde{T}$, let $\pi(t)$ be the possibility distribution function, and $p(t)$ be its associated probability distribution function. According to the proportional transformation method, we can obtain the induced probability distribution as follows:

$$p(t) = \frac{\pi(t)}{\int_t \pi(t)dt}. \tag{4.8}$$

In the following, we shall extend the two probabilistic target-oriented decision models into the fuzzy target-oriented decision analysis case.

4.4.1 Cumulative Distribution Function Based Method

Probabilistic target-oriented decision model suggests using the following ranking of alternatives be obtained by using the value function defined by

$$\text{Val}(A^m) = \sum_x \Pr(x \succeq T) p_{A^m}(x)$$

where p_{A^m} is the probability distribution for the random consequence X^m associated with an act A^m, T is an uncertain target having a random distribution on D, $\Pr(x \succeq T)$ is the probability of meeting the uncertain target T and T is stochastically independent of X^m.

As we mentioned in Chapter 3, there are three type of target preferences: benefit target, cost target, and equal/range level target. Based on probabilistic target-oriented decision model and the induced pdf of fuzzy target $\widetilde{T}$, we can obtain the probability of meeting target as follows:

$$\Pr(x \succeq \widetilde{T}) = \frac{\int_{X_{\min}}^{X_{\max}} u(x,t)\pi(t)dt}{\int_{X_{\min}}^{X_{\max}} \pi(t)dt}. \tag{4.9}$$

In case of benefit target preference, we can obtain the target achievement function as

$$\begin{aligned} \Pr(x \succeq \widetilde{T}) &= \Pr(x \geq \widetilde{T}) \\ &= \frac{\int_{X_{\min}}^{x} \pi(t)dt}{\int_{X_{\min}}^{X_{\max}} \pi(t)dt} \end{aligned} \tag{4.10}$$

In case of cost target preference, we can obtain the target achievement function as

$$\begin{aligned}\Pr(x \succeq \widetilde{T}) &= \Pr(x \leq \widetilde{T}) \\ &= \frac{\int_{x}^{X_{\max}} \pi(t)dt}{\int_{X_{\min}}^{X_{\max}} \pi(t)dt}.\end{aligned} \tag{4.11}$$

When the DM has equal/range level target preference, as the target-oriented decision model views the mode value of the pdf as a reflection point, there will be added loss of value for missing the reference point on the low side, or added loss for exceeding the reference point. We first consider the situation that the pdf is unimodal, the mode value is denoted by T_m. Then the fuzzy target-oriented decision model with respect to equal target preference can be defined as

$$\Pr(x \cong \widetilde{T}) = \begin{cases} \frac{\int_{X_{\min}}^{x} \pi(t)dt}{\int_{X_{\min}}^{T_m} \pi(t)dt}, & \text{if } x < T_m; \\ 1, & \text{else if } x = T_m; \\ \frac{\int_{x}^{X_{\max}} \pi(t)dt}{\int_{T_m}^{X_{\max}} \pi(t)dt}, & \text{otherwise.} \end{cases} \tag{4.12}$$

Generally speaking, the reference point T_m may have a interval range, such that $T_m \equiv [T_{ml}, T_{mu}]$. In this case, the induced value function is defined as follows:

$$\Pr(x \cong \widetilde{T}) = \begin{cases} \frac{\int_{X_{\min}}^{x} \pi(t)dt}{\int_{X_{\min}}^{T_{ml}} \pi(t)dt}, & \text{if } x < T_{ml}; \\ 1, & \text{else if } x \in [T_{ml}, T_{mu}]; \\ \frac{\int_{x}^{X_{\max}} \pi(t)dt}{\int_{T_{mu}}^{X_{\max}} \pi(t)dt}, & \text{if } x > T_{mu}. \end{cases} \tag{4.13}$$

4.4.2 Level Set Based Approach

In Chapter 3, a level set based approach has been proposed to deal with the target-oriented decision making with probabilistic uncertainty. The first step is to converse the fuzzy target $\widetilde{T}$ into probability distribution such that

$$p(t) = \frac{\pi(t)}{\int_{x} \pi(t)}.$$

Let T be the induced probabilistic target having a random pdf over the bounded domain $D = [X_{\min}, X_{\max}]$. Let σ be any given probability level, where $0 \leq \sigma \leq \sup T$ ($\sup T$ denotes the support of the pdf of uncertain target), T_σ consists of all the elements whose probabilities are greater than or equal to σ such that

$$T_\sigma = \{t \in D | p(t) \geq \sigma\}, \tag{4.14}$$

T_σ is called the σ-level set of random target T. It should be noted that target-oriented decision analysis assumes that the uncertain target has a unimodal pdf, thus we can express as $T_\sigma = [T_\sigma^l, T_\sigma^r]$, where T_σ^l and T_σ^r are the left and right bound of level cut, respectively.

Based on the distribution function of level sets of pdf provided before, similar but different from Garcia et al. [45], we define the following function:

$$\Pr(x \succeq T) = \int_0^{\sup T} u(x, T_\sigma) T_\sigma d\sigma, \tag{4.15}$$

where $u(x, T_\sigma)$ indicates the degree that the target achievement in the level set T_σ, $\sup T = \frac{1}{\int_t \pi(t)dt}$ denotes the support of the pdf of uncertain target, and $u(x, T_\sigma) \in [0, 1]$, $\sup u(x, T_\sigma) = 1$.

Considering different target preferences, we further define

$$\Pr(x \succeq T) = \begin{cases} \Pr(x \geq T) = \int_0^{\sup T} u(x \geq T_\sigma) T_\sigma d\sigma, & \text{benefit target preference;} \\ \Pr(x \leq T) = \int_0^{\sup T} u(x \leq T_\sigma) T_\sigma d\sigma, & \text{cost target preference;} \\ \Pr(x \cong T) = \int_0^{\sup T} u(x \cong T_\sigma) T_\sigma d\sigma, & \text{equal/range target preference.} \end{cases} \tag{4.16}$$

The second step is to calculate the target achievement with respect to different target preferences based on the induced probability distribution function.

Benefit target preference

If the DM has a monotonically increasing target preference, for an interval $T_\sigma = [T_\sigma^l, T_\sigma^r]$, to ensure that $u(x, T_\sigma) \in [0, 1]$ and $\sup u(x, T_\sigma) = 1$, we define

$$u(x \geq T_\sigma) = \frac{\int_{T_\sigma^l}^{T_\sigma^r} u(x, t) p(t) dt}{\int_{T_\sigma^l}^{T_\sigma^r} p(t) dt} \tag{4.17}$$

As target-oriented model assumes that there are only two levels of utility (1 or 0), thus we define

$$u(x, t) = \begin{cases} 1, & \text{if } x \geq t; \\ 0, & \text{otherwise.} \end{cases}$$

where $u(x, t)$ denotes whether the attribute level achieves target level or not. Then we can obtain $u(x \geq T_\sigma)$ as follows:

$$u(x \geq T_\sigma) = \begin{cases} 0, & \text{if } x < T_\sigma^l; \\ \frac{\int_{T_\sigma^l}^{x} p(t) dt}{\int_{T_\sigma^l}^{T_\sigma^r} p(t) dt}, & \text{if } T_\sigma^l \leq x \leq T_\sigma^r; \\ 1, & \text{if } x > T_\sigma^r. \end{cases} \tag{4.18}$$

By substituting Eq. (4.18) into the general representation of level set based target-oriented decision model Eq. (4.16), we can obtain the probability of meeting uncertain target T.

Cost target preference

Similarly, in case of cost target preference, we define

$$u(x, t) = \begin{cases} 1, & \text{if } x \leq t; \\ 0, & \text{otherwise.} \end{cases}$$

and then we can obtain $u(x \leq T_\sigma)$ as follows:

$$u(x \leq T_\sigma) = \begin{cases} 1, & \text{if } x < T_\sigma^l; \\ \frac{\int_x^{T_\sigma^r} p(t)dt}{\int_{T_\sigma^l}^{T_\sigma^r} p(t)dt}, & \text{if } T_\sigma^l \leq x \leq T_\sigma^r; \\ 0, & \text{if } x > T_\sigma^r. \end{cases} \tag{4.19}$$

It is clear that $u(x \leq T_\sigma) = 1 - u(x \geq T_\sigma)$, thus we obtain

$$\begin{aligned} \Pr(x \leq T) &= \int_0^{\sup T} u(x \leq T_\sigma) T_\sigma d\sigma \\ &= \int_0^{\sup T} (1 - u(x \geq T_\sigma)) T_\sigma d\sigma \\ &= \int_0^{\sup T} T_\sigma d\sigma - \int_0^{\sup T} u(x \geq T_\sigma) T_\sigma d\sigma \\ &= 1 - \Pr(x \geq T) \end{aligned} \tag{4.20}$$

Equal/range target preference

In case of non-monotonic target preference, there exists an "ideal" level. Recall that target-oriented decision analysis views the modal value T_m of the pdf as reference point (reflection point), then there will be added loss of value for missing the reference point on the low side, or added loss for exceeding the reference point. In other words, when $x = T_m$ the probability of meeting target should be equivalent to one; when $x < T_m$ it can be viewed as pseudo benefit attribute; and when $x > T_m$ it can be viewed as pseudo cost attribute. Due to this observation, we can define the following function:

1. When $x < T_m$,

$$u(x \cong T_\sigma) = \begin{cases} 0, & \text{if } x < T_\sigma^l; \\ \frac{\int_{T_\sigma^l}^{x} p(t)dt}{\int_{T_\sigma^l}^{T_m} p(t)dt}, & \text{otherwise.} \end{cases} \tag{4.21}$$

2. When $x = T_m$,

$$u(x \cong T_\sigma) = \frac{\int_{T_m}^{x} p(t)dt}{\int_{T_m}^{T_m} p(t)dt} = 1 \tag{4.22}$$

3. When $x > T_m$,

$$u(x \cong T_\sigma) = \begin{cases} 0, & \text{if } x > T_\sigma^r; \\ \frac{\int_x^{T_\sigma^r} p(t)dt}{\int_{T_m}^{T_\sigma^r} p(t)dt}, & \text{otherwise.} \end{cases} \tag{4.23}$$

It should be noted that if the mode value T_m is an interval range, such that $T_m \equiv [T_{ml}, T_{mu}]$, then we can define $u(x \cong T_\sigma) = 1$ if $T_{ml} \leq x \leq T_{mu}$. Typical examples of this case are the trapezoidal distributions.

In the following subsection, we shall use four commonly used fuzzy targets in decision making to illustrate the proposed model.

4.5 Illustrative Examples

Fuzzy decision analysis has been widely studied in the literature. Some commonly used fuzzy targets in decision problems are shown in Fig. 4.1.

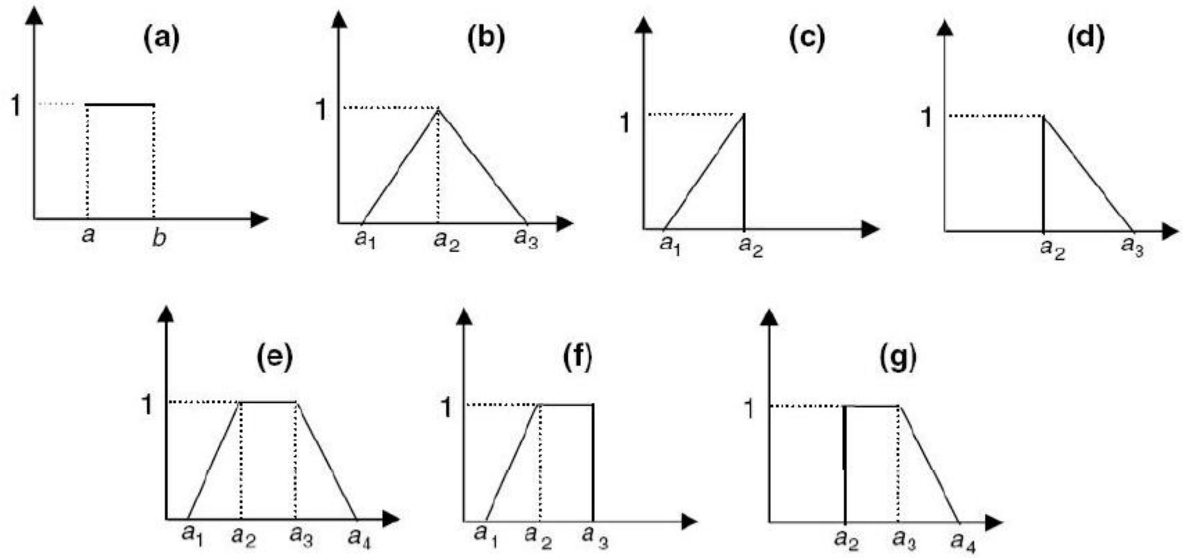

Figure 4.1: Different fuzzy targets used in decision making problems

In decision analysis involving fuzzy targets, there are four types of commonly used fuzzy targets: "**fuzzy min** T_m", "**fuzzy max** T_m", "**fuzzy equal** T_m" and "**fuzzy range/interval**: from T_{ml} to T_{mu}". For computational efficiency, trapezoidal or triangular fuzzy numbers are used to represent the above uncertain targets. In the following, we shall consider these four types of fuzzy targets to illustrate our proposed models.

4.5.1 Fuzzy min

We first assume that the DM assesses his fuzzy target $\widetilde{T}$ as **fuzzy min** T_m distributed over the domain D, where T_m is viewed as the reference point. In this case the DM has a monotonically increasing preference, the fuzzy number can be represented as

$$\pi(t) = \begin{cases} \frac{t - X_{\min}}{T_m - X_{\min}}, & \text{if } X_{\min} \leq t \leq T_m \\ 1, & \text{if } T_m < t \leq X_{\max}. \end{cases} \tag{4.24}$$

Secondly, we use those two approaches mentioned above to obtain the induced probability of meeting this target. Fig. 4.2 graphically depicts the membership function of the **fuzzy min** target, its associated probability distribution and the corresponding probability of meeting the target.

As illustrated, no matter which approach is chosen, the fuzzy min target induces the S-shaped function, according to which

1. The target divides the space of outcomes into regions of gains and loss relative the reflection point T_m.

2. People tend to be risk averse over gains and risk seeking over looses.

3. Outcomes that encoded as losses are more painful than similar sized gains are pleasurable. In Kahneman and Tversky's [70] words, "losses loom larger than gains".

It is clearly seen from Fig. 4.2 that the portraits of $\Pr_{\mathtt{I}}(x \geq \widetilde{T})$ and $\Pr_{\mathtt{II}}(x \geq \widetilde{T})$ have similar shapes for this corresponding target. However, the behavior of $\Pr_{\mathtt{II}}(x \geq \widetilde{T})$ is steeper towards the modal value of this target than that of $\Pr_{\mathtt{I}}(x \geq \widetilde{T})$. This practically implies that the value function defined with $\Pr_{\mathtt{II}}(x \geq \widetilde{T})$ reflects a stronger decision attitude towards the target than that defined with $\Pr_{\mathtt{I}}(x \geq \widetilde{T})$.

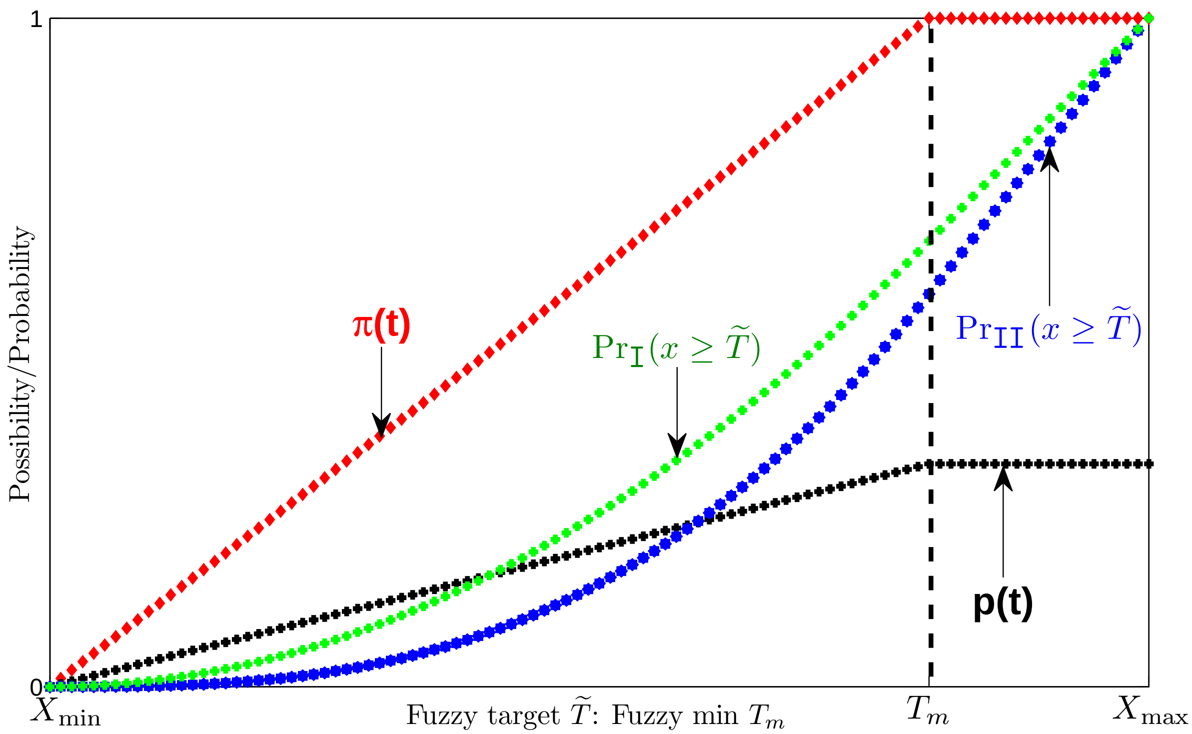

Figure 4.2: Target achievements under *fuzzy min* target

4.5.2 Fuzzy max

Similarly, we now assume that the DM assesses as the membership function for his target of at most about T_m, where T_m is viewed as the reference point. And we get the membership function for this target as follows:

$$\pi(t) = \begin{cases} 1, & \text{if } X_{\min} \leq t \leq T_m; \\ \frac{X_{\max}-t}{X_{\max}-T_m}, & \text{if } T_m < t \leq X_{\max}. \end{cases} \tag{4.25}$$

In this case, the DM has a monotonically decreasing target preference, and then we obtain the associated probability density distribution by the proportional transformation approach and the induced target achievement by means of those two approaches mentioned above. The related functions of this target are graphically illustrated in Fig. 4.3.

As illustrated, we obtain the inverse S-shaped function, according to which

1. The target divides the space of outcomes into regions of gains and loss relative to the reflection point T_m. The consequence x below the reference point can be viewed as a loss, whereas the consequence x upper than the reference point is viewed as loss.

2. People tend to be risk averse over gains and risk seeking over looses.

3. Outcomes that encoded as losses are more painful than similar sized gains are pleasurable.

It can be also seen from Fig. 4.3 that the value function $\Pr_{\mathtt{II}}(x \leq \widetilde{T})$ induced by the level set based method also reflects a stronger decision attitude towards the target than that defined with $\Pr_{\mathtt{I}}(x \leq \widetilde{T})$ induced by the cdf based method.

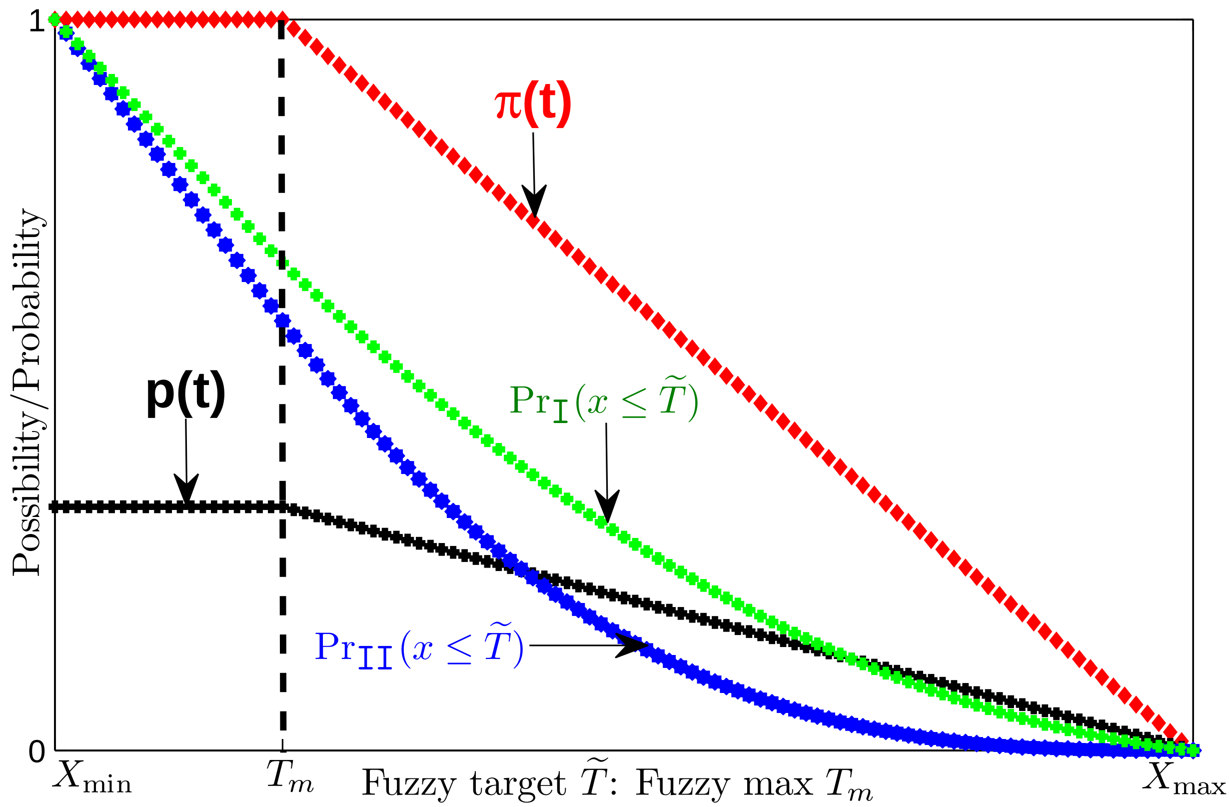

Figure 4.3: Target achievements under *fuzzy max* target

4.5.3 Fuzzy equal

Another fuzzy target is "fuzzy equal". In this case, the target values are fairly fixed and not subject to much change, i.e., too much or too little is not acceptable. Let us assume that the DM assesses the membership function for his target about T_m as

$$\pi(t) = \begin{cases} \frac{t - X_{\min}}{T_m - X_{\min}}, & \text{if } X_{\min} \leq t < T_m; \\ 1, & \text{if } t = T_m; \\ \frac{X_{\max} - t}{X_{\max} - T_m}, & \text{if } T_m < t \leq X_{\max}. \end{cases} \tag{4.26}$$

This fuzzy target characterizes the situation at which the DM establishes a modal target value T_m as the mostly likely target and assesses the possibilistic uncertain target as distributed around it. We call this target the unimodal. And then we obtain the associated probability density distribution by the proportional transformation approach and the induced target achievement by means of those two approaches mentioned above. Fig. 4.4 graphically depicts the membership function of the **fuzzy equal** target, its associated probability distribution and the corresponding probability of meeting the target.

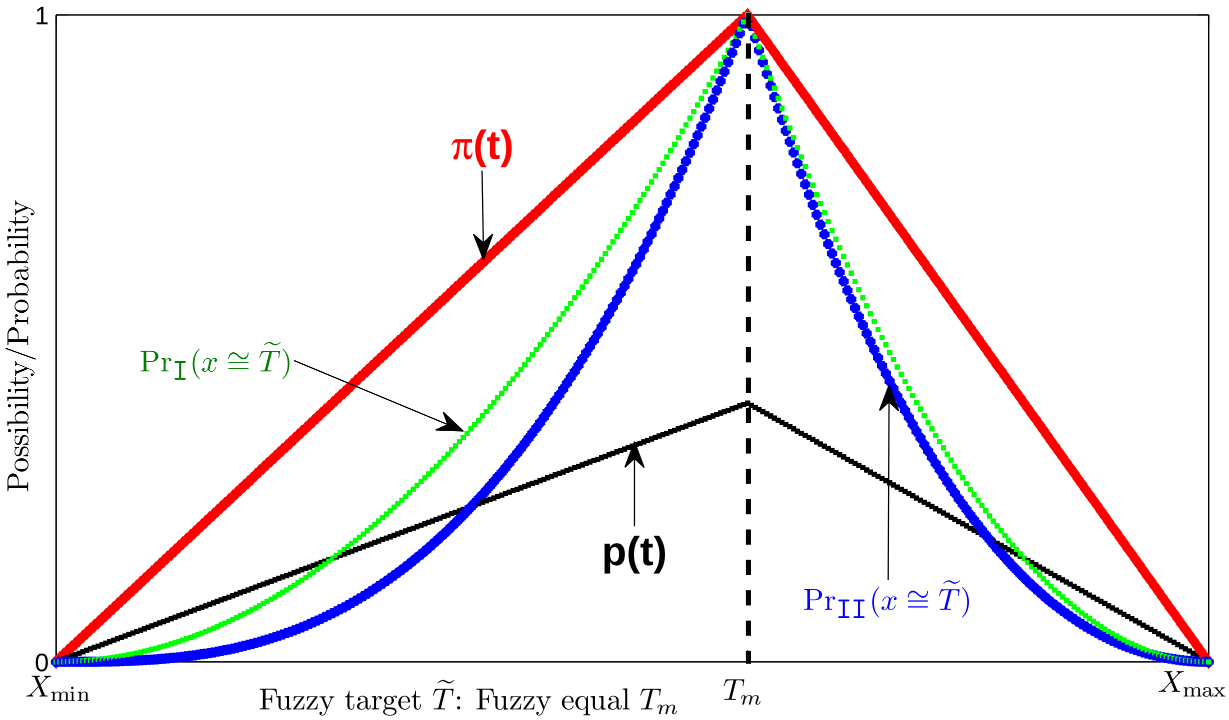

Figure 4.4: Target achievements under *fuzzy equal* target

Looking at Fig. 4.4, as the DM assesses T_m as the mostly likely target, he/she will feel losses with respect to the modal value. The unimodal target induces the convex value function when the possible attribute values are below or upper the mode value. The value function defined with $\Pr_{\mathtt{II}}(x \cong \widetilde{T})$ also reflects a stronger decision attitude towards the target than that defined with $\Pr_{\mathtt{I}}(x \cong \widetilde{T})$.

Remark It should be noted that, the fuzzy equal target can have other types of semantics. For example, Huynh *et al.* [62] have also considered this fuzzy target by assuming monotonically increasing target preference. Generally speaking, we suggest that each fuzzy number can have three types of target preference depending on DM's preferences. To be consistent with Bellman-Zadeh paradigm the same semantics of fuzzy numbers are assumed.

4.5.4 Fuzzy interval

The "fuzzy equal" target is a special case of the "fuzzy interval" target. In this case, the DM assesses target ranges. An example is a manufacturing process where any dimension for manufactured component within a tolerance is equally acceptable. The fuzzy interval target can be defined as

$$\pi(t) = \begin{cases} \frac{t - X_{\min}}{T_{ml} - X_{\min}}, & \text{if } X_{\min} \le t < T_m; \\ 1, & \text{if } T_{ml} \le t \le T_{mr}; \\ \frac{X_{\max} - t}{X_{\max} - T_{mr}}, & \text{if } T_m < t \le X_{\max}. \end{cases} \tag{4.27}$$

Fig. 4.5 graphically shows the possibility distribution, induced probability distribution, and its associated probability function of meeting targets. As illustrated, the fuzzy interval target induces the convex probability function when the possible attribute values are below T_{ml} or upper than T_{mu}. The value function defined with $\Pr_{\mathtt{II}}(x \cong \widetilde{T})$ also reflects a stronger decision attitude towards the target than that defined with $\Pr_{\mathtt{I}}(x \cong \widetilde{T})$.

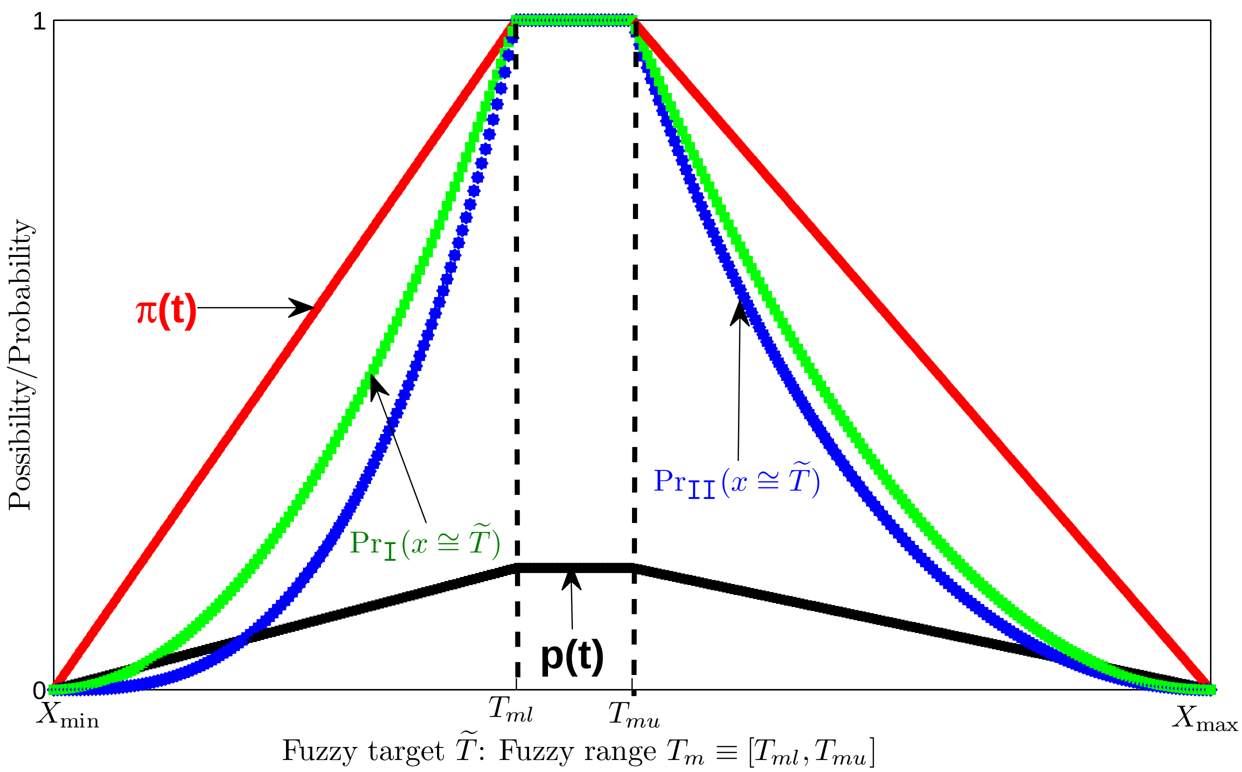

Figure 4.5: Target achievements under *fuzzy interval* target

4.6 Comparison with Bellman-Zadeh's Paradigm

In their pioneering work on fuzzy multi-attribute decision making, Bellman and Zadeh [12] suggest that a attribute can be represented as a fuzzy subset over the alternatives. In particular, if x is a attribute we can represent this as a fuzzy subset x over A such that $A(x)$ is the degree to which this criterion is satisfied, where $\forall\ A(x) \in [0, 1]$. They use the fuzzy membership function to represent the degree of preference (utility). Both the Bellman-Zadeh's paradigm and our approach use fuzzy subset to model decision making involving targets. The main differences between our approach and Bellman-Zadeh's paradigm are threefold.

1. The semantics of membership functions of fuzzy sets are different. Bellman and Zadeh view the membership function of fuzzy sets as a kind of utilities, whereas in our approach the membership function of fuzzy sets is viewed as a kind of uncertainty representations, possibility distribution. In fact, according to the context of problems, membership degrees can be interpreted as similarity, preference, or uncertainty [41]. As pointed out by Beliakov and Warren [11]:

   > *In fuzzy set theory, membership functions of fuzzy sets play the role similar to utility functionsthe role of degrees of preference. Many authors, including Zadeh himself, refer to membership functions as 'a kind of utility functions'. The equivalence of utility and membership functions extends from semantical to syntactical level. Although this is not the only possible interpretation of membership functions, it allows one to formulate and solve problems of multiple attributes decision making using the formalism of fuzzy set theory.*

   In our approach, besides random uncertainty, we also consider fuzzy uncertainty, whereas Bellman and Zadeh only considers fuzzy uncertainty.

2. The semantics of fuzzy numbers are different. In our approach, even the same fuzzy number can have more than one semantic depending on DM's preferences. Whereas, Bellman and Zadeh consider only one semantic. For example, for the fuzzy number $T = (X_{\min}, T_m, X_{\max})$, in the above discussion, we view this fuzzy number as fuzzy equal target. Huynh et al. [62] have also considered this fuzzy target by assuming monotonically increasing target preference. Generally speaking, we suggest that each fuzzy number can have three types of target preference depending on DM's preferences. In this study, we only listed the three types of fuzzy targets commonly used in Bellman-Zadeh paradigm. To be consistent with Bellman-Zadeh paradigm the same semantics of fuzzy numbers are assumed.

3. Our approach can model the psychological preferences of DM, in which the utility functions can have four shapes: S-shaped, inverse S-shaped, convex, and concave.

4.7 Conclusions

Target-oriented model assumes that target has a random probability distribution. It is now more and more widely acknowledged that all facets of uncertainty cannot be captured by a single probability distribution. Moreover, it is usually not easy to find the probability distribution of the uncertain target. In many applications, fuzzy subsets provide a very convenient object for the representation of uncertain information. The contribution of this chapter is to propose two fuzzy target-oriented decision models with respect to different target preferences. The proportional approach is used to transform a possibility distribution into the probability distribution. Some widely used fuzzy targets used in Bellman-Zadeh paradigm [12] are selected to illustrate the fuzzy target-oriented model.

Chapter 5

Non-Additive Multi-Attribute Target-Oriented Decision Analysis

Abstract: In many decision-making situations, multiple attributes are of interest, so it is necessary to consider multi-attribute target-oriented decision analysis. In the literature, several researches have extended the target-oriented decision model into multi-attribute case. In their model, multi-additive value function (MAVF) is used to aggregate partial target achievements while assuming the independence between different targets. However, it is recognized that in many decision problems targets are interdependent. On the other hand, even if, in an objective sense the targets are mutually independent (probabilistically mutually independent), they are not necessary considered to be independent from the decision maker (DM)'s subjective viewpoint.

Due to this observation, the main focus of this chapter is to model the interdependence between different targets based on the fuzzy measure and fuzzy integral. In our research, several similarities between multi-attribute target-oriented model and non-additive fuzzy integral have been discovered. Hence, the λ-fuzzy measure is used as a technique to induce the possible combinations of indices of meeting targets and fuzzy integral is used to model the non-additive multi-attribute model. Compared with previous research, our method can model the interdependence from DM's subjective viewpoint as well as be of simple use in real applications.

5.1 Introduction

Target-oriented decision analysis presumes that a decision maker (DM)'s utility may depend not on the absolute level of performance on an attribute, but rather on whether that level of performance meets a target, in which case the DM is said to be target oriented. For example, typical attributes in new product development include cost, quality, and features, and the corresponding targets might be the best performance on these attributes by competing products. In previous chapters, we considered target-oriented decision analysis with different target preferences under single attribute case. In many decision making situations, multiple attributes are of interest [75], so it is important to know whether the basic target-oriented results extend to the multi-attribute case. In this chapter, we focus on multi-attribute target-oriented decision analysis.

Bordley and Kirkwood [17] consider situations in which a target-oriented approach is natural and define a target-oriented decision maker for a single attribute as one with a utility that depends only on whether a target for that attribute is achieved. They extend this definition to targets for multiple attributes, requiring that the DM's utility for a multidimensional outcome depend only on the subset of attributes for which targets are met, and they develop a target-oriented approach to assess a multi-attribute preference function. Abbas and Howard [2] introduce a class of multi-attribute utility functions called attribute dominance utility functions that can be manipulated like joint probability distributions and allow the use of probability assessment methods in utility elicitation. Taking a different tack, Tsetlin and Winkler [131] consider decision making in a multi-attribute target-oriented setting and study the impact on expected utility of changes in parameters of performance and target distributions (i.e., location, spread, and dependence) by making use of statistics theory. Furthermore, Tsetlin and Winkler [130] also consider the equivalent target-oriented formulations for multi-attribute utility function.

In the literature, multi-additive value function (MAVF) is used to simplify the decision problems while assuming the mutual independence between different targets, e.g. [17]. Although mutual independence assumption leads to convenient and simple use in real applications, the interdependence/interaction phenomena between different targets is very natural in many applications. Toward this end, Tsetlin and Winkler [131] consider the interdependence in multi-attribute target-oriented decision model by means of statistics analysis. They firstly assume targets have some predefined probability distributions, and then model the interaction among targets using a function of *correlations* through an example, in which two targets are both normally distributed. However, as targets may have different probability distributions, their approach will be too complex in real applications. Furthermore, even if, in an objective sense the targets are mutually independent (probabilistically mutually independent), they are not necessary considered to be independent from the DM's subjective viewpoint. As discussed in previous chapters, if the DM specifies fuzzy targets, there is no interdependence between targets from the probabilistic viewpoint, however, the targets may be interdependent from the DM's subjective viewpoint. In this sense, Tsetlin and Winkler's approach will not be suitable, traditional analytic methods are inadequate and not applicable for modeling such complex situations.

Due to the above observations, the main focus of this chapter is to develop a non-additive multi-attribute target-oriented decision model. In the literature, the use of fuzzy measure and fuzzy integral in multi-attribute decision analysis (MADA) enables us to model some interaction phenomena existing among different attributes. As we shall see, there are some similarities between the fuzzy measure and multi-attribute target-oriented decision model. In addition, the fuzzy integral model does not need to assume the independence of each target, thus it can be used in non-linear situations. Based on these two reasons, we shall use the λ-fuzzy measure and Choquet fuzzy integral to model the interdependence between different targets in multi-attribute target-oriented decision analysis.

The rest of this chapter is organized as follows. Section 5.2 formulates the multi-attribute target-oriented decision model. In Section 5.3 we recall some basic knowledge of fuzzy measure and fuzzy integral. In Section 5.4 we propose a non-additive multi-attribute target-oriented decision model based on the λ-fuzzy measure and Choquet integral. Section 5.5 gives an example to illustrate the proposed model. Finally, some concluding remarks are given in Section 5.6.

5.2 Formulation of Multi-Attribute Target-Oriented Function

Now let us consider the multi-attribute decision matrix mentioned in Chapter 2. $\mathcal{A} = \{A^1, \cdots, A^m, \cdots, A^M\}$ is the set of alternatives, and $\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}$ is the set of attributes/criteria. The consequence on attribute X_n of alternative A^m is expressed as $X_n(A^m)$, which can be shortened to X_n^m when there is no possibility of confusion. Note that there are M alternatives and N attributes altogether. In this chapter, for denotational simplicity, for an alternative we shall use x_n instead of X_n^m to denote the consequence data of an alternative.

According to Bordley and Kirkwood [17], with the set of N attributes and N targets $\mathrm{T} = (T_1, \cdots, T_n, \cdots, T_N)$, a DM is defined to be *target oriented* if his or her utility for outcome (alternative) $x = (x_1, \cdots, x_n, \cdots, x_N)$ depends only on which targets are met by that outcome, where there is a single target for each attribute. [1]

If the DM cares only about meeting the targets, his/her utility function should reflect that. The utility function for a target-oriented DM is completely specified by 2^N constants where these constants are the utilities of achieving specific combinations of the various targets. Therefore, to calculate expected utilities it is necessary to know the probability for each of the 2^N different possible combinations of target achievement as a function of the levels for the N attributes. Define $\mathrm{I} = (I_1, \cdots, I_n, \cdots, I_N)$ as a set of indicator variables for the outcome (alternative) [2], where

$$I_n = \begin{cases} 1, & \text{if } x_n \succeq T_n; \\ 0, & \text{otherwise.} \end{cases} \tag{5.1}$$

Then a target-oriented DM has a function $U_I(\mathrm{I})$ assigning utilities to the 2^N possible values of I [130, 131]. Let $U_I(\mathrm{I}) = \nu_R$, where R is the set of indices $\{n | I_n = 1\}$ corresponding to the attributes in I for which the targets are met.

For example, $U_I(1, 0, \cdots, 0) = \nu_1$, $U_I(0, 1, 1, \cdots, 0) = \nu_{2,3}$ and so on. If $R_1 \subseteq R_2$, then $\nu_{R1} \leq \nu_{R2}$; utility can never be reduced by meeting additional targets [130, 131].

[1] In Chapter 3 and 4, we consider probabilistic target and fuzzy target. In this chapter, without denotation danger, we assume that the probabilistic target and fuzzy target are both represented as T.

[2] It should be noted that $\succeq$ can have three semantics: *greater than or equal to* $\geq$, *less than or equal to* $\leq$, and *equal/range level* $\cong$.

We also know that $0 \leq \nu_R \leq 1$ for all R, with $\nu_\varnothing = U_I(0, \cdots, 0, \cdots, 0)$ and $\nu_{1,\cdots,n,\cdots,N} = U_I(1, \cdots, 1, \cdots, 1) = 1$, leaving $2^N - 2$ utilities ν_R to be assessed.

Let us consider a simple example with $N = 2$, we know

$$U_I(\mathrm{I}) = \nu_\varnothing I_\varnothing + \nu_1 I_1 + \nu_2 I_2 + \nu_{12} I_1 I_2 \tag{5.2}$$

Recall that I_n depends on whether $x_n \succeq T_n$, $\nu_\varnothing = 0$, and $\nu_{12} = 1$ thus by integrating the uncertainty about T, we can get

$$\begin{aligned} \Pr(\mathrm{x} \succeq \mathrm{T}) &= \nu_1 \left(\Pr_1 - \Pr_{1,2}\right) + \nu_2 \left(\Pr_2 - \Pr_{1,2}\right) + \Pr_{1,2} \\ &= \nu_1 \Pr_1 + \nu_2 \Pr_2 + (1 - \nu_1 - \nu_2) \Pr_{1,2} \end{aligned} \tag{5.3}$$

where $\Pr_{1,2}$ is the joint probability of meeting targets T_1 and T_2, $\Pr_1$ and $\Pr_2$ are the probability of meeting targets T_1 and T_2 respectively. If targets are mutually independent

$$\Pr(x \succeq T) = \nu_1 \Pr_1 + \nu_2 \Pr_2 + (1 - \nu_1 - \nu_2) \Pr_1 \Pr_2 \tag{5.4}$$

Extending this approach to the case of N targets, the target-oriented function for the outcome x is as follows [17]

$$\Pr(x \succeq T) = \sum_R \nu_R \Pr_R \tag{5.5}$$

where ν_R is the DMs utility function over R. As pointed out by Bordley and Kirkwood [17], there is a descriptive formulation that is equivalent to Eq. (5.5), just as there is a descriptive formulation for the single attribute case. Specially, if ν_R in Eq. (5.5) is interpreted as the probability that a particular set of target achievements R is "good enough" with respect to the entire set of targets, then the right side of gives the probability for the decision maker to select this alternative.

Assessment of 2^N possible ν_R could be time consuming and the mutual dependence among targets will lead to complexity and inconvenience in real applications. Thus, Bordley and Kirkwood [17] assume that the targets are mutually independent. Some special cases of multi-attribute target-oriented function with mutual independent targets are as follows:

1. **Independent preference**
   Targets are independent if the DM's probability of achieving the target on any attribute depend only on the value of attribute, and not on whether targets for other attributes are achieved. A target-oriented DM with independent targets is strategically equivalent to multi-linear utility function [130].

$$\Pr(x \succeq T) = \sum_{n=1}^{N} \nu_n \Pr_n + \sum_{n=1}^{N} \sum_{m>n} \nu_{nm} \Pr_n \Pr_m + \cdots + \nu_{123\cdots N} \prod_{n=1}^{N} \Pr_n. \tag{5.6}$$

2. **Additive target preferences**
   The target-oriented preference function for a target-oriented DM with independent targets and additive independent preferences is strategically equivalent to additive utility function [17, 130].

$$\Pr(x \succeq T) = \sum_{n=1}^{N} \nu_n \Pr_n. \tag{5.7}$$

In many applications, MAVF is used to simplify the decision problems. Although independence assumption leads to convenient and simple use in real applications, interdependence/interaction phenomena among the targets is very natural. Toward this end, Tsetlin and Winkler [131] consider the interdependence in multi-attribute target-oriented decision model by means of statistics analysis. They firstly assume targets have some predefined probability distribution, and then model the interaction among targets using a function of correlations through an example, in which two targets are both normally distributed. However, as targets may have different probability distributions, their approach is too complex in real applications. Furthermore, even if, in an objective sense the targets are mutually independent (probabilistically mutually independent), they are not necessary considered to be independent from the DM's subjective viewpoint. Finally, as discussed previously, if the DM specifies fuzzy targets, Tsetlin and Winkler's approach will not be suitable. In this regard, traditional analytic methods are inadequate and not applicable for modeling such complex situations.

The use of fuzzy measures and fuzzy integral in MADA enables us to model some interaction phenomena existing among different attributes; see [47, 50, 51]. As we shall see, multi-attribute target-oriented function has a similar structure with fuzzy measure and fuzzy integral does not assume the independence, thus we shall use fuzzy measure and fuzzy integral to model the interaction between different targets.

5.3 Fuzzy Measure and Fuzzy Integral

5.3.1 General fuzzy measure

Fuzzy measure is an assessment for representing the membership degree of objects in candidate sets. It assigns a value to each crisp set in the universal set and signifies the degree of evidence or belief about the element's membership in the set. Let $\mathcal{X}$ be a universal set. The fuzzy measure is then defined by the following function $\nu : P(\mathcal{X}) \rightarrow [0, 1]$. That assigns each crisp subset of $\mathcal{X}$ a number in the unit interval $[0, 1]$. The definition of function ν is the power set $P(\mathcal{X})$. When a number is assigned to a subset $R \in P(\mathcal{X})$, ν_R represents the degree of available evidence or the subject's belief that a given element in $\mathcal{X}$ belongs to the subset R [34]. This particular element is most likely found in the subset assigned the highest value.

For quantifying a fuzzy measure, function ν must satisfy several properties. Conventionally, function ν is assumed to have met the conditions of the axiom of probability theory, a probability theory measurement. However, actual practice always goes against this assumption. It is a fuzzy measurement in reality that should be defined by weaker axioms. The probability measure becomes a special type of fuzzy measure. Axioms of the fuzzy measure should include the following:

- Axiom 1: boundary conditions, $\nu_\varnothing = 0$ ($\varnothing$ is the empty set) and $\nu_\mathcal{X} = 1$.
- Axiom 2: monotonic, if $R \subseteq S$, then $\nu_R \leq \nu_S$, $\forall R, S \in P(\mathcal{X})$.

If the universal set is infinite, it is necessary to add continuous axiom. It is quite implicit that the elements in question are not within an empty set but within the universal set, regardless of the amount the evidence from the boundary conditions in Axiom 1. Axiom 2 refers to the necessary evidence for particular elements to belong to a certain set. There would have to be equivalent evidence required for the subset belonging to a set, making this monotonic.

For each subset of the attributes $R \subseteq P(\mathcal{X})$, ν_R can then be interpreted as the weight or the importance of the coalition R. The monotonicity of ν means that the weight of a subset of the attributes can only increase when one adds new attributes to it. The fuzzy measure is often defined by an even more general function $\nu : \beta \to [0, 1]$, where $R \in P(\mathcal{X})$ so that $\varnothing \in \beta$ and $\mathcal{X} \in \beta$. The set β is usually called a Borel field. The triplet $(\mathcal{X}, \beta, \nu)$ is called a fuzzy measure space if ν is a fuzzy measure in a measurable space $(\mathcal{X}, \beta)$.

In actual practice, it is sufficient to consider the finite set. Let $\mathcal{X}$ be a finite attributes set such that $\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}$. The power set $P(\mathcal{X})$ be a class of all of the subsets of $\mathcal{X}$. It can be noted that $\nu_{\{X_n\}}$ for a subset with a single element, $\{X_n\}$ is called a fuzzy density. For purposes of simplicity, in the following we shall use ν_n to represent $\nu_{\{X_n\}}$.

To differentiate the proposed model from other fuzzy measure models (such as λ-fuzzy measure, F-additive measure, classical probability measure), a general fuzzy measure is used to designate a fuzzy measure that is monotonic and only required to satisfy the boundary conditions. A general fuzzy measure has the fewest number of constraints and is the most general measure pattern.

5.3.2 λ-fuzzy measure

Since the specification for fuzzy measures requires the values of a fuzzy measure for all subsets in $\mathcal{X}$, Sugeno and Terano [122] incorporated the λ-additive axiom in order to reduce the difficulty of collecting information. In a fuzzy measure space $(\mathcal{X}, \beta, \nu)$, let $\lambda \in (-1, \infty)$. If $R \in \beta, S \in \beta$, $R \subset S = \varnothing$, and $\nu_{R\cup S}(\lambda) = \nu_R + \nu_S + \lambda\nu_R\nu_S$, then the fuzzy measure ν is λ-additive.

1. If $\nu_{R\cup S}(\lambda) < \nu_R + \nu_S$ then ν satisfies the substitutive effects.
2. If $\nu_{R\cup S}(\lambda) > \nu_R + \nu_S$ then ν satisfies the multiplicative effects.
3. If $\nu_{R\cup S}(\lambda) = \nu_R + \nu_S$ then ν satisfies the mutual independence.

This particular fuzzy measure is termed λ fuzzy measure because it must fulfil λ-additive. It is known as Sugeno measure. Then the λ-fuzzy measure of the finite set can be derived from fuzzy densities, as indicated in the following equation

$$\nu_{\{X_1,X_2\}}(\lambda) = \nu_1 + \nu_2 + \lambda\nu_1\nu_2 \tag{5.8}$$

where ν_1 and ν_2 represent the fuzzy density. Extending this to N attributes, we obtain

$$\begin{aligned} \nu_\lambda\left(\{X_1, \cdots, X_n, \cdots, X_N\}\right) &= \sum_{n=1}^{N} \nu_n + \lambda \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} \nu_n\nu_m + \cdots + \lambda^{n-1} \prod_{n=1}^{N} \nu_n \\ &= \frac{1}{\lambda}\left[\prod_{n=1}^{N} (1 + \lambda\nu_n)\right] \end{aligned} \tag{5.9}$$

where $-1 < \lambda < \infty$, and λ is the parameter showing the relationship among the related attributes (if $\lambda = 0$ it is an additive form; if $\lambda \neq 0$, it is a non-additive form) [29, 34]. In a later publication by Sugeno, the value of the parameter λ is allowed to include -1, i.e., $-1 \leq \lambda < \infty$.

5.3.3 Choquet fuzzy integral

When using a fuzzy measure to model the importance of each subset of attributes, a suitable aggregation operator that generalizes the weighted arithmetic mean is the discrete Choquet integral which is defined as follows [34]

Definition Consider a fuzzy measure space $(\mathcal{X}, \beta, \nu)$. Let h be a measurable function from X to $[0,1]$, Assuming that $h(X_1) \geq \cdots \geq h(X_n) \geq \cdots \geq h(X_N)$, then the fuzzy integral is defined as follows:

$$(C)\int h d\nu = \sum_{n=1}^{N} \nu_{H_n}\left[h(X_n) - h(X_{n+1})\right] \tag{5.10}$$

where $H_1 = \{X_1\}$, $H_2 = \{X_1, X_2\}$, $\cdots$, $H_N = \{X_1, \cdots, X_n, \cdots, X_N\}$, and $h(X_{N+1}) = 0$.

The Choquet integral takes into account the interaction among attributes by means of the fuzzy measure ν. As soon as ν is additive, that is, as soon as attributes are independent, the Choquet integral collapses into the multi-additive value function (MAVF), i.e.,

$$(C)\int h d\nu = \sum_{n=1}^{N} h(X_n)\nu_n \tag{5.11}$$

It should be noted that an axiomatic characterization of the Choquet integral as an aggregation operator has been proposed by Marichal [97]. Although there are other forms of integrals defined over fuzzy measures, such as the Sugeno integral, we considered only the Choquet integral for our analysis.

5.4 Multi-Attribute Target-Oriented Decision Analysis Based on λ-Fuzzy Measure and Choquet Fuzzy Integral

Recall that DM's utility function ν_R in Eq. (5.5) satisfy the axioms of the fuzzy measure: boundary conditions, $\nu_\varnothing = 0$ ($\varnothing$ is the empty set) and $\nu_{1,2,\cdots,N} = 1$; monotonic, if $R_1 \subseteq R_2$, then $\nu_{R_1} \leq \nu_{R_2}$. Here R_1 and R_2 are two sets of indices $\{n|I_n = 1\}$ corresponding to the attributes in I for which the targets are met. Thus we can model DM's utility function ν_R over R via fuzzy measure. Particularly, the λ-fuzzy measure is used to induce the utility for the DM.

The fact that fuzzy integral model does not need to assume the independence of each target, means it can be used in non-linear situations. As pointed out by [34], even if, in an objective sense any two targets are independent, they are not necessarily considered to be independent from the DM's subjective viewpoint. This explains why a fuzzy integral is more appropriate. Furthermore, even if one target is probabilistically independent from another, fuzzy integral can be used to measure the relations between each target in the same group.

Due to these observations, we shall use λ-fuzzy measure and Choquet fuzzy integral to model the dependence in multi-attribute target-oriented decision analysis. With a set of N attributes $\mathcal{X} = (X_1, \cdots, X_n, \cdots, X_N)$ and N targets $\mathrm{T} = (T_1, \cdots, T_n, \cdots, T_N)$, for each attribute, we assume that the importance weight is given by

$$W = (w_1, \cdots, w_n, \cdots, w_N)).$$

For an outcome

$$x = (x_1, \cdots, x_n, \cdots, x_N),$$

we obtain the probability of meeting target for each attribute, denoted as

$$\mathrm{Pr} = (\mathrm{Pr}_1, \cdots, \mathrm{Pr}_n, \cdots, \mathrm{Pr}_N).$$

And then we proceed as follows:

1. Reorder the partial target achievements $\mathrm{Pr} = (\mathrm{Pr}_1, \cdots, \mathrm{Pr}_n, \cdots, \mathrm{Pr}_N)$ as $\mathrm{Pr}_1^{'} \geq \cdots \mathrm{Pr}_n^{'} \geq \cdots \geq \mathrm{Pr}_N^{'}$, where $\mathrm{Pr}_n^{'}$ is the n-th largest value in the collection Pr. We also reorder the importance weights W according to the order of $\mathrm{Pr}^{'}$, denoted as $W^{'} = (w_1^{'}, \cdots, w_n^{'}, \cdots, w_N^{'})$.

2. Using λ-fuzzy measure to express the fuzzy measures of each individual attributes group

   (a) Obtain the individual attributes group $H_1 = \{X_1'\}$, $H_2 = \{X_1', X_2'\}$, $\cdots$, $H_N = \{X_1^{'}, \cdots, X_n^{'}, \cdots, X_N^{'}\}$.

   (b) Specify a λ value

   (c) Identify the fuzzy measures of individual attributes group with a given λ value according to Algorithm 1.

Algorithm 1 A bisection search algorithm to find $\nu_\lambda(H_N) = 1$

1: Normalize weights where $\max(w_n^{'}) = 1$
2: Initialize `lower` $= 0$, $\kappa = 0.5$, `upper` $= 1$
3: Specify $\nu_{\{X_n'\}} = \kappa \cdot w_n^{'}$
4: **for** $1 \leq n \leq N$ **do**
5:     $\nu_{H_n} = \nu_{H_{n-1}} + \nu_{\{X_n'\}} + \lambda \nu_{H_{n-1}} \nu_{\{X_n'\}}$ where $n = 2, \cdots, N$
6:     **if** $\nu_{H_n} > 1$ **then**
7:         `upper` $= v$, $\kappa = ($`lower` $+$ `upper`$)/2$, go to 3
8:     **else**
9:         continue
10:     **end if**
11: **end for**
12: **if** $\nu_{H_n} < 1$ **then**
13:     `lower` $= v$, $\kappa = ($`lower` $+$ `upper`$)/2$, go to 3
14: **else if** $\nu_{H_n} = 1$ **then**
15:     Specify $\nu_{\{X_n'\}} = \kappa \cdot w_n^{'}$
16: **end if**

---

3. The fuzzy integral of the fuzzy measure $\nu_{(\cdot)}$ and $\mathrm{Pr}_{(\cdot)}^{'}$ on X can be defined using

$$(C) \int \mathrm{Pr}^{'} d\nu = \sum_{n=1}^{N} [\mathrm{Pr}_n^{'} - \mathrm{Pr}_{n+1}^{'}] \nu_{H_n},$$

   where $\mathrm{Pr}_{N+1}^{'} = 0$.

5.5 Illustrative Example-New Products Development Problem

In this section we shall consider the following example borrowed from [17, 74] to illustrate the effectiveness and advantages of our proposed approach.

5.5.1 Problem descriptions

A company wants to assess how prospective customers would evaluate a proposed new tester for very large-scale integrated circuits. They identified four categories of evaluation criteria (technical, economic, software, and vendor support) with a total of 17 evaluation attributes, as shown in the first column of Table 5.1. The preference monotonicity for each evaluation attribute is shown in the third column of the table, and the performance scores on each of the evaluation attributes are shown from the fourth through the sixth column of the table for the `OR 9000`, which was the proposed new tester, and its two competitors, the `J941` and the `Sentry 50`.

Table 5.1: New product development: Data

Evaluation attribute	Weight	Monotonicity	Tester ratings		
			`OR 9000`	`J941`	`Sentry 50`
Technical X_1	0.52				
Pin capacity X_{11}	0.15	Increasing	160	96	256
Vector depth X_{12}	0.20	Increasing	0.128	0.256	0.064
Data rate X_{13}	0.10	Increasing	50	20	50
Timing accuracy X_{14}	0.35	Decreasing	1,000	1,000	600
Pin capacitance X_{15}	0.10	Decreasing	55	50	40
Programmable measurement units X_{16}	0.10	Increasing	8	2	4
Economic X_2	0.14				
Price X_{21}	0.50	Decreasing	1.4	1	2.8
Uptime X_{22}	0.20	Increasing	98	95	95
Delivery time X_{23}	0.30	Decreasing	3	6	6
Software X_3	0.32				
Software translator X_{31}	0.15	Increasing	90	90	90
Networking: Communications X_{32}	0.20	Increasing	1	1	1
Networking: Open X_{33}	0.20	Increasing	1	0	0
Development time X_{34}	0.30	Decreasing	3	4	4
Data analysis software X_{35}	0.15	Increasing	1	1	1
Vendor support X_4	0.02				
Vendor service X_{41}	0.30	Decreasing	2	4.75	6
Vendor performance X_{42}	0.30	Decreasing	4	4	4
Customer applications X_{43}	0.40	Increasing	1	1	1

5.5.2 Previous research

Keeney and Lilien [74] assessed the measurable value function for a lead user at a primary customer company company for this testing equipment. This lead user first assessed a minimum acceptability level and a maximum desirability level for each attribute. They then confirmed that the user's preference were describable by an additive measurable value function, and assessed a single dimensional value function and an importance weight for each attribute. Keeney and Lilien used a two-stage process to assess weights for the evaluation attributes with each of four evaluation categories were assessed, and then weights were assessed for each of the four categories so that the overall weight for each evaluation attribute was the product of its category weight and its within-category weight. Both the within-category weights and the category weights (which are 0.52, 0.14, 0.32, and 0.02) are shown in the third column of Table 5.1. The assessed additive measurable value function was then used to evaluate the `OR 9000` against `J941` and `Sentry 50`, and the result served as input to determine that the proposed new tester `OR 9000` was not competitive enough to market.

It is natural to think this decision problem in terms of performance targets because the explicit purpose of the analysis is to determine whether the `OR 9000` was attractive against the `J941` and the `Sentry 50` or not. Thus, *the performance of these two testers sets targets against which the `OR 9000` is judged.* Toward this end, Bordley and Kirkwood [17] use the performance targets to valuate the multi-attribute performance analysis. As there is no random uncertainty about the performance of the three testers, they specify crisp target for each attribute (see Column 2 of Table 5.2).

The target for each attribute is specified by using the following function

$$T_{(\cdot)} = \begin{cases} \max\{X_{(\cdot)}^{\texttt{J941}}, X_{(\cdot)}^{\texttt{Sentry 50}}\}, & \text{increasing preference,} \\ \min\{X_{(\cdot)}^{\texttt{J941}}, X_{(\cdot)}^{\texttt{Sentry 50}}\}, & \text{decreasing preference.} \end{cases} \tag{5.12}$$

For example, consider the attribute **Pin capacity** X_{11}, the performance scores of `J941` and `Sentry 50` are $X_{11}^{\texttt{J941}} = 96$ and $X_{11}^{\texttt{Sentry 50}} = 256$, respectively. As X_{11} is a benefit attribute, thus

$$\begin{aligned} T_{11} &= \max\{X_{11}^{\texttt{J941}}, X_{11}^{\texttt{Sentry 50}}\} \\ &= \max\{96, 256\} \\ &= 256 \end{aligned}$$

According to Bordley and Kirkwood, we know the target achievement method of benefit target is $\Pr_{(\cdot)} = 1, \ \texttt{if} \ x_{(\cdot)} \geq T_{(\cdot)}; \ 0, \ \texttt{otherwise}.$[3] The cost attribute has a contrary function. Using the performance targets and target achievement computation method given by Bordley and Kirkwood, we can obtain the target achievements for each attribute with respect to those three testers, as shown in Columns 4-6 of Table 5.2.

For purposes of simplicity, Bordley and Kirkwood assume that additive independence holds and then based on multi-additive value function, they suggest that `OR 9000` was not competitive enough against the `J941` and the `Sentry 50` with the specified targets. The overall values and ranking result of these three testers are

$$\texttt{Sentry 50} = 0.820 \succ \texttt{J941} = 0.584 \succ \texttt{OR 9000} = 0.514.$$

[3] In Section 5.2, we already mentioned that x_n can be used to represent X_n^m.

Usually, it is difficult for the DM(s) to specify exactly their target values. There are two basic reasons why such parameters or inputs cannot be assigned precise values. First, some quantities are subject to intrinsic variability. Another reason for uncertainty is the plain lack of knowledge about relevant parameters. This lack of knowledge may stem from a partial lack of data, either because this data is impossible to collect, or too expensive to collect, or because the measurement devices have limited precision, or yet because only human experts can provide some imprecise information [10]. In addition, dependence among the targets is very natural.

These two observations lead us to use the fuzzy target-oriented decision model and the non-additive multi-attribute target-oriented decision model to solve this problem.

Table 5.2: New product development: Target-oriented analysis

Attribute	Target	Fuzzy target	Bordley-Kirkwood			Fuzzy target-oriented		
			`OR 9000`	`J941`	`Sentry 50`	`OR 9000`	`J941`	`Sentry 50`
X_1								
X_{11}	256	$(96, 256, 256)$	0	0	1	**0.160**	0.000	1.000
X_{12}	0.256	$(.064, .256, .256)$	0	1	0	**0.111**	1.000	0.000
X_{13}	50	$(20, 50, 50)$	1	0	1	1.000	0.000	1.000
X_{14}	600	$(600, 600, 1000)$	0	0	1	0.000	0.000	1.000
X_{15}	40	$(40, 40, 50)$	0	0	1	0.000	0.000	1.000
X_{16}	4	$(2, 4, 4)$	1	0	1	1.000	0.000	1.000
X_2								
X_{21}	1	$(1, 1, 2.8)$	0	1	0	**0.467**	1.000	0.000
X_{22}	95	$(95, 95, 95)$	1	1	1	1.000	1.000	1.000
X_{23}	6	$(6, 6, 6)$	1	1	1	1.000	1.000	1.000
X_3								
X_{31}	90	$(90, 90, 90)$	1	1	1	1.000	1.000	1.000
X_{32}	1	$(1, 1, 1)$	1	1	1	1.000	1.000	1.000
X_{33}	0	$(0, 0, 0)$	1	1	1	1.000	1.000	1.000
X_{34}	4	$(4, 4, 4)$	1	1	1	1.000	1.000	1.000
X_{35}	1	$(1, 1, 1)$	1	1	1	1.000	1.000	1.000
X_4								
X_{41}	4.75	$(4.75, 4.75, 6)$	1	1	0	1.000	1.000	0.000
X_{42}	4	$(4, 4, 4)$	1	1	1	1.000	1.000	1.000
X_{43}	1	$(1, 1, 1)$	1	1	1	1.000	1.000	1.000

5.5.3 Non-additive fuzzy target-oriented decision analysis

In this subsection, we shall use the fuzzy target-oriented decision model discussed in Chapter 4 and the non-additive multi-attribute target-oriented decision model to solve this problem.

Fuzzy target-oriented decision analysis

Similar with Bordley and Kirkwood [17], the first step in our approach is to specify a target for each attribute. As there is no random uncertainty about the performance scores, Bordley and Kirkwood specified crisp performance target for each attribute; see Eq. (5.12). This specification method is too arbitrary and too strict. For example, the performance scores of `J941` and `Sentry 50` with respect to the attribute **Pin capacity** X_{11} are $X_{11}^{\texttt{J941}} = 96$ and $X_{11}^{\texttt{Sentry 50}} = 256$. Using Bordley and Kirkwood's method, 256 is the performance target value of attribute X_{11}. Recall that the explicit purpose of the analysis was to determine whether the `OR 9000` was attractive against the `J941` and the `Sentry 50` or not. Thus, the performance of these two testers sets targets against which the `OR 9000` is judged [17]. Although there is no random uncertainty about the performance scores of the three testers, there exists some fuzzy uncertainty. If 256 is the target value, how about the performance scores of `J941` regarding X_{11} or a possible value 200? To integrate the fuzzy uncertainty, we shall use fuzzy subsets to represent the uncertainty of target.

As discussed previously, the fuzzy targets can have different types. However, For the sake of simplicity and to be consistent with Bordley and Kirkwood, it is assumed that every benefit attribute should be **"fuzzy min"** target, and for a cost attribute the fuzzy target is **"fuzzy max"** target. We will use Bordley and Kirkwood's crisp target values as the reference point of each fuzzy target. The reservation and utopia points are specified according to the min and max values of $X_{(\cdot)}^{\texttt{J941}}$ and $X_{(\cdot)}^{\texttt{Sentry 50}}$.

For example, consider the attribute **Pin capacity** X_{11}, as it is a benefit attribute, we can define fuzzy target as $(96, 256, 256)$ [4]. The fuzzy targets specified for attributes are shown in Column 3 of Table 5.2. Then by making use of the performance data in Table 5.1 and the fuzzy targets in Column 3 of Table 5.2, we can obtain the probability of meeting targets for benefit and cost attributes, respectively [5]. The satisfaction degrees calculated by the target-oriented decision model are shown from column 7 to 9 of Table 5.2.

From Table 5.2 it is clearly seen that there are three different target achievements between our approach and Bordley and Kirkwood's approach, the bold numbers in Column 7 of Table 5.2. Taking the attribute **Pin capacity** X_{11} as an example, the crisp target defined by Bordley and Kirkwood is 256. It is clearly that

$$X_{11}^{\texttt{J941}} < X_{11}^{\texttt{OR 9000}} < X_{11}^{\texttt{Sentry 50}} = 256,$$

thus `OR 9000` performs better than `J941`, but worse than `Sentry 50` with respect to X_{11}. However, According to Bordley and Kirkwood's approach, we know that there is

[4] It should be noted that when $X_{(\cdot)}^{\texttt{J941}} = X_{(\cdot)}^{\texttt{Sentry 50}}$, there will be no uncertainty about the performance target. In this case, we will obtain a crisp target. Generally speaking, the crisp target can be viewed as a special case of fuzzy target.

[5] In this example, the cumulative distribution function (cdf) based target-oriented decision model, which has been proposed in Chapter 4, is used.

no difference between OR 9000 and J941 regarding X_{11}. This is the main reason why we utilize fuzzy target-oriented decision model.

Non-additive aggregation

Both Keeney and Lilien [74] and Bordley and Kirkwood [17] use the multi-additive value function (MAVF) assuming the independence of targets. However, the interdependence among different targets is quite natural. Thus we shall use λ-fuzzy measure and fuzzy integral to aggregate the partial target achievements according to the procedure in Section 5.4. The areas of the λ value is $-1 \leq \lambda < \infty$. The research sets the λ value from -1 to 100 (Table 5.3, Fig. 5.1 and Fig. 5.2) to acquire different effective values and varying ranking. To clearly distinguish the differences between aggregation values regarding different λ values, we divide the λ value domain as two groups: $[-1, 0]$ and $[0, 100]$.

Table 5.3: Sensitivity scores of three testers

Three testers	λ value									
	-1.0	-0.5	0.0	1.0	5.0	10.0	20.0	50.0	100.0	(MAVF)
OR 9000	0.9758	0.6462	0.5707	0.4929	0.3751	0.317	0.2624	0.2003	0.1621	0.5707
J941	0.9835	0.6621	0.584	0.5027	0.3782	0.317	0.2581	0.1919	0.1512	0.584
Sentry 50	1.0	0.8672	0.82	0.7655	0.6696	0.6152	0.5583	0.4848	0.433	0.820

Table 5.3 shows the sensitivity scores of three testers regarding the parameter λ. Based on the analysis in Table 5.3, we discover that

1. when $-1 \leq \lambda < 0$, there are substitutive effects among the three testers;
2. when $\lambda = 0$, there is no interdependence;
3. when $0 < \lambda \leq 100$, there is multiplicative effects.

When $\lambda = 0$, the non-additive multi-attribute target-oriented decision model is consistent with the multi-additive value function.

Looking at Fig. 5.1, it is known that OR 9000 is not competitive enough to market. However, it lacks consistent ranking.

1. When $-1 \leq \lambda < 10$, the ranking is Sentry 50 $\succ$ J941 $\succ$ OR 9000;
2. when $\lambda = 10$, the ranking is Sentry 50 $\succ$ J941 $\sim$ OR 9000;
3. when $10 < \lambda \leq 100$, the ranking is Sentry 50 $\succ$ OR 9000 $\succ$ J941.

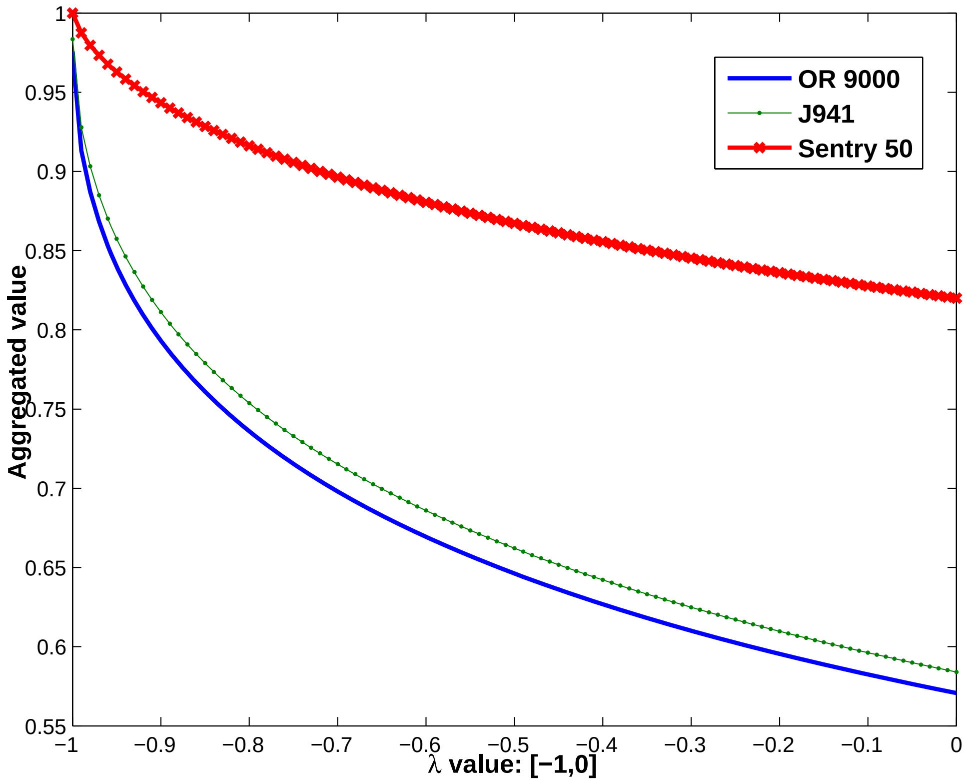

Figure 5.1: Aggregation values of three testers with different λ: λ value from -1 to 0

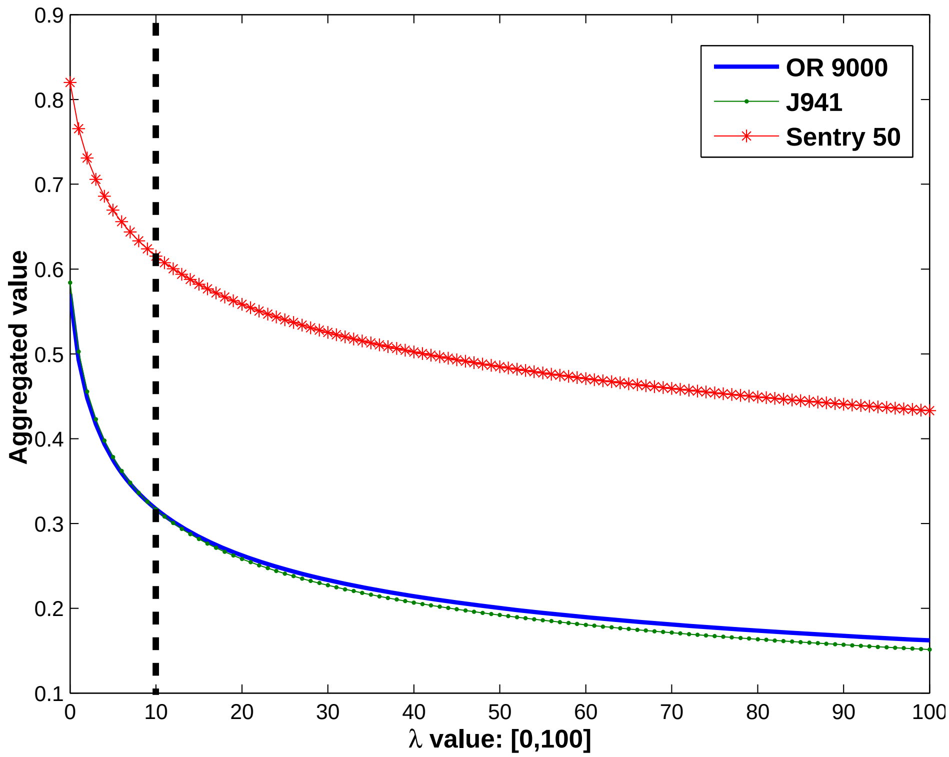

Figure 5.2: Aggregation values of three testers with different λ: λ value from 0 to 100

Discussions

Although in this example the competitive shortcomings of the `OR 9000` can be identified with all the three analysis approaches. Slightly changing the performance scores of `OR 9000` may induce different results.

The Keeney and Lilien [74]'s measurable value analysis requires ranges for all the evaluation attributes and mid-values for those ranges, while both Bordely and Kirkwood's approach and our approach require that a target be specified for each attribute. These three methods require that attribute weights be assessed.

Both Bordley and Kirkwood's approach and our approach consider that the performance of these two testers sets targets against which the `OR 9000` is judged. However, as `J941` and `Sentry 50` are competitors of `OR 9000`, there should be some fuzzy uncertainty about the target itself. The main advantage of our approach is that it can capture the fuzzy quantities of target achievement. In addition, it is quite natural to consider interdependence among different targets, which is missed in both Keeney and Lilien's approach and Bordely and Kirkwood's approach. Our approach can model the dependence phenomena via fuzzy measure and fuzzy integral.

5.6 Summary

In this chapter, we develop a non-additive multi-attribute target-oriented decision model based on fuzzy measure and fuzzy integral. Particularly, the λ-fuzzy measure is used as a technique to induce the possible combinations of indices of meeting targets and fuzzy integral is used to model the non-additive multi-attribute model. A bisection search algorithm is also designed to identify the fuzzy measures of individual attributes group with a given λ value. An illustrative example borrowed from the literature is also given to compare our work with previous research. Compared with previous research, our method can model the interdependence from DM's subjective viewpoint as well as be of simple use in real applications.

In future, we would like to consider the bipolar scale in this non-additive multi-attribute target-oriented decision model. Target-oriented decision model assumes that the target divides the outcomes into gains and loss, thus the outcomes below or exceeding the reference point should have different impacts on the aggregation of partial target achievements. In this case, from the point view of aggregation, multi-attribute target-oriented decision model satisfies the conditions that the values to be aggregated lie on different bipolar scales, where 0 is the worst score, 1 is the best score, and there exists different reference points, denoted as e. For different attributes, the values e are probably different as different attributes may have different target distributions. The resulting continuous piecewise linear aggregation function has the ability to represent decisional behaviors that depend on the "positive" or "negative" satisfaction of some of the attributes.

Chapter 6

Prioritized Multi-Attribute Target-Oriented Decision Analysis

Abstract: In multi-attribute target-oriented decision model, the importance information associated with different targets is important as some targets are more important than others. In general, the importance information plays a fundamental role in the comparison between alternatives by overseeing the tradeoffs between respective satisfactions of different targets. A concept closely related to the importance of targets is the priority of targets. Simply speaking, by saying target T_1 has a higher priority than target T_2, it indicates that we are not willing to tradeoff satisfaction of target T_1 until perhaps we attain some level of satisfaction of target T_2.

The main objective of this chapter is to study the prioritized multi-attribute target-oriented decision model, where there exists a prioritization of different targets. To do so, firstly, the ordered weighted averaging (OWA) operator will be used to obtain the satisfaction degree for each priority level. Secondly, we suggest that roughly speaking any t-norm can be used to model the priority relationships between the targets in different priority levels. To keep the slight change of priority weight, strict Archimedean t-norms perform better in inducing priority weight. As Hamacher family of t-norms provide a wide class of strict Archimedean t-norms ranging from the product to weakest t-norm, Hamacher t-norms are selected to induce the priority weight for each priority level. Thirdly, considering decision maker (DM)'s requirement toward the higher priority levels, a *benchmark* based approach is proposed to induce priority weight for each priority level, i.e., "*the satisfactions of the higher priority attributes are larger than or equal to the DM's requirements*". We suggest that the weights of lower priority level should depend on the benchmark achievement of all the higher priority levels.

6.1 Introduction

For a multi-attribute decision analysis (MADA) problem, with N attributes and N targets, a decision maker (DM) is defined to be *target oriented* if his or her utility for outcome (alternative) depends only on which targets are met by that outcome, where there is a single target for each attribute. As our research is based on the value-focused model [75] and we divide MATODA problems into two main steps: (1) on the target achievement of single attribute, (2) aggregation of partial target achievements into a global value. In this chapter, we assume that the target achievements of different attributes have already been obtained according to the target-oriented decision models proposed in Chapter 3 and Chapter 4. Thus we can simply view the MATODA problems as an aggregation problems, and MATODA can be viewed as a special case of MADA problems, where there exists a target for each attribute. In fact, traditional MADA problems assume that there exists one utility function for each attribute. Target-oriented decision model presumes a target-oriented utility. From now on, we shall use MATODA and MADA interchangeably.

In Chapter 5, several cases of MATODA problems have been introduced and discussed: (1) general representation; (2) independent case; (3) additive preference case; and (4) non-additive case based on fuzzy measure and fuzzy integral. In all these four cases, the importance information associated with different targets/attributes is important as some targets/attributes are more important than others. In this case, the DM usually associates different importance weights with different targets/attributes. There are several approaches to incorporating and/or assigning weights to different targets/attributes [23, 79, 94, 101, 125, 126, 141, 144]. Typical is some form of weighted arithmetic mean, such as quasi-arithmetic means, weighted arithmetic means, weighted quasi-arithmetic means [23]. These aggregation operations work well in situations in which any differences are viewed as being in conflict because the operator reflects a form of compromise behavior among the various targets/attributes [94, 137]. In general, the importance information associated with different attributes plays a fundamental role in the comparison between alternatives by overseeing tradeoffs between respective satisfactions of different targets [148, 150].

A concept closely related to the importance of attributes is the priority of attributes [36, 148]. In practical decision making situations, it is usual for DMs to consider different priorities of targets/attributes. A typical example is in the case of buying a car based upon the attributes of *safety* and *cost*. Assume that the DM specifies two targets; $T_{\texttt{safety}}$ and $T_{\texttt{cost}}$. In this case, usually we may not allow compensation between the target achievements of *cost* and *safety*. Simply speaking, by saying target $T_{\texttt{safety}}$ has a higher priority than target $T_{\texttt{cost}}$, it indicates that we are not willing to tradeoff satisfaction of target $T_{\texttt{cost}}$ until perhaps we attain some level of satisfaction of target $T_{\texttt{safety}}$, This kind of MATODA, so-called prioritized MATODA, will be studied in this chapter.

Many studies have attempted to include different priorities of attributes into MADA problems in the literature. Generally speaking, approaches to prioritized MADA can be classified into two categories according to our knowledge. Approaches belonging to the first class aim to use non-monotonic intersection operator [58, 142] and triangular norms (t-norms) to model the priority relationships among attributes. For example, Yager [143] uses the non-monotonic intersection operator to deal with MADA problems and presents a type of attribute, called second order attribute. Yager [145] uses the weighted conjunction of fuzzy sets and fuzzy modeling to develop the operators in fuzzy information structures. Chen and Chen [30] extend the non-monotonic intersection operator to present a prioritized multi-attribute fuzzy decision making problems based on the similarity measure of generalized fuzzy numbers. Luo et al [95] give five methods to construct the priority operators that are used for calculating the global degree of satisfaction of a prioritized fuzzy constraint problem based on Dubois et al [39]. The second class of approaches tend to use weighted aggregation operators to model the prioritized MADA. For example, Yager [148] shows that the prioritization of attributes can be modeled by using importance weights in which the weights associated with the lower priority attributes are related to the satisfaction of the higher priority attributes. Moreover, they provide some models that allow for the formalization of these prioritized MADA problems using both the Bellman-Zadeh paradigm [12] for MADA and the ordered weighted averaging (OWA) operator. To develop this concept further, Yager [150] proposes a prioritized averaging/scoring aggregation operator with a strict/weak priority order by means of the product t-norm. Furthermore, taking DM's requirements into account, Wang and Chen [31, 136] suggest that the weights of the lower priority attributes depend on *whether* each alternative satisfies the requirements of all the higher priority attributes *or not*.

In this study, we focus on the second class of prioritized MADA, i.e., priority weighted MADA [31, 136, 148, 150]. Although previous research has greatly advanced the priority weighted MADA, there are still some limitations and drawbacks in previous works.

1. Firstly, in prioritized MADA we will have a prioritization of attributes. Attributes in the same priority level should allow different tradeoffs. However, as we shall see in Section 6.3, Yager's method [148, 150] does not preserve this property.

2. Secondly, as suggested by Yager [148, 150], the product triangular norm is used to induce the priority weight for each priority level. However, as there are many types of t-norms available, can any t-norm be used to induce the priority weight? If so, which type of t-norms are better?

3. Thirdly, DM(s) may have a requirement toward the higher priority levels. The method of inclusion of DM's requirements into satisfaction function proposed by Wang and Chen [31, 136] will be too strict for DM to make decision under prioritized environments. In addition, due to the vagueness or impreciseness of knowledge, it is difficult for DMs to estimate their requirements with precision.

Motivated by the above observations, the objective of this paper is to propose a prioritized aggregation operator to overcome the limitations and drawbacks of previous works [31, 136, 148, 150]. Toward this end, firstly, similar with Yager [148, 150] and Wang and Chen [31, 136], the OWA operator will also be used to obtain the degree of satisfaction for each priority level. To preserve the tradeoffs among the attributes in the same priority level, the degree of satisfaction for each priority level is viewed as a pseudo attribute. Secondly, we suggest that roughly speaking any t-norm can be used to model the priority relationships between the attributes in different priority levels. To keep the slight change of priority weight, strict Archimedean t-norms perform better in inducing priority weight. As Hamacher family of t-norms provide a wide class of strict Archimedean t-norms ranging from the product to weakest t-norm [110], Hamacher t-norms are selected to induce the priority weight for each priority level. Thirdly, considering DM's requirement toward the higher priority levels, a *benchmark* based approach is proposed to induce priority weight for each priority level, i.e., "*the satisfactions of the higher priority attributes are* *larger than or equal to the DM's requirements*". We suggest that the weights of lower priority level should depend on the benchmark achievement of all the higher priority levels. In particular, Łukasiewicz implication is utilized to compute benchmark achievement for crisp requirements. In case of fuzzy uncertain requirements, as target-oriented decision analysis [17, 18] lies in the philosophical root of Simon's bounded rationality [120] as well as represents the S-shaped value function [70], fuzzy target-oriented decision analysis [62] is utilized to obtain the benchmark achievement.

The rest of this chapter is organized as follows. In Section 6.2 we recall some basic knowledge of aggregation operators in MADA problems operator and t-norms. In Section 6.3 we propose a prioritized weighted aggregation operator based on OWA operator and t-norms, we also compare our method with Yager's prioritized aggregation operator [148, 150]. In Section 6.4, we propose a benchmark based approach to induce the priority weight for each priority level by taking DM's requirement toward higher priority levels in account. Considering the uncertainties of DM's requirements, crisp and fuzzy uncertain benchmarks are studied. Comparative analysis with [31, 136] are also given to show the effectiveness and advantages of our proposed approach. Finally, we provide some concluding remarks and future work in Section 6.5.

6.2 Theoretical Background

In this section, we shall recall some basic knowledge about aggregation operators in MADA problems operator and t-norms.

6.2.1 Aggregation in multi-attribute decision analysis

A MADA problem consists of a set of alternatives $\mathcal{A} = \{A^1, \cdots, A^m, \cdots, A^M\}$ and a set of attributes $\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}$ to evaluate each alternative and rank or select the best alternatives. Assume that the satisfaction degrees with respect to each attribute are denoted as c_n^m, where $\forall c_n^m \in [0,1]$. By using this, an aggregation function F is used to aggregate each c_n^m into an overall degree of satisfaction $\mathrm{Val}(A^m)$ with respect to the set of criteria C such that

$$\mathrm{Val}(A^m) = F\left(c_1^m, \cdots, c_n^m, \cdots, c_N^m\right). \tag{6.1}$$

The choice of the form for F models the DM's desired imperative and individual preference for combining the criteria [94, 148]. As suggested by Bellman and Zadeh [12],

1. if the relationship is that we desire **all** attributes be satisfied then we can use $\mathrm{Val}(A^m) = \min_n[c_n^m]$;
2. when we need **only one** attribute satisfied then we can model this as $\mathrm{Val}(A^m) = \max_n[c_n^m]$.

Yager [141] introduced the ordered weighted averaging (OWA) operator to provide a method for aggregating multiple inputs that lie between the min and max operators. An OWA operator of dimension N is a mapping $F : R^N \to R$ that has an associated weighting vector $W = (w_1, \cdots, w_n, \cdots, w_N)$ such that

$$w_n \in [0,1], \sum_{n=1}^{N} w_n = 1, \text{ for } n = 1, 2, \cdots, N.$$

and

$$\text{OWA}\,(c_1^m, \cdots, c_n^m, \cdots, c_N^m) = \sum_{n=1}^{N} b_n^m \cdot w_n$$

where b_n^m is the n-th largest element in the collection $\mathcal{X}$.

The OWA operator provides a class of averaging operators parameterized by the weighting vector W. The type of average is determined by the weighting vector W. Some notable examples are

1. If $W = W_*$ where $w_N = 1$ and $w_n = 0$ for $n \neq N$, then

$$\text{OWA}\,(c_1^m, \cdots, c_n^m, \cdots, c_N^m) = \min_n [c_n^m]$$

2. If $W = W^*$ where $w_1 = 1$ and $w_n = 0$ for $n \neq 1$, then

$$\text{OWA}\,(c_1^m, \cdots, c_n^m, \cdots, c_N^m) = \max_n [c_n^m]$$

3. If $W = W_N$ where $w_n = \frac{1}{N}$, then

$$\text{OWA}\,(c_1^m, \cdots, c_n^m, \cdots, c_N^m) = \frac{1}{N} \sum_{n=1}^{N} c_n^m$$

Central to the OWA operator is how to obtain OWA weights. Many techniques are available to calculate the OWA weights [44]. We could resolve a mathematical programming problem [44, 138, 139], associate it with a linguistic quantifier [44, 141], or obtain OWA weights via analytic method [43]. In the first part, Yager [141] introduced two characterizing measures associated with the weighting vector W of an OWA operator. The first one, *orness measure* of the aggregation, is defined as

$$\Omega = \text{orness}(W) = \sum_{n=1}^{N} \frac{N-n}{N-1} \cdot w_n$$

and it characterizes the degree to which the aggregation is like an ***or*** operation. It is clear that $\Omega(W) \in [0, 1]$ holds for any weighting vector. Recently, the "orness" of OWA operator is also called "attitudinal character" [147, 149], as it associates with the subjective preference in decision making. In this study, we prefer using "attitudinal character".

The second one, the *dispersion measure* of the aggregation, is defined as

$$\text{disp}(W) = -\sum_{n=1}^{N} w_n \cdot \ln w_n$$

and it measures the degree to which W takes into account all information in the aggregation.

O'Hagan [108] suggested a maximum entropy method to determined OWA operator weights, which formulates the OWA operator weight problem as a constrained nonlinear optimization model with a predefined degree of orness (attitudinal character) as its constraint and the entropy as its objective function. This approach is based on the solution of the following mathematical programming problem:

$$\text{Maximize } -\sum_{n=1}^{N} w_n \cdot \ln w_n \tag{6.2a}$$

$$\text{subject to } \sum_{n=1}^{N} \frac{N-n}{N-1} \cdot w_n = \Omega, \ 0 \leq \Omega \leq 1 \tag{6.2b}$$

$$\sum_{n=1}^{N} w_n = 1, \ w_n \in [0,1], \ n = 1, 2, \cdots, N. \tag{6.2c}$$

An Operations Research software package called *LINDO*[1] can be used to solve this mathematical programming problem.

6.2.2 Triangular norms

Definition A triangular norm (t-norm for short) $\mathbb{T}$ is a mapping from $[0,1]^2$ to $[0,1]$, which is increasing in both arguments, commutative, associative and fulfilling the boundary condition: $\forall x \in [0,1], \mathbb{T}(x,1) = x$ [9, 36, 110].

The definition of t-norms does not imply any kind of continuity. Nevertheless, such a property is desirable from theoretical as well as practical points of view.

Definition A t-norm is is said to be continuous if it is continuous as a two-place function.

T-norms can be classified as follows:

- A t-norm $\mathbb{T}$ is called Archimedean if it is continuous and $\mathbb{T}(x,x) < x$, for all $x \in (0,1)$.
- An Archimedean t-norm $\mathbb{T}$ is called strict if it is strictly increasing in each variable for $x, y \in (0,1)$.
- An Archimedean t-norm $\mathbb{T}$ is called nilpotent if it is not strictly increasing in each variable for $x, y \in (0,1)$.

Typical examples of t-norm operators are listed as below [78, 79, 95].

1. Minimum operator: $\mathbb{T}_M(x,y) = \min(x,y)$
2. Product operator: $\mathbb{T}_P(x,y) = x \cdot y$
3. Łukasiewicz operator: $\mathbb{T}_L(x,y) = \max(x+y-1, 0)$

These basic t-norms have some remarkable properties. The minimum t-norm $\mathbb{T}_M$ is the largest t-norm. The product t-norm $\mathbb{T}_P$ and the Łukasiewicz t-norm $\mathbb{T}_L$ are prototypical examples of two important subclasses of t-norms (of strict Archimedean and nilpotent Archimedean t-norms, respectively).

[1] http://www.lindo.com/.

6.3 Prioritized Multi-Attribute Target-Oriented Decision Analysis

Let us consider a multi-attribute target-oriented decision matrix, as shown in Chapter 2. Assume that the set of alternatives is $\mathcal{A} = \{A^1, \cdots, A^m, \cdots, A^M\}$, and $\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}$ is the set of attributes. The consequence on attribute X_n of alternative A^m is expressed as $X_n(A^m)$, which can be shortened to X_n^m when there is no possibility of confusion. In addition, a DM has specified N targets, such that $\mathrm{T} = (T_1, \cdots, T_n, \cdots, T_N)$. For an alternative A^m, we obtain the probability of meeting target for each attribute, denoted as

$$\mathrm{Pr}^m = (\mathrm{Pr}_1^m, \cdots, \mathrm{Pr}_n^m, \cdots, \mathrm{Pr}_N^m).$$

In the following, for denotational simplify, we shall use $(\cdot)$ to represent the m alternative.

Assume that a set of attributes X are partitioned into Q distinct priority levels, $\mathcal{H} = \{H_1, \cdots, H_q, \cdots, H_Q\}$, such that $H_q = \{X_{q1}, \cdots, X_{qk}, \cdots, X_{qN_q}\}$, where N_q is the attributes number in priority level H_q, and X_{qk} is the k-th target in priority level H_q. We also assume a prioritization of these priority levels is $H_1 \succ \cdots \succ H_q \succ \cdots \succ H_Q$. The prioritization of different targets indicates that we have a prioritization of the attributes. From now on, we shall use the prioritization of targets and prioritization of attributes interchangeably.

In this prioritization, we have for each attribute X_{qk}, a value Pr_{qk} indicating the target achievement regarding attribute X_{qk}. Table 6.1 shows the priority hierarchy structure of the set of attributes X.

Table 6.1: Priority hierarchy of a set of attributes $\mathcal{X}$

Priority level	Attributes
H_1	$X_{11}, \cdots, X_{1k}, \cdots, X_{1N_1}$
$\vdots$	$\vdots$
H_q	$X_{q1}, \cdots, X_{qk}, \cdots, X_{qN_q}$
$\vdots$	$\vdots$
H_Q	$X_{Q1}, \cdots, X_{Qk}, \cdots, X_{QN_Q}$

Yager [148, 150] classified this priority hierarchy into two cases:

strict priority order if each priority level has only one attribute or one target, i.e. $N_q = 1$ for $q = 1, \cdots, Q$;

weakly ordered prioritization if more one target/attribute exists in each priority level.

6.3.1 A prioritized OWA aggregation operator

As an OWA operator is similar to a weighted mean, but with the values of the variables previously ordered in a decreasing way [79, 91]. Thus, contrary to the weighted means, the weights are not associated with concrete variables. Consequently, OWA operators satisfy symmetry. Moreover, OWA operators generalize the arithmetic mean and the median, and they also exhibit some other interesting properties such as monotonicity, idempotence, and compensativeness (i.e., the value of an OWA operator is located between the minimum and the maximum values of the variables). Due to these properties, the OWA operator will be used to obtain degree of satisfaction for each priority level.

Given DM's attitudinal character Ω_q toward priority level H_q, according to O'Hagan's OWA weight determination method as shown in Eq. (6.2), we can associate with priority level H_q an OWA weighting vector such that $U_q = (u_{q1}, \cdots, u_{qk}, \cdots, u_{qN_q})$, where $u_{qk} \in [0,1]$ and $\sum_{k=1}^{N_q} u_{qk} = 1$. In addition, let $B_q^{(\cdot)} = (b_{q1}^{(\cdot)}, \cdots, b_{qN_q}^{(\cdot)})$ be the reordered vector of $\mathrm{Pr}_q^{(\cdot)} = (\mathrm{Pr}_{q1}^{(\cdot)}, \cdots, \mathrm{Pr}_{qN_q}^{(\cdot)})$, where $b_{qk}^{(\cdot)}$ is the k-th largest in priority level H_q. Using this we can calculate the satisfaction degree in priority level H_q for an alternative as

$$\begin{aligned} \mathrm{Sat}_q^{(\cdot)} &= \mathrm{OWA}_{\Omega_q}[H_q] \\ &= \sum_{k=1}^{N_q} b_{qk}^{(\cdot)} \cdot u_{qk} \end{aligned} \tag{6.3}$$

To model the priority relationship, as suggested by Yager [148, 150], the lower priority targets/attributes will become important with the higher degree of satisfaction of higher priority level, i.e., the priority weights are dependent upon the satisfaction of higher priority levels. Motivated by this observation, we will associate with each priority level a **priority weight** $Z_q^{(\cdot)}$, which is derived from the degree of satisfaction of all the higher priority levels. Furthermore, as t-norms do not allow low values to be compensated by high values [9, 36, 110], t-norms are used to induce the priority weight $Z_q^{(\cdot)}$ for each priority level.

In particular, for priority level H_1, we have $Z_1^{(\cdot)} = 1$. For priority level H_2, we express the priority weight as $Z_2^{(\cdot)} = \mathbb{T}\left(Z_1^{(\cdot)}, \mathrm{Sat}_1^{(\cdot)}\right)$. For priority level H_3, we express the priority weight as $Z_3 = \mathbb{T}\left(Z_2^{(\cdot)}, \mathrm{Sat}_2^{(\cdot)}\right)$. More succinctly and generally, we can induce the priority weight for priority level H_q as

$$\begin{aligned} Z_q^{(\cdot)} &= \mathbb{T}\left(Z_{q-1}^{(\cdot)}, \mathrm{Sat}_{q-1}^{(\cdot)}\right) \\ &= \mathbb{T}_{l=0}^{q-1} \mathrm{Sat}_l^{(\cdot)} \end{aligned} \tag{6.4}$$

with the understanding that $Z_0^{(\cdot)} = \mathrm{Sat}_0^{(\cdot)} = 1$.

We now see that for priority level H_q, we have a priority weight $Z_q^{(\cdot)}$. In addition, for each attribute in priority level H_q, we have a local OWA weight. To preserve the tradeoffs between attributes in the same priority level, we shall view the degree of satisfaction of each priority level as a pseudo attribute. In this way, we can get an aggregated value for each alternative under these prioritized attributes as [2]

$$\mathrm{Val}(\cdot) = \sum_{q=1}^{Q} Z_q^{(\cdot)} \cdot \mathrm{Sat}_q^{(\cdot)} \tag{6.5}$$

[2] It should be emphasized that $\mathrm{Sat}_q^{(\cdot)}$ and $Z_q^{(\cdot)}$ are used to denote the satisfaction degree and the priority weight in the q-th priority level for any alternative. If we use A^m to express the alternative, we can express them as Sat_q^m and Z_q^m, respectively.

No matter what type of t-norms is selected, the priority weight $Z_q^{(\cdot)}$ of a priority level depends upon the satisfaction of all the higher priority levels, such that $Z_q^{(\cdot)} = \mathrm{T}_{l=0}^{q-1}\mathrm{Sat}_l^{(\cdot)}$, thus poor satisfaction of all the higher priority levels leads to lower priority weights for the current priority level. In addition, the OWA operator are used to aggregate the criteria in the same priority level. Based on these two features, we shall call the proposed aggregation operator as *Prioritized OWA operator*.[3]

The prioritization of the attributes induces a priority weighting schema such that the attributes gain more importance only if all the higher priority attributes are higher satisfied. If one wants to raise the global degree of satisfaction of all attributes, a attribute with a relatively high priority level must be sufficiently satisfied prior to the attributes in relatively low priority levels. This is accordance with the meaning of the word *priority* in English dictionaries [111]. In fact, the concept of priority has the following two characteristics [95]:

1. It measures the relative importance among things in a group to determine only their relative precedence, and
2. the higher the priority of one thing, the earlier the thing should be handled or the more preferred is the thing.

Consequently, the higher the priority of level, the more preference should be given when finding a solution. That is precisely the reason why this kind of aggregation is called prioritized aggregation.

6.3.2 Properties of proposed prioritized OWA operator

Proposition 6.3.1. *The proposed prioritized aggregation operator is monotonic regarding any attribute X_{lk}.*

Proof. For monotonicity to hold, for any attribute X_{lk} we have to prove $\frac{\partial \mathrm{Val}(\cdot)}{\partial \mathrm{Pr}_{lk}^{(\cdot)}} \geq 0$.

Not all the $\mathrm{Sat}_q^{(\cdot)}$ and $Z_q^{(\cdot)}$ change with $\mathrm{Pr}_{lk}^{(\cdot)}$, thus we express $\mathrm{Val}(\cdot)$ as

$$\mathrm{Val}(\cdot) = \sum_{i=1}^{l-1} Z_i^{(\cdot)}\mathrm{Sat}_i^{(\cdot)} + Z_l^{(\cdot)}\mathrm{Sat}_l^{(\cdot)} + \sum_{j=l+1}^{Q} Z_j^{(\cdot)}\mathrm{Sat}_j^{(\cdot)}$$

And then we can obtain $\frac{\partial \mathrm{Val}(\cdot)}{\partial \mathrm{Pr}_{lk}^{(\cdot)}}$ as

$$\begin{aligned}\frac{\partial \mathrm{Val}(\cdot)}{\partial \mathrm{Pr}_{lk}^{(\cdot)}} &= 0 + \frac{\partial \mathrm{Sat}_l^{(\cdot)}}{\partial \mathrm{Pr}_{lk}^{(\cdot)}} Z_l^{(\cdot)} + \sum_{j=l+1}^{Q}\left[\frac{\partial Z_j^{(\cdot)}}{\partial \mathrm{Pr}_{lk}^{(\cdot)}}\mathrm{Sat}_j^{(\cdot)}\right] \\ &= \frac{\partial \mathrm{Sat}_l^{(\cdot)}}{\partial \mathrm{Pr}_{lk}^{(\cdot)}} Z_l^{(\cdot)} + \sum_{j=l+1}^{Q}\left[\frac{\partial Z_j^{(\cdot)}}{\partial \mathrm{Sat}_l^{(\cdot)}}\frac{\partial \mathrm{Sat}_l^{(\cdot)}}{\partial \mathrm{Pr}_{lk}^{(\cdot)}}\mathrm{Sat}_j^{(\cdot)}\right]\end{aligned}$$

Since we use the OWA operator to obtain the degree of satisfaction for each priority level and OWA is monotonic, thus we know that $\frac{\partial \mathrm{Sat}_l^{(\cdot)}}{\partial \mathrm{Pr}_{lk}^{(\cdot)}} \geq 0$.

[3] It should be noted that each priority level H_q may have a different attitudinal character Ω_q. Here for purposes of simplicity, we assume that each priority level H_q has the same attitudinal character $\Omega_q = \Omega$.

Since $Z_j^{(\cdot)} = \mathbb{T}\left(Z_{j-1}^{(\cdot)}, \mathrm{Sat}_{j-1}^{(\cdot)}\right) \geq 0$ and t-norm increases in both arguments, thus $\frac{\partial Z_j^{(\cdot)}}{\partial \mathrm{Sat}_l^{(\cdot)}} \geq 0$. The product of monotonic operators is also monotonic, hence we know that

$$\frac{\partial \mathrm{Val}(\cdot)}{\partial \Pr_{lk}^{(\cdot)}} \geq 0.$$

□

Proposition 6.3.2. *Our proposed prioritized OWA operator guarantees monotonicity regarding DM's attitudinal character* Ω.

Proof. To prove our proposed prioritized OWA operator is monotonic regarding DM's attitudinal character, we have to prove that

$$\frac{\partial \mathrm{Val}(\cdot)}{\partial \Omega} = \frac{\partial \left(\sum_{q=1}^{Q} Z_q^{(\cdot)} \mathrm{Sat}_q^{(\cdot)}\right)}{\partial \Omega} \geq 0$$

We know that

$$\frac{\partial \left(\sum_{q=1}^{Q} Z_q^{(\cdot)} \mathrm{Sat}_q^{(\cdot)}\right)}{\partial \Omega} = \sum_{q=1}^{Q} \left(\mathrm{Sat}_q^{(\cdot)} \frac{\partial Z_q^{(\cdot)}}{\partial \Omega} + Z_q^{(\cdot)} \frac{\partial \mathrm{Sat}_q^{(\cdot)}}{\partial \Omega} \right)$$

According to the properties of OWA operator, it is clear that $\frac{\partial \mathrm{Sat}_q^{(\cdot)}}{\partial \Omega} \geq 0$ [52]. In addition, we know $Z_q(\cdot) = \mathbb{T}\left(Z_{q-1}^{(\cdot)}, \mathrm{Sat}_{q-1}^{(\cdot)}\right)$ and t-norm increases in both arguments, thus $\frac{\partial Z_q^{(\cdot)}}{\partial \Omega} \geq 0$, hence $\frac{\partial \mathrm{Val}(\cdot)}{\partial \Omega} \geq 0$. □

6.3.3 Illustrative examples

We shall apply the proposed prioritized OWA aggregation operator to deal with a car selection problem, adapted from [31, 136].

Example Assume that John wants to buy a new car considering the following attributes "X_1 ***Safety***", "X_2 ***Price***", "X_3 ***Appearance***" and "X_4 ***Performance***". We also assume that there are four alternatives of cars A^1, A^2, A^3, A^4 and the degrees in which each alternative satisfies each attribute are shown in Table 6.2. We also assume that the priority hierarchy specified by John is $H_1 = \{X_1\}$, $H_2 = \{X_2\}$, $H_3 = \{X_3, X_4\}$, and

$$H_1 \succ H_2 \succ H_3.$$

For purposes of simplicity, for each priority level H_q we shall specify the same attitudinal character, such that $\Omega_q = \Omega$, where $q = 1, 2, 3$. We assume that $\Omega = 0.5$. The minimum t-norm $\mathbb{T}_M$ is the largest t-norm. The product t-norm $\mathbb{T}_P$ and the Łukasiewicz t-norm $\mathbb{T}_L$ are prototypical examples of two important subclasses of t-norms (of strict Archimedean and nilpotent Archimedean t-norms, respectively). In this example, these three prototypical t-norms are used to induce the priority weights. Taking car A^1 as an example. We first consider product t-norm $\mathbb{T}_P$, we proceed as follows:

Table 6.2: Satisfaction degree of each attribute regarding each alternative: car selection

Alternatives	Attributes			
	X_1	X_2	X_3	X_4
A^1	0.95	0.60	0.70	0.80
A^2	0.91	0.75	0.50	0.90
A^3	0.95	0.70	0.80	0.70
A^4	0.945	0.75	0.30	0.70

1. We first calculate the degree of satisfaction for each priority level via OWA operator as follows:

   $$\mathrm{Sat}_1^1 = \mathrm{OWA}_{0.5}\{\mathrm{Pr}_{11}^1\} = \mathrm{OWA}_{0.5}\{0.95\} = 0.95$$
   $$\mathrm{Sat}_2^1 = \mathrm{OWA}_{0.5}\{\mathrm{Pr}_{12}^1\} = \mathrm{OWA}_{0.5}\{0.60\} = 0.6$$
   $$\mathrm{Sat}_3^1 = \mathrm{OWA}_{0.5}\{\mathrm{Pr}_{13}^1, \mathrm{Pr}_{14}^1\} = \mathrm{OWA}_{0.5}\{0.70, 0.80\} = 0.75$$

2. Then, according to Eq. (6.4), we calculate the priority weight for each priority level by using product t-norm as follows:

   $$Z_1^1 = \mathbb{T}_P(Z_0^1, \mathrm{Sat}_0^1) = \mathbb{T}_P(1, 1) = 1$$
   $$Z_2^1 = \mathbb{T}_P(Z_1^1, \mathrm{Sat}_1^1) = \mathbb{T}_P(1, 0.95) = 0.95$$
   $$Z_3^1 = \mathbb{T}_P(Z_2^1, \mathrm{Sat}_2^1) = \mathbb{T}_P(0.95, 0.6) = 0.57$$

3. According to Eq. (6.5), we obtain the global prioritized aggregated value as follows:

   $$\mathrm{Val}(A^1) = \sum_{q=1}^{3} Z_q^1 \cdot \mathrm{Sat}_q^1 = 1 \cdot 0.95 + 0.95 \cdot 0.6 + 0.57 \cdot 0.75 = 1.9475$$

Similarly, the prioritized aggregation values for cars A^2, A^3, and A^4 by product t-norm can be obtained. We can also obtain the aggregated value with Minimum t-norm and Łukasiewicz t-norm, as shown in Table 6.3. From Table 6.3, it is clearly seen that car A^3 is the best choice whatever the t-norm is. The ranking order of prioritized aggregation values are as $A^3 \succ A^2 \succ A^4 \succ A^1$, where $\succ$ denotes "*prefer to*".

Table 6.3: Prioritized aggregation with different t-norms under attitudinal character $\Omega = 0.5$

$\mathbb{T}$-norms	Alternatives			
	A^1	A^2	A^3	A^4
Minimum t-norm $\mathbb{T}_M$	1.9700	2.1175	**2.1400**	2.0286
Product t-norm $\mathbb{T}_P$	1.9475	2.0703	**2.1137**	2.0080
Łukasiewicz t-norm $\mathbb{T}_L$	1.9325	2.0545	**2.1025**	2.0012

6.3.4 Discussion: Choosing a suitable t-norm

In the previous example, we used minimum t-norm $\mathbb{T}_M$, Łukasiewicz t-norm $\mathbb{T}_L$ and product t-norm $\mathbb{T}_P$ to induce the priority weight for each priority level. As mentioned previously, in general any t-norm can be used to induce the priority weight. Are there any differences between these t-norms? Which type of t-norms perform better in inducing priority weight? To illustrate this point, we shall discuss this topic from the following two aspects.

A Special Case

Firstly, let us consider a special case, where only two levels of priority hierarchy exists, i.e. $Q = 2$. We observed that in this case, no matter which t-norm is used, we always obtain the priority weight $Z_1^{(\cdot)} = 1$ for priority level H_1 and a priority weight $Z_2^{(\cdot)} = \text{Sat}_1^{(\cdot)}$ for priority level H_2. The main reasons for this are as follows:

1. we assume there exists a pseudo hierarchy level H_0 with $Z_0^{(\cdot)} = \text{Sat}_0^{(\cdot)} = 1$.
2. Moreover, any t-norm has the property such that $\mathbb{T}(1, x) = x$.

An Example

Assume that there are two alternatives A^1 and A^2. For the q-th priority level, the priority weight and degree of satisfactions are shown in Columns 2-3 of Table 6.4. According to Eq. (6.4) and the three typical t-norms, we can calculate three induced priority weights for priority level $q+1$, as shown in Columns 4-6 of Table 6.4.

Table 6.4: The priority weight and degree of satisfactions of q-th priority level and its induced priority weights

Alt.	q-th priority level		Induced priority weights Z_{q+1}		
	Priority weight Z_q	Satisfaction degree	Minimum $\mathbb{T}_M$	Łukasiewicz $\mathbb{T}_L$	Product $\mathbb{T}_P$
A^1	0.3	0.5	0.3	0	0.15
A^2	0.3	0.6	0.3	0	0.18

It is clear that the priority weights of $q+1$-th priority level induced by Minimum t-norm and Łukasiewicz t-norm do not reflect the changes of the priority weight and degree of satisfactions of q-th priority level. We want to preserve the slight change of priority weight as well as do not want to ignore the slight change, thus non-Archimedean t-norms and nilpotent t-norms are not suitable to induce the priority weight for each priority level. The product t-norm is the prototypical example of strict Archimedean t-norms and can catch the slight change of of priority weight. This is perhaps the main reason why Yager [150] and Wang and Chen [31, 136] both use the product t-norm to induce the priority weight.

Based on the above observations, strict Archimedean t-norms perform well in reducing the priority weight. As Hamacher family of t-norms provide a wide class of strict

Archimedean t-norms ranging from the product to weakest t-norm [110], we shall use Hamacher parameterized t-norm to induce the priority weight such that

$$\begin{aligned}\mathbb{T}_H^{\gamma} &= \frac{Z_{q-1}^{(\cdot)}\mathrm{Sat}_{q-1}^{(\cdot)}}{\gamma + (1-\gamma)\left(Z_{q-1}^{(\cdot)} + \mathrm{Sat}_{q-1}^{(\cdot)} - Z_{q-1}^{(\cdot)}\mathrm{Sat}_{q-1}^{(\cdot)}\right)} \\ &= \mathbb{T}_{H\,l=0}^{\gamma\;q-1}\mathrm{Sat}_l^{(\cdot)}\end{aligned} \tag{6.6}$$

where $\gamma \geq 0$

If $\gamma = 0$, then we can obtain the priority weight inducing method as

$$Z_q^{(\cdot)} = \frac{Z_{q-1}^{(\cdot)}\mathrm{Sat}_{q-1}^{(\cdot)}}{Z_{q-1}^{(\cdot)} + \mathrm{Sat}_{q-1}^{(\cdot)} - Z_{q-1}^{(\cdot)}\mathrm{Sat}_{q-1}^{(\cdot)}} \tag{6.7}$$

When $\gamma = 1$, the induced priority weight is represented as

$$Z_q^{(\cdot)} = Z_{q-1}^{(\cdot)}\mathrm{Sat}_{q-1}^{(\cdot)} = \prod_{l=0}^{q-1}\mathrm{Sat}_l^{(\cdot)} \tag{6.8}$$

with the understanding that $Z_0^{(\cdot)} = \mathrm{Sat}_0^{(\cdot)} = 1$.

Now let us reconsider the example as shown in Table 6.2 via the Hamacher parameterized t-norm. According to the three steps of our proposed prioritized aggregation, we obtain the prioritized aggregated values as shown in Fig. 6.1, where γ is set to $\in [0, 1000]$.

Obviously T_H^{γ} is non-increasing with respect to γ [110]. In addition, the ranking order of prioritized aggregation values may be different with different γ values. From Fig. 6.1, it is clearly that the prioritized aggregation values are non-increasing with respect to γ. In our example, there are two γ values changing the ranking order of the four alternatives, $\gamma_{\mathrm{I}} \approx 16.8295$ and $\gamma_{\mathrm{II}} \approx 145.8237$. Thus five ranking orders can be obtained:

- When $0 \leq \gamma < \gamma_{\mathrm{I}}$, the ranking result is $A^3 \succ A^2 \succ A^4 \succ A^1$;
- When $\gamma = \gamma_{\mathrm{I}}$, the ranking result is $A^3 \succ A^2 \sim A^4 \succ A^1$;
- when $\gamma_{\mathrm{I}} < \gamma < \gamma_{\mathrm{II}}$, the ranking result is $A^3 \succ A^4 \succ A^2 \succ A^1$;
- When $\gamma = \gamma_{\mathrm{II}}$, the ranking result is $A^3 \sim A^4 \succ A^2 \succ A^1$;
- when $\gamma_{\mathrm{II}} < \gamma \leq 1000$, the ranking result is $A^4 \succ A^3 \succ A^2 \succ A^1$.

Remark From Fig. 6.1 we know that different γ values may lead to different ranking orders. Then a natural question that arises is how the DM selects optimal alternative with different γ values. From the theoretical point of view, we show that the strict Archimedean t-norms perform well in reducing the priority weight for each priority level. As Hamacher's family of t-norms supplies a wide class of t-norm operators ranging from the probabilistic product to the weakest t-norm, extending the product t-norm into Hamacher's family of t-norms provides a generalization of previous research. In our prioritized aggregation, the γ value can be viewed as an index to represent strongness of the prioritization. The larger the γ value is, the stronger of prioritization the DM prefers. From the applicable point of view, it is not easy to specify a γ value. In fact, we can do sensitivity analysis of prioritized aggregation with respect to different γ values. When DM's preferred γ value falls into a range, we can approximately know the best alternative. If the DM does not provide his/her subjective preference, two commonly used cases are $\gamma = 0$ and $\gamma = 1$ (product t-norm). We can set $\gamma = 1$ for default.

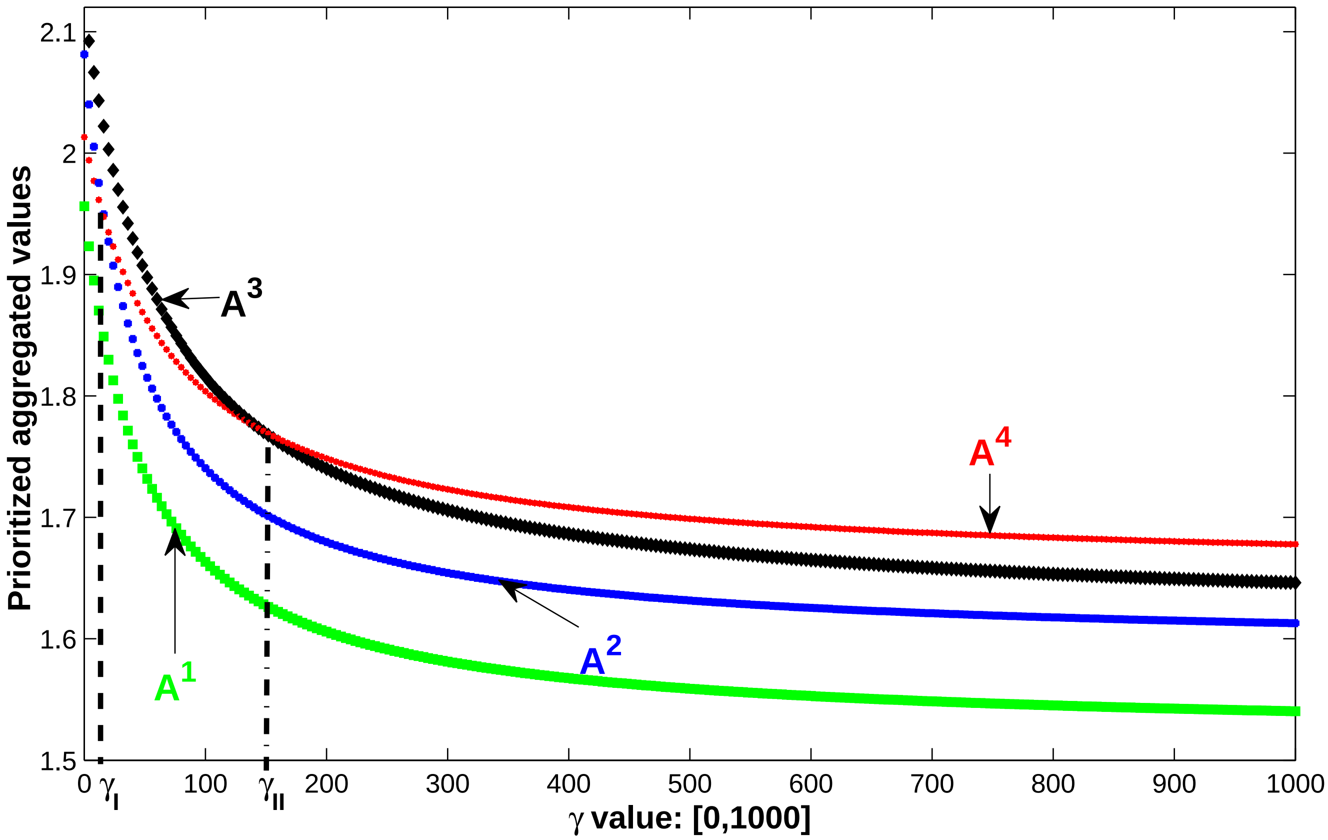

Figure 6.1: Prioritized aggregated values by means of Hamacher's t-norm: $\gamma \in [0, 1000]$

6.3.5 Comparison with previous research

To illustrate the effectiveness and advantages of our formulation of prioritized aggregation, we shall compare our approach with previous work. Yager [150] proposed a prioritized aggregation operator according to the following three steps:

1. To calculate the degree of satisfaction for each priority level by OWA operator as follows:
$$\mathrm{Sat}_q^{(\cdot)} = \mathrm{OWA}_\Omega[H_q]$$

2. The product t-norm is used to calculate the priority weight $Z_q^{(\cdot)}$ for priority level H_q
$$Z_q^{(\cdot)} = \mathbb{T}_P\left(Z_{q-1}^{(\cdot)}, \mathrm{Sat}_{q-1}^{(\cdot)}\right) = \prod_{l=0}^{q-1} \mathrm{Sat}_l^{(\cdot)} \tag{6.9}$$
where $Z_0^{(\cdot)} = \mathrm{Sat}_0^{(\cdot)} = 1$.

3. To calculate the overall degree of satisfaction

   (a) For strict priority order, where $N_q = 1$ and $q = 1, \cdots, N$, an averaging aggregation is used as follows:

$$\begin{aligned} \mathrm{Val}(\cdot) &= \frac{\sum_{q=1}^{Q}\left[\sum_{k=1}^{N_q} Z_q^{(\cdot)}\,\mathrm{Pr}_{qk}^{(\cdot)}\right]}{\sum_{q=1}^{Q} Z_q^{(\cdot)}} \\ &\Rightarrow \frac{\sum_{q=1}^{Q} Z_q^{(\cdot)}\,\mathrm{Pr}_{q}^{(\cdot)}}{\sum_{q=1}^{Q} Z_q^{(\cdot)}} \end{aligned} \tag{6.10}$$

   (b) For weak priority order, a scoring aggregation is used as follows:

$$\mathrm{Val}(\cdot) = \sum_{q=1}^{Q}\left[\sum_{k=1}^{N_q} Z_q^{(\cdot)}\mathrm{Pr}_{qk}^{(\cdot)}\right] \tag{6.11}$$

In the first step of both Yager's prioritized aggregation operator and our prioritized OWA operator, the OWA operator is used to obtain the degree of satisfaction for each priority level. To find out some limitations of Yager's prioritized operator, we compare our operator with Yager's operator from the following three aspects:

T-norm selection

In Yager's aggregation operator, the product t-norm is used to induce the priority weight for each priority level. As mentioned previously, we suggested that roughly speaking, any t-norm can be used to induce the priority weight for each priority level. To preserve the slight change of priority weight as well as do not want to ignore the slight change, we suggest using Hamacher parameterized t-norm. In this view, Yager's operator is one special case of our operator.

Tradeoffs of attributes in the same priority level

Let us consider a special case. If the DM does not specify the priority hierarchy, it means that the DM agrees the tradeoffs among all the criteria. In this case, only one priority level is considered, all the criteria have the same priority level, we shall use P_n instead of P_{1n} to represent the n-th criterion. Our proposed prioritized OWA operator Eq. (6.5) reduces to the OWA operator [141] such that

$$\begin{aligned} \mathrm{Val}(\cdot) &= \text{PRI-OWA}(H_1) \\ &= \mathrm{OWA}_\Omega\left(\mathrm{Pr}_1^{(\cdot)}, \cdots, \mathrm{Pr}_n^{(\cdot)}, \cdots, \mathrm{Pr}_N^{(\cdot)}\right). \end{aligned} \tag{6.12}$$

Whereas, Yager's prioritized aggregation operator reduces to the summation of all the degrees of satisfactions of all the attributes, such that

$$\mathrm{Val}(\cdot) = \sum_{n=1}^{N} \mathrm{Pr}_n^{(\cdot)} \tag{6.13}$$

In this regard, Yager's prioritized aggregation operator only allows summation tradeoffs for each priority level, whereas our approach allows OWA tradeoffs.

Yager [150](p.267) pointed out that under strictly ordered prioritization (only one criterion in each priority level), the prioritized aggregation should be represented as the normalized form; under weakly ordered prioritization, the prioritized aggregation should be represented as the scoring type. The main reason is that the normalized form under weakly ordered prioritization dose not always guarantee monotonicity.

However, as the strictly ordered prioritization and weakly ordered prioritization are just two special cases of the prioritization hierarchy, hence they should have the same properties and type. To keep the general properties of aggregation operators as well as to keep the original priority changes, similar with [22, 77, 78] we shall use the scoring type of aggregation.

6.4 Including Benchmark into Prioritized OWA Aggregation

We now turn to a possible variation of our formulation for prioritized OWA aggregation

$$\mathrm{Val}(\cdot) = \sum_{q=1}^{Q} Z_q^{(\cdot)} \mathrm{Sat}_q^{(\cdot)}$$

where

$$Z_q^{(\cdot)} = \mathbb{T}_H^{\gamma}\left(Z_{q-1}^{(\cdot)}, \mathrm{Sat}_{q-1}^{(\cdot)}\right) = \mathbb{T}_{H\,l=0}^{\gamma\;q-1} \mathrm{Sat}_l^{(\cdot)}$$

and

$$\mathrm{Sat}_q(\cdot) = \mathrm{OWA}_{\Omega}[H_q].$$

In this formulation, the priority weight $Z_q^{(\cdot)}$ directly depends upon the satisfactions of the attributes in all higher priority levels. As suggested by Yager [148], as a variation of this, we can let $Z_q^{(\cdot)}$ depend on some function of the satisfactions of the higher priority attributes. Without loss of generality, we assume that for any priority level, the DM specifies a requirement. In particular, we can let $\mathrm{E} : [0,1] \to [0,1]$ such that $\mathrm{E}(0) = 0$, $\mathrm{E}(1) = 1$, and $\mathrm{E}(x) \geq \mathrm{E}(y)$ if $x \geq y$. Using this we can express

$$\begin{aligned} \mathrm{Z}_q^{(\cdot)} &= \mathbb{T}_H^{\gamma}(\mathrm{Z}_{q-1}^{(\cdot)}, \mathrm{E}(\mathrm{Sat}_{q-1}^{(\cdot)})) \\ &= \mathbb{T}_H^{\gamma}\left(\mathrm{E}\left(\mathrm{Sat}_0^{(\cdot)}\right), \mathrm{E}\left(\mathrm{Sat}_1^{(\cdot)}\right), \cdots, \mathrm{E}\left(\mathrm{Sat}_{q-1}^{(\cdot)}\right)\right) \\ &= \mathbb{T}_{H\,l=0}^{\gamma\;q-1} \mathrm{E}\left(\mathrm{Sat}_1^{(\cdot)}\right) \end{aligned} \tag{6.14}$$

Roughly speaking, we can view $\mathrm{E}\left(\mathrm{Sat}_1^{(\cdot)}\right)$ as some kind of effective or pseudo satisfaction. Here then the score value associated with attributes can be different in its evaluation of induced priority weights and the satisfaction degree used in the aggregation. With the introduction of the use of function E to transform levels of satisfaction, we are able to model the linguistically expressed DM's requirements by means of Zadeh's paradigm of computing with words [165, 166]. Due to this observation, DM's requirement can be viewed as *benchmark or reference level* for the degrees of satisfaction of each priority level, i.e., "***the satisfactions of the higher priority criteria are larger than or equal to the DM's requirements***" expressed as $\mathbb{E}\left(\mathrm{Sat}_q^{(\cdot)} \geq G_q\right) = \mathrm{E}\left(\mathrm{Sat}_1^{(\cdot)}\right)$. We shall consider two possible DM's requirements such that

1. The DM specifies a certain requirement G_q for priority level H_q,
2. The DM specifies an uncertain requirement $\widetilde{G}_q$ for priority level H_q.

In the following, we will deal with these two types of benchmarks respectively.

6.4.1 Crisp requirements

Before discussing crisp requirements, we shall recall some knowledge of R-implications.

R-implication

Definition An implication operator $\mathbb{I}$ is a mapping: $[0,1]^2 \to [0,1]$, such that [24, 110]

- $\mathbb{I}$ is non-increasing with respect to its first argument;
- $\mathbb{I}$ is non-decreasing with respect to its second argument;
- $\mathbb{I}(0,0) = \mathbb{I}(0,1) = \mathbb{I}(1,1) = 1$, $\mathbb{I}(1,0) = 0$.

Basically, there are many implication operators. We shall use the R-implications. R-implications $\mathbb{I}$ are based on the idea that implication reflects partial ordering on proposition, i.e., $\mathbb{I}(x,y) = 1$ if and only if $x \leq y$. In the standard semantics of t-norm based fuzzy logics, where conjunction is interpreted by a t-norm, the residuum plays the role of implication (often called R-implication). R-implications can be obtained by residuum of a continuous t-norm $\mathbb{T}$ [110] as follows,

$$x \to y = \sup\{z \in [0,1] | \mathbb{T}(x,z) \leq y\}, \text{ for all } x,y,z \in [0,1]. \tag{6.15}$$

These implications arise from the intutionistic logic formalism [24]. Typical examples of R-implication operators are

1. Kleene-Dienes implication:
$$\mathbb{I}(x,y) = \max(1-x,y) \tag{6.16}$$
2. Łukasiewicz implication:
$$\mathbb{I}(x,y) = \min(1-x+y,1) \tag{6.17}$$
3. Gödel implication:
$$\mathbb{I}(x,y) = \begin{cases} 1, & \text{if } x \leq y, \\ y, & \text{otherwise.} \end{cases} \tag{6.18}$$

Benchmark based on R-implication

If both the degree of satisfaction and the requirement are both crisp numbers, we can implement $\mathbb{E}\left(\mathrm{Sat}_q^{(\cdot)} \geq G_q\right)$ using the strict implication operator. It is clear that this will be very sensitive to small changes of both arguments. However, we can still sustain the benchmark character if we use an R-implication operator to transform the degrees of satisfaction. As Łukasiewicz implication is the one that satisfies most of the properties pertaining to the logical implication operators [110], in this study Łukasiewicz implication was used as a technique to compute benchmark satisfaction for crisp requirements. Given DM's requirement G_q of priority level H_q, according to Łukasiewicz implication Eq. (6.17), we can define

$$\begin{aligned}\mathbb{E}\left(\mathrm{Sat}_q^{(\cdot)} \geq G_q\right) &= G_q \rightarrow \mathrm{Sat}_q^{(\cdot)} \\ &= \min\{1 - G_q + \mathrm{Sat}_q^{(\cdot)}, 1\}\end{aligned} \tag{6.19}$$

It is of interest noting that in Section 6.3, $Z_q^{(\cdot)}$ directly depends on the satisfaction of higher priority criteria. In this case, the DM's requirement can be modeled as $G_q = 1$. According to Eq. (6.19), we know that

$$\begin{aligned}\mathbb{E}\left(\mathrm{Sat}_q^{(\cdot)} \geq G_q\right) &= \min\{1 - G_q + \mathrm{Sat}_q^{(\cdot)}, 1\} \\ &= \min\{\mathrm{Sat}_q^{(\cdot)}, 1\} \\ &= \mathrm{Sat}_q^{(\cdot)}\end{aligned}$$

6.4.2 Uncertain requirements

Due to the vagueness or impreciseness of knowledge, it is difficult for DM(s) to estimate their requirements with precision. In many applications, fuzzy subsets [162] provide a very convenient object for the representation of uncertain information [151]. The subjective assessments provided by DM(s) are usually conceptually vague, with uncertainty that is frequently represented in linguistic forms [129]. To help people easily express their subjective assessments, the linguistic variables [163, 167] are used to linguistically express requirements.

A fuzzy number $\widetilde{G}$ can be conveniently represented by the canonical form [76]

$$\mu_{\widetilde{G}(g)} = \begin{cases} f_{\widetilde{G}(g)}, & g_1 \leq g \leq g_2, \\ 1, & g_2 \leq g \leq g_3, \\ h_{\widetilde{G}(g)}, & g_3 \leq g \leq g_4, \\ 0, & \text{otherwise.} \end{cases} \tag{6.20}$$

where $\mu_{\widetilde{G}(g)}$ denotes the membership function of fuzzy number $\widetilde{G}$, $f_{\widetilde{G}(g)}$ is a real-valued function that is monotonically increasing, and $h_{\widetilde{G}(g)}$ is a real-valued function that is monotonically decreasing. In addition, like most applications, we assume that functions $f_{\widetilde{G}(g)}$ and $h_{\widetilde{G}(g)}$ are continuous. If $f_{\widetilde{G}(g)}$ and $h_{\widetilde{G}(g)}$ are linear functions then $\widetilde{G}$ is called a trapezoidal fuzzy number and denoted by $\widetilde{G} = (g_1, g_2, g_3, g_4)$. In particular, $\widetilde{G}$ becomes a triangular fuzzy number if $g_2 = g_3$.

Fuzzy target-oriented decision analysis is used to compute $\mathrm{E}(\mathrm{Sat}_q(\cdot))$ for uncertain requirements $\widetilde{G}_q$. Firstly, we define

$$\mathbb{E}\left(\mathrm{Sat}_q^{(\cdot)} \geq \widetilde{G}_q\right) = \Pr\left(\mathrm{Sat}_q^{(\cdot)} \geq \widetilde{G}_q\right) \tag{6.21}$$

where $\Pr\left(\mathrm{Sat}_q^{(\cdot)} \geq \widetilde{G}_q\right)$ denotes the probability of "meeting the fuzzy benchmark $\widetilde{G}_q$". Then we can obtain the probability of "meeting the fuzzy benchmark $\widetilde{G}_q$" as follows

$$\Pr\left(\mathrm{Sat}_q(\cdot) \geq \widetilde{G}_q\right) = \frac{\int_0^{\mathrm{Sat}_q(\cdot)} \mu_{\widetilde{G}(g)} dg}{\int_{g_1}^{g_4} \mu_{\widetilde{G}(g)} dg} \tag{6.22}$$

Remark It is of interest noting that in a different context, Carlsson and Fullér [24] concentrated on the issue of linguistic importance weighted aggregations, where the importance is interpreted as *benchmarks*. In their study, when both importance weights and ratings of criteria are given as crisp numbers, Łukasiewicz implication is also used to compute the benchmark achievement. In case of fuzzy numbers, a possibilistic approach is presented to compute the benchmark achievement. In addition, Carlsson and Fullér's method only focus on *symmetric triangular fuzzy numbers*. The benchmark used in our aggregation operator has similar but different meaning with Carlsson and Fullér's method [24]. Firstly, as there are various types of fuzzy numbers, specifying only *symmetric triangular fuzzy numbers* will not be appropriate in practical applications. Secondly, instead of the possibility interpretation, the benchmark has a probability interpretation lying in the philosophical root of Simon's bounded rationality [120] as well as represents the S-shaped value function [70]. Finally, benchmark in Carlsson and Fullér's work [24] and ours has different meanings. Carlsson and Fullér viewed importance weight as benchmark of criteria satisfaction, whereas our method considered DM's requirements.

6.4.3 A comparative analysis

To show the effectiveness and advantages of our approach, we shall use the same example (car selection) introduced in Section 6.3 to compare our approach with related research.

Wang and Chen's approach

Wang and Chen [31, 136] suggested that the weights of lower priority criteria depends on ***whether*** each alternative satisfies the requirements of all the higher priority criteria ***or not***. They proposed two benchmark achievement according to two cases.

1. For criteria in priority level H_q, a degree of satisfaction $\mathrm{Sat}_q^{(\cdot)}$ is calculated as follows

$$\mathrm{Sat}_q^{(\cdot)} = \mathrm{OWA}_\Omega[H_q] \tag{6.23}$$

2. Then an importance weight $Z_q^{(\cdot)}$ for priority level H_q by means of product t-norm is calculated as follows

$$Z_q^{(\cdot)} = \prod_{l=0}^{q-1} \mathrm{E}(\mathrm{Sat}_l^{(\cdot)}) = Z_{q-1}(\cdot)\mathrm{E}(\mathrm{Sat}_{q-1}^{(\cdot)}) \tag{6.24}$$

where $Z_0^{(\cdot)} = \text{Sat}_0^{(\cdot)} = 1$. To obtain the benchmark achievement $\text{E}(\text{Sat}_q^{(\cdot)})$, Wang and Chen considered two cases:

(a) The DM wants that a good solution must have *at least* G_q degree of satisfaction such that

$$\text{E}(\text{Sat}_q^{(\cdot)}) = \begin{cases} 1, & \text{if } \text{Sat}_q^{(\cdot)} \geq G_q, \\ 0, & \text{otherwise.} \end{cases} \tag{6.25}$$

(b) The decision maker wants that a good solution must have *at least* G_q and *as high as possible* degree of satisfaction such that

$$\text{E}\left(\text{Sat}_q^{(\cdot)}\right) = \begin{cases} \text{Sat}_q, & \text{if } \text{Sat}_q^{(\cdot)} \geq G_q, \\ 0, & \text{otherwise.} \end{cases} \tag{6.26}$$

3. To calculate the overall degree of satisfaction as follows

$$\text{Val}(\cdot) = \sum_{q=1}^{Q} Z_q^{(\cdot)} \text{Sat}_q^{(\cdot)} \tag{6.27}$$

A comparative analysis

In both our approach and Wang and Chen's work [31, 136], OWA operator is used to obtain the degrees of satisfaction for each priority level. For purposes of simplicity, we shall assume that DM's attitudinal character $\Omega = 0.5$. The main difference between our approach and Wang and Chen are twofold.

1. Firstly, more than product t-norm, we proposed using Hamacher parameterized t-norm to induce the priority weight for each priority level. As product t-norm is one special case of Hamacher parameterized t-norm where $\gamma = 1$, and in previous section we have already discussed the t-norm selection problem, here in order to distinguish the main difference between our approach from Wang and Chen's approach, we shall just use product t-norm.

2. Secondly, instead of strict threshold method, we propose using Łukasiewicz implication to compute benchmark achievement. In addition, due to the uncertainty of DM's requirements, target-oriented decision decision analysis is used to solve the fuzzy requirement.

Let us reconsider the same example as shown in Table 6.2. Assume that John wants to buy a car having a requirement for the satisfaction of criterion *safety*. We also assume that John specifies his attitudinal character as $\Omega = 0.5$. Considering the uncertainty of requirement, we do comparative analysis from the following three aspects.

Benchmark: at least G_1

Assume that John specifies his requirement toward the criterion *safety* as G_1, here specify three possible values, as shown in the first column of Table 6.5. According to Wang and Chen's approach (Eq. (6.23), Eq. (6.24), Eq. (6.25) and Eq. (6.27)), we can obtain the aggregation values with different G_1 values, as shown in Columns 2-5 of Table 6.5. With the three steps of our prioritized OWA aggregation operator and the benchmark achievement Eq. (6.19) we can easily obtain the aggregation values as shown in Columns 6-9 of Table 6.5.

Looking at the second row of Table 6.5 where $G_1 = 0.91$, it is clearly seen that both Wang and Chen's approach and our approach get the same result as $A^2 \succ A^3 \succ A^4 \succ A^1$. Secondly, we consider the case $G_1 = 0.96$, the fourth row of Table 6.5. We can see that the prioritized aggregation values are different. Taking alternative A^1 and A^3 as an example, from Table 6.2 we know that the degrees of satisfaction of criterion *safety* are $X_1^1 = X_1^3 = 0.95$. Using Wang and Chen's approach we know that satisfaction of criterion *safety* does not satisfy the requirement $G_1 = 0.95$ at all, i.e. the priority weights of lower priority criteria are all 0, thus A^1 and A^2 induce the same aggregation value. However, the satisfactions of lower priority criteria of alternative A^3 are higher than those alternative A^1, and 0.95 is slightly less than $G_1 = 0.96$, thus Wang and Chen's approach [31, 136] is too strict. By using our approach, it is clearly seen that the ranking order of alternatives is $A^3 \succ A^2 \succ A^4 \succ A^1$.

Table 6.5: Prioritized aggregated value under different crisp requirements

Benchmark: at least G_1	Wang and Chen				Our proposed method			
	A^1	A^2	A^3	A^4	A^1	A^2	A^3	A^4
$G_1 = 0.91$	2.0000	**2.1850**	2.1750	2.0700	2.0000	**2.1850**	2.1750	2.0700
$G_1 = 0.95$	2.0000	0.9100	**2.1750**	0.9450	2.0000	2.1340	**2.1750**	2.0644
$G_1 = 0.96$	**0.9500**	0.9100	**0.9500**	0.9450	1.9895	2.1213	**2.1628**	2.0531

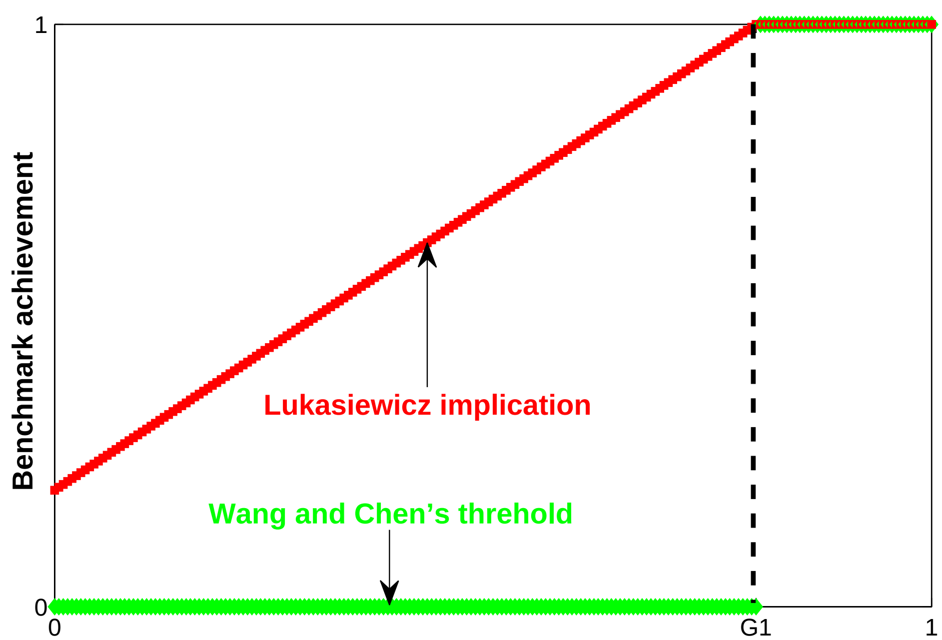

Figure 6.2: Benchmark achievement: at least G_1

The main difference of benchmark achievement in inducing priority weight between our approach and Wang and Chen's approach is illustrated in Fig. 6.2.It is clear that when the degree of satisfaction of criterion *safety* is higher than requirement G_1, we will obtain the same result with Wang and Chen. If the degree of satisfaction of criterion *safety* is less than requirement G_1, Wang and Chen's approach will be too strict.

Benchmark: at least G_1 and as high as possible

Now let us consider the second type of requirement. Column 1 of Table 6.6 shows the requirement values. According to Wang and Chen's approach (Eq. (6.23), Eq. (6.24), Eq. (6.26) and Eq. (6.27)), we can obtain the aggregation value with different G_1 values, as shown in Columns 2-5 of Table 6.6.

Table 6.6: Prioritized aggregated value under different fuzzy uncertain requirements

at least and as high as possible	Wang and Chen's method				Our proposed method			
	A^1	A^2	A^3	A^4	A^1	A^2	A^3	A^4
$G_1 = 0.91$	1.9475	2.0703	**2.1137**	2.0081	1.4167	0.9100	**1.4944**	1.3825
$G_1 = 0.95$	1.9475	0.9100	**2.1137**	0.9450	**0.9500**	0.9100	**0.9500**	0.9450
$G_1 = 0.96$	**0.9500**	0.9100	**0.9500**	0.9450	**0.9500**	0.9100	**0.9500**	0.9450

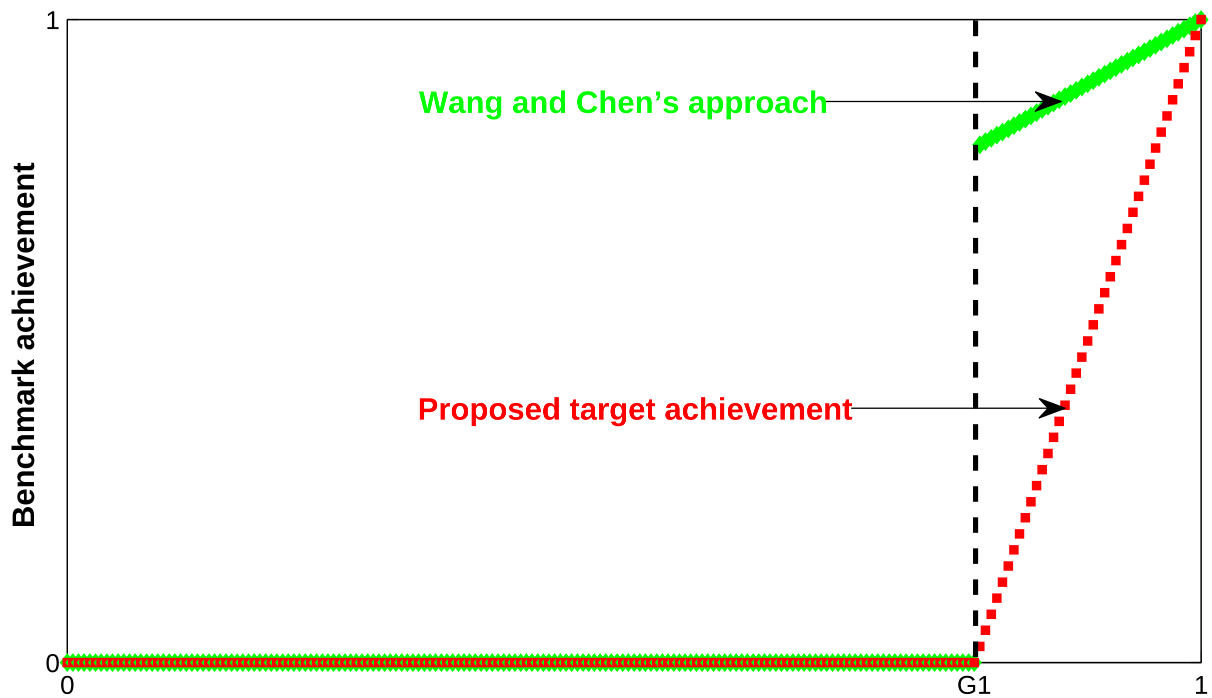

Figure 6.3: Benchmark achievement: at least G_1 and as high as possible

To model at least G_1 and as high as possible, we can use fuzzy number to represent this uncertainty, denoted as $\tilde{G}_1 = (G_1, G_1, 1, 1)$. It is in fact an interval number $[G_1, 1]$. And then according to Eq. (6.22) and our prioritized aggregation operator we can obtain the prioritized aggregation values as shown in Columns 6-10 of Table 6.6.

We will analysis the difference between our approach and Wang and Chen via Fig. 6.3. The ***red*** line in Fig. 6.3 shows the benchmark achievement by means of fuzzy target-oriented decision analysis under the requirement $\tilde{G}_1 = (G_1, G_1, 1, 1)$. The ***green*** line in Fig. 6.3 represents Wang and Chen's approach. According to Fig. 6.3, it is clear that when the degree of satisfaction of criterion ***safety*** is slightly less than the benchmark G_1, the induced degree of satisfaction $\mathrm{E}\left(\mathrm{Sat}_1^{(\cdot)}\right)$ will be zero, which will be too strict; when the degree of satisfaction of criterion ***safety*** is more than the benchmark G_1, the induced degree of satisfaction $\mathrm{E}\left(\mathrm{Sat}_1^{(\cdot)}\right)$ will be $\mathrm{Sat}_1^{(\cdot)}$. Whereas our approach is more consistent than Wang and Chen's approach. In addition, our approach usually constrain the benchmark achievement into an interval range $[0, 1]$.

Benchmark: Fuzzy at least G_1

Due to the vagueness or impreciseness of knowledge, it is difficult for DMs to estimate their requirements with precision. Fuzzy min target (fuzzy at least) is the target commonly used in decision making. We can model the fuzzy min G_1 as $\tilde{G}_1 = (0, G_1, 1, 1)$. In the previous case, we considered the requirement "at least G_1 and as high as possible" via target-oriented decision analysis. The fuzzy target $(G_1, G_1, 1, 1)$ is a special case of fuzzy at least G_1. And then according to Eq. (6.22) we obtain the target achievement function. Finally, according to our prioritized aggregation operator, we can easily obtain the aggregation values as shown in Columns 2-5 of Table 6.7, in which A^3 is always the optimal alternative.

Table 6.7: Prioritized aggregated value under different fuzzy min requirements

Fuzzy min G_1	Our proposed method			
	A^1	A^2	A^3	A^4
$(0, 0.91, 1, 1)$	1.9037	1.9744	**2.0626**	1.9565
$(0, 0.95, 1, 1)$	1.9000	1.9685	**2.0583**	1.9522
$(0, 0.96, 1, 1)$	1.8991	1.9675	**2.0573**	1.9513

Fig. 6.4 graphically depicts the fuzzy requirement and its associated target achievement function. It is clear from the ***red line*** in Fig. 6.4, the benchmark achievement reflects Simon's bounded rationality and the S-shaped value function. This is main reason why we utilize fuzzy target-oriented decision model to calculate fuzzy benchmark achievement.

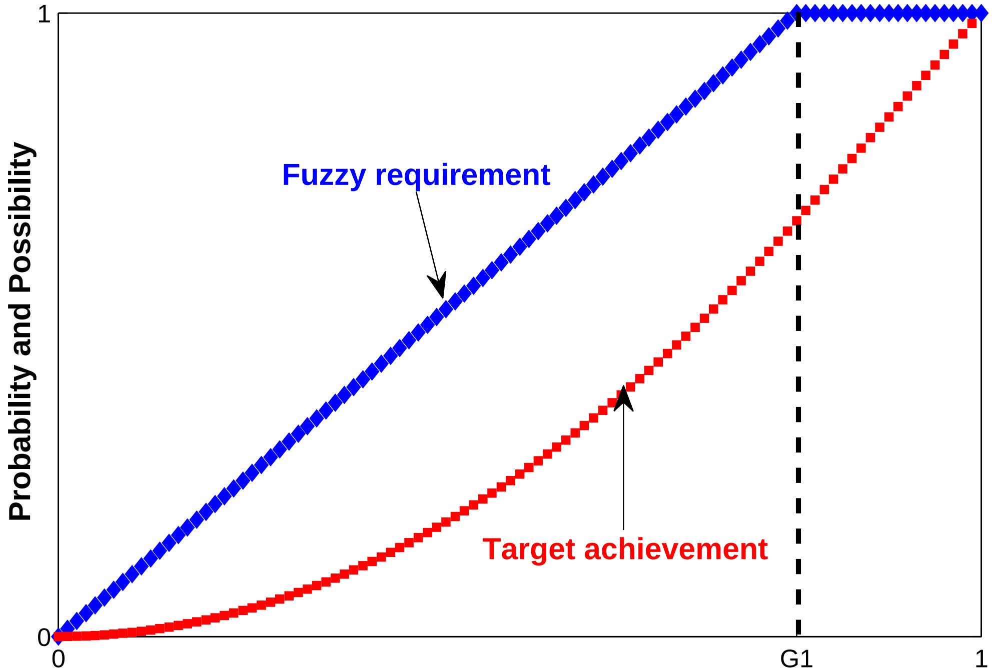

Figure 6.4: Benchmark achievement: fuzzy min G_1

6.5 Conclusions

In this paper, we have concerned ourselves with multi-attribute decision analysis (MADA) problems where there exists a prioritization of attributes, in which the priority weights associated with the lower priority are related to the satisfactions of the higher priority attributes. We have builded upon the work of [31, 136, 148, 150] and extended it in a number of directions.

1. Firstly, the OWA operator is used to obtain the degree of satisfaction for each priority level. To preserve the tradeoffs between the attributes in the same priority level, the degree of satisfaction regarding each priority level is viewed a pseudo attribute.

2. Secondly, we suggest that roughly speaking any t-norms can be used to model the priority relationships between the attributes in different priority levels. To preserve slight change of the priority weight, strict Archimedean t-norms perform better in inducing priority weight. As Hamacher family of t-norms provide a wide class of strict Archimedean t-norms ranging from the product to weakest t-norm [110], Hamacher t-norms are selected to induce the priority weight.

3. Thirdly, considering DM's requirement toward higher priority levels, a *benchmark* based approach has been proposed to induce priority weight for each priority level. In particular, Łukasiewicz implication is used as a technique to compute benchmark achievement for crisp requirements. In case of fuzzy uncertain requirements, as target-oriented decision analysis lies in the philosophical root of Simon's bounded rationality [120] as well as represents the S-shaped value function [70], fuzzy target-oriented decision analysis [62] is utilized to obtain the benchmark achievement. In

contrast to Wang and Chen's [31, 136] work, our approach can catch the slight changes of DM's requirement as well as coincide with the intuition of DM.

Multi-criteria decision analysis (MCDA) problems can be categorized into two classes: discrete and continuous MCDA [127], in this study we focused on discrete MCDA (MADA). In the context of continuous MCDA (multi-objective decision analysis), Ogryczak and Śliwiáski [107] proposed using the OWA operator to solve linear programming problems. Furthermore, it is practical to consider that there are different importances and priorities of objectives [28, 60, 86, 87]. In this regard, we believe that our proposed prioritized OWA operator will provide a general and convenient tool for multi-objective decision making with multiple priorities. However, this is left for the future work.

Chapter 7

Kansei Evaluation Based on Prioritized Multi-Attribute Fuzzy Target-Oriented Decision Analysis

Abstract: In this chapter, we focus on the evaluation problems using Kansei data, taking consumers' preferences on Kansei attributes into consideration. This chapter aims at proposing and developing a Kansei evaluation model based on multi-attribute target-oriented decision analysis. To do so, firstly, like the traditional Kansei evaluation method, a preparatory experiment study is conducted in advance to select Kansei attributes by means of semantic differential (SD) method. In order to obtain Kansei data of products, a number of subjects are selected to assess products regarding these Kansei attributes. Differed from the previous research, linguistic variables are used to represent the uncertain assessments. Secondly, these Kansei data are used to generate Kansei profiles for evaluated products by means of the voting statistics. Thirdly, as consumers' preferences on Kansei attributes vary from people to people and the consumers may have prioritization of Kansei attributes, we can view the current Kansei evaluation problem as a prioritized multi-attribute target-oriented decision analysis problem. Based on the proposed Kansei evaluation model, the consumers can select or choose the products according to their preferences.

7.1 Introduction

In today's increasingly competitive market place, satisfying consumers' needs and tastes has become a great concern of almost every company [59, 68, 129]. Consumers have put more emphasis not only on functional requirements of products, defined objectively, but also on psychological needs and feelings, by essence subjective [112]. Moreover, with the development of global markets and modern technologies, it is likely that many similar products will be functionally equivalent [68], thus consumers may find that it is difficult to distinguish and choose their desired product(s). In this regard, consumers' psychological needs and feelings must be considered in choice of products.

Kansei engineering has been developed as a methodology to deal with consumers' subjective impressions (called Kansei in Japanese) regarding a product into the design elements of a product [103, 104, 105]. There is no corresponding term to Kansei in English. The term Kansei is imbedded in the Japanese culture in a way that is difficult to translate into words. A specific Kansei arises when a human is subjected to an artifact in a certain environmental context [118]. Kansei may be easier to experience than define by a western person. Looking at picture or artifact may evoke a certain "good feeling" that is difficult to describe. This is what Kansei is about. According to Nagamachi [103] and Schütte [118],

> Kansei is an individual subjective impression from a certain artifact, environment or situation using all the senses of *sight, hearing, feeling, smell, taste, recognition and balance.*

For building a Kansei database on psychological feelings regarding products, the most commonly used method is to choose Kansei words (bipolar subjective words) first, and then ask people to express their feelings using those words often by means of the semantic differential (SD) method [109] or its modifications [53, 118].

In this chapter, we focus on the evaluation problems using Kansei data, taking consumers' preferences on Kansei attributes into consideration. The evaluation would be of great help for marketing or recommendation purposes, and particularly in the era of e-commerce, where recommendation systems have become an important research area [4]. It should be emphasized that many studies of Kansei engineering or other consumer-oriented design techniques have involved an evaluation process, in which a design could be selected for production, e.g., [112].

This chapter aims at proposing and developing a Kansei evaluation model based on multi-attribute target-oriented decision analysis. To do so, firstly, like the traditional Kansei evaluation method, a preparatory experiment study is conducted in advance to select Kansei attributes by means of semantic differential (SD) [109] method. In order to obtain Kansei data of products, a number of subjects are selected to assess products regarding these Kansei attributes. Differed from the previous research, linguistic variables are used to represent the uncertain assessments. Secondly, these Kansei data are used to generate Kansei profiles for evaluated products by means of the voting statistics. Thirdly, as consumers' preferences on Kansei attributes vary from people to people and the consumers may have prioritization of Kansei attributes, we can view the current Kansei evaluation problem as a prioritized multi-attribute target-oriented decision analysis problem. In particular, three main types of fuzzy targets are defined to represent consumers' uncertain preferences on Kansei attributes. Based on the fuzzy target-oriented decision analysis model discussed in Chapter 4, we can obtain the satisfaction degrees (probabilities of meeting targets) regarding the Kansei attributes selected by consumers for all the evaluated products. And then, considering prioritization of the Kansei attributes, the so-called prioritized OWA aggregation operator discussed in Chapter 6 is used to aggregate the partial satisfaction degrees for the evaluated products. Based on the proposed Kansei evaluation model, the consumers can select or choose the products according to their preferences.

The rest of this chapter is organized as follows. In Section 7.2 we recall some basic knowledge of Kansei evaluation and give the motivations of our work. In Section 7.3 we put forward a Kansei evaluation model based the fuzzy target-oriented decision model discussed in Chapter 4 and the prioritized OWA aggregation operator proposed in Chapter 6. In Section 7.4 we give some discussions of the our model and give some effective analysis. Finally, some concluding remarks are given in Section 7.5.

7.2 Literature Review of Kansei Evaluation and Motivations

In this section, we shall given some basic knowledge of Kansei evaluation and its related work, and then review some approaches to Kansei evaluation. Finally, we will given our motivations of current research.

7.2.1 Definition of Kansei evaluation

Kansei engineering has been developed as a methodology to deal with consumers' subjective impressions (called Kansei in Japanese) regarding a product into the design elements of a product [103, 104, 105]. It is a methodology that integrates a affective elements already in the developing process [118]. Kansei engineering has been applied to home equipment, architecture, packagings design. Most Kansei studies are done in Japan and Korea. There is no corresponding term to Kansei in English. The term "Kansei" is embedded in Japanese culture in a way that is difficult to translate into words. Several definitions of Kansei can be trailed from the literature. A notable definition of Kansei is given by Nagamachi [103] and Schütte [118] as "Kansei is an individual subjective impression from a certain artifact, environment or situation using all the sense of sight, hearing, feeling, smell, taste, recognition and balance."

Kansei engineering is also sometimes referred to as "*sensory engineering*" or "*emotional usability*" [53]. Kansei engineering can be either used by designers as a design aid to develop products that are able to match consumers' Kansei or used by consumers to select products based on their Kansei requirements [103]. Among Kansei engineering, Kansei evaluation is an important process in which a product design may be selected for production or design [27, 85, 90, 92, 112, 129]. In this chapter, we focus on Kansei evaluation process based on consumers' Kansei requirements, the very early process in Kansei engineering.

In the literature, Kansei evaluation is also called as sensory evaluation. A classical definition of sensory evaluation is given by Stone and Sidel [121] and Dijksterhuis [37] as follows: " Sensory evaluation is a scientific discipline used to evoke, measure, analyze, and interpret reactions to those characteristics of products or materials as they are perceived by the senses of **sight, smell, taste, touch and hearing**". The term "subjective evaluation" is also used in the literature, e.g., [100, 106, 129]. The term "subjective" means private "mental" stuff: sensations, beliefs, feelings, emotions, opinions, etc.; whereas objective (the opposite of subjective) is the public "physical" stuff: publicly-observable things, events, knowledge, facts [1]. In this thesis, we shall reserve the term "Kansei" instead of "sensory" and "subjective", as the research context is closely related to Japanese culture.

7.2.2 Approaches to Kansei evaluation

Many studies have attempted to solve Kansei evaluation [8, 27, 85, 90, 92, 102, 112] in the literature. Generally speaking, there are two types of approaches to Kansei evaluation.

1. **Statistical methods**
   Statistical analysis plays an important role and is widely accepted as the most systematic tool for Kansei evaluation. For example, Hsu et al. [59] used multivariate analysis to analyze consumers' perceptions and to build conceptual models for telephone design. Llinares and Page [92] performed statistical analysis to quantify purchaser perceptions in housing assessment to identify main attributes which describe consumers' perception. To reduce dimensionality, principal component analysis (PCA) and fuzzy PCA are also used [92, 106] in Kansei evaluation. Moreover, Barone et al. [8] proposed a weighted regression approach by means of conjoint analysis, in which attribute importance weights are estimated by using respondent choice time in controlled interviews. Petiot and Yannou [112] proposed an integrated approach which rates and ranks the new product prototypes according to their closeness to the specified "ideal product", in which three types of satisfaction utility functions are defined and a multi-additive model is used to obtain the global satisfaction utility.

2. **Decision analysis methods**
   In addition to these methods, in closely similar and related studies on sensory evaluation or subjective evaluation, decision analysis has also been utilized in the evaluation problemsFor example, Martínez [98] proposed a sensory evaluation model based on linguistic decision analysis by using the linguistic 2-tuple representation model [56, 57], in which knowledge used for sensory evaluation is acquired from a panel of experts by means of the five senses of *sight, taste, touch, smell and hearing*. The sensory evaluation model [98] considers the evaluation problem as a multi-expert/multi-attribute decision problem, assuming a consistent order relation on the quantitative evaluation scale treated as the linguistic term set of a linguistic variable [163, 167]. More studies of sensory evaluation based on the linguistic 2-tuple representation model [56, 57] can be found in the literature [99, 115, 169]. The additive or multiplicative utility model has also been used for subjective evaluations [69, 112].

[1] `http://instruct.westvalley.edu/lafave/subjective_objective.html`

7.2.3 Motivations

Previous studies have significantly advanced the issue of Kansei and Kansei-related evaluations. However, there are still some drawbacks.

1. Firstly, consumers' preferences on Kansei attributes vary from person to person according to character, feeling, aesthetic and so on. For example, a Kansei attribute *fun* having left and right Kansei words as <solemn, funny>. Some consumers may prefer *solemn*, others may prefer *funny*, and there are also some consumers preferring *neither solemn nor funny*. In this regard, in contrast to the sensory evaluation model [98, 99, 115, 169], we will have *inconsistent order relations* on Kansei attributes.

2. Furthermore, as pointed out by Bordley and Kirkwood [17], empirical evidence indicates that conventional concave attribute utility function often does not provide a good description of individual preference, and usually it is difficult for consumers to determine their utility functions for Kansei attributes.

3. Finally, a consumer usually may have a priority order of the Kansei attributes, i.e., some Kansei attributes may be necessary to be satisfied.

These considerations lead us to solve Kansei evaluation based on multi-attribute fuzzy target-oriented decision analysis and prioritized aggregation. In their pioneering work Kahneman and Tversky [70] proposed an S-shaped value function to substitute for utility function. Heath et al. [54] suggested that the reference point in this S-shaped value function can be interpreted as a target. Developing this concept further, target-oriented decision analysis [18] suggested that instead of maximizing the utility, the decision makers try to maximize the probability of meeting target. In general, target-oriented decision analysis lies in the philosophical root of Simon's bounded rationality [120] as well as represents the S-shaped value function [70]. Particularly, in Kansei evaluation, due to vagueness and uncertainty of consumers' preferences, fuzzy targets can be used to represent consumers' uncertain preferences. In addition, multiple Kansei attributes are usually considered. To model the prioritization of Kansei attributes, the prioritized OWA aggregation operator proposed in Chapter 6 induces a weighting schema by using priority weights in which the weights associated with the lower priority Kansei attributes are related to the satisfaction of the higher priority Kanesi attributes.

7.3 A Kansei Evaluation Model Based on Prioritized Multi-Attribute Target-Oriented Decision Analysis

In this section we shall propose a Kansei evaluation model, based on the assumption that a consumer will be only interested in products that best meet her/his psychological needs from an aesthetic point of view. Our proposed Kansei evaluation model consists of the following steps, as shown in Fig. 7.1.

The dashed rectangle I in Fig. 7.1 shows the preparatory experiment study phase, a common process in Kansei engineering, which is used to identify and measure Kansei attributes first and then to obtain Kansei data of the products to be evaluated. The dashed rectangle II in Fig. 7.1 shows the target-oriented decision analysis phase, in which fuzzy target-oriented decision analysis is used to compute degrees of satisfaction for the Kansei attributes selected by consumers, and a prioritized aggregation operator is used to aggregate partial degrees of satisfaction under a given priority hierarchy. In the following subsections, we will describe our model in more detail.

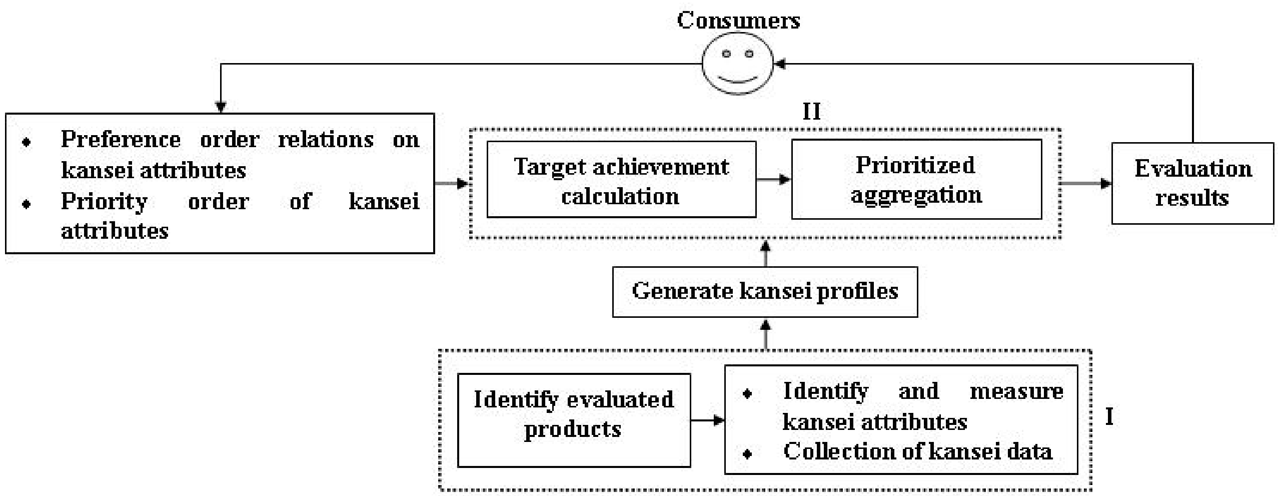

Figure 7.1: Proposed Kansei evaluation process

7.3.1 Identification and measurement of Kansei attributes

Let $\mathcal{A}$ be set of products to be evaluated and M is the cardinality of products, i.e., $M = |\mathcal{A}|$. Once having identified and selected the products to be evaluated, we have to identify and measure Kansei attributes used by people to express their psychological feelings regarding the products to be evaluated. Usually Kansei attributes are identified by a panel of experts (experts means people familiar with the product type and Kansei engineering) via a brainstorming process [53].

A person's Kansei will be expressed through physiological functions. There are different ways of measuring the Kansei:

1. Words
2. Physiological response (Heart rate, EMG, EEG)
3. People's behaviors and actions
4. Facial and body expressions

The words reflect elements of the Kansei. They are just external descriptions of the Kansei within a person's mind. Elements of the Kansei may be absent because we do not have words to describe all emotions. The words are not the Kansei itself. Facial and body expressions have been used within emotional design outside Kansei Engineering as well.

Most of the Kansei Engineering studies which have been published in English use words when measuring the Kansei. In this study, words are also used to measure the Kansei.

Each Kansei attribute is defined by a bipolar pair of Kansei words. The bipolar pairs of Kansei words describing the product domain can be collected from many sources, such as *Magazines, Literature, Manuals, Experts, Experienced users, Ideas, and Visions* [53]. It is important to also include words from ideas and visions so that potential new solutions also are included. This collection of words goes on until no new words occur. A good result depends on that all important words are included, so it is better to include a few more words than necessary. Although identification of Kansei words in practice is a difficult task, it is a necessary and important process in Kansei engineering.

Particularly, the Kansei attributes can be expressed as follows:

1. Let $\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}$ be set of Kansei attributes of products, where N denotes the total number of Kansei attributes;

2. Let $\mathrm{KW}_n = \langle \mathrm{KW}_n^-, \mathrm{KW}_n^+ \rangle$ be the opposite pair of Kansei words with respect to Kansei attribute $X_n, n = 1, 2, \cdots, N$. For example, a Kansei attribute *fun* can be denoted as bipolar Kansei words as <solemn, funny>.

In addition, a questionnaire is designed by means of the SD method [109] to collect subjective assessments provided by a number of subjects (respondents for the questionnaire). The questionnaire consists of listing Kansei attributes, each of which corresponds to a bipolar pair of Kansei words with a $2K + 1$-point odd qualitative scale. For example, the odd qualitative scale of Kansei attributes can be 5-point scale [106], 7-point scale [102], and 9-point scale [53].

The subjective assessments provided by the subjects are usually conceptually vague, with uncertainty that is frequently represented in linguistic forms [129]. To help people easily express their subjective assessments, the linguistic variables [163, 167] are used to linguistically assess the products to be evaluated. In order to establish the linguistic term set for each Kansei attribute, we have to choose syntax and semantics [55, 56] as follows

1. The cardinality of each linguistic term set for each Kansei attribute corresponds to the semantic scale of each Kansei attribute, i.e., the cardinality of each linguistic term set is $2K + 1$.

2. Similar to the linguistic decision analysis [55, 56], ordered structure approach has been used to choose linguistic descriptors for Kansei attributes. For example, the linguistic terms "*fairly*" and "*very*" are used to describe the Kansei linguistic variables.

3. Fuzzy numbers are used to represent the Kansei linguistic variables. Fuzzy numbers can have a variety of shapes. In practical applications, for simplicity, the triangular or trapezoidal form of the membership function is used most often for representing fuzzy numbers [57, 76]. In this study, triangular fuzzy numbers are used to represent the Kansei linguistic variables.

In this way, we can establish a linguistic term set for each Kansei attribute, denoted as

$$L_n = \{L_n^{-K}, L_n^{-(K-1)}, \cdots, L_n^k, \cdots, L_n^{(K-1)}, L_n^K\},$$

where $k = -K, -(K-1), \cdots, 0, \cdots, (K-1), K$.

Example Assume a Kansei attribute *fun* having left and right Kansei words <solemn, funny> with a 7-point ($K = 3$) scale, similar to the linguistic variables in [90, 98], the linguistic term set for this Kansei attribute can be defined as

$$
\begin{aligned}
L &= \{L^{-3}, L^{-2}, L^{-1}, L^{0}, L^{1}, L^{2}, L^{3}\} \\
&= \{\textit{Very solemn, Solemn, Fairly solemn, Neutral, Fairly funny, Funny, Very funny}\} \\
&= \{(-3,-3,-2), (-3,-2,-1), (-2,-1,0), (-1,0,1), (0,1,2), (1,2,3), (2,3,3)\}
\end{aligned}
$$

Fig. 7.2 shows the semantics and fuzzy numbers of Kansei linguistic variables for Kansei attribute *fun*.

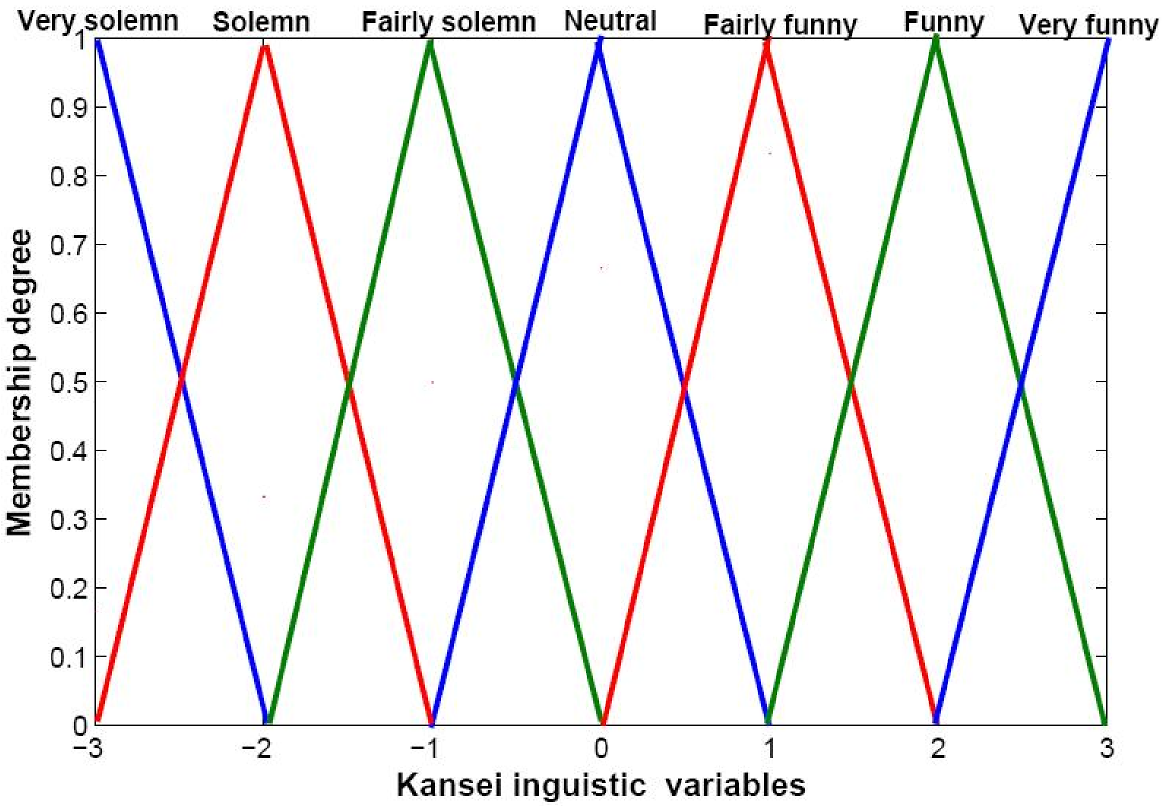

Figure 7.2: Linguistic variables for Kansei attribute *fun*

It should be noted that the Kansei linguistic term set L_n for each Kansei attribute X_n here we used is different from that used in the sensory evaluation model [98, 99, 115, 169]. The sensory evaluation model considers the linguistic term set having a *consistent order relation*. However, for the linguistic term set of a Kansei attribute, the order relation depends on the consumers' preferences, in this sense, we have *inconsistent order relations*. Now we will take the Kansei attribute *fun* represented in Fig. 7.2, as an example to illustrate the *inconsistent order relations*. Generally, three types of order relations can be considered

1. Some consumers may prefer *solemn*, then the linguistic order relation is

$$L^{-3} \succ L^{-2} \succ L^{-1} \succ L^{0} \succ L^{1} \succ L^{2} \succ L^{3};$$

2. Other consumers prefer *neutral*, then the linguistic order relation is

$$L^{-3} \prec L^{-2} \prec L^{-1} \prec L^{0} \succ L^{1} \succ L^{2} \succ L^{3};$$

3. There are also some consumers preferring *funny*, then the linguistic order relation is

$$L^{-3} \prec L^{-2} \prec L^{-1} \prec L^{0} \prec L^{1} \prec L^{2} \prec L^{3}.$$

7.3.2 Generation of Kansei profiles

The questionnaire is then assigned to a number J of subjects $\mathcal{S}$, who are selected to linguistically express their subjective assessments regarding the Kansei attributes in a simultaneous way. Formally, we can model the Kansei data of each product A^m according to Kansei attributes obtained from the assessment of subjects S_j, as shown in Table 7.1, where $X_n^m(S_j) \in L_n$, for $j = 1, \cdots, J = |\mathcal{S}|$ and $k = -K, -(K-1), \cdots, 0, \cdots, (K-1), K$.

Table 7.1: Kansei linguistic assessment data of product A^m

Subjects	Kansei attributes				
	X_1	$\cdots$	X_n	$\cdots$	X_N
S_1	$X_1^m(S_1)$	$\cdots$	$X_n^m(S_1)$	$\cdots$	$X_N^m(S_1)$
S_2	$X_1^m(S_2)$	$\cdots$	$X_n^m(S_2)$	$\cdots$	$X_N^m(S_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_j	$X_1^m(S_j)$	$\cdots$	$X_n^m(S_j)$	$\cdots$	$X_N^m(S_j)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_J	$X_1^m(S_J)$	$\cdots$	$X_n^m(S_J)$	$\cdots$	$X_N^m(S_J)$

Having obtained the Kansei assessments given by the subjects, we can obtain Kansei profiles as follows. For evaluated product $A^m, m = 1, 2, \cdots, M$, we define for Kansei attribute X_n, $n = 1, 2, \cdots, N$, a probability distribution function $p_n^m : L_n \to [0, 1]$ as follows

$$p_n^m(L_n^k) = \frac{|\{S_j \in \mathcal{S} : X_n^m(S_j) = L_n^k\}|}{|\mathcal{S}|} \qquad (7.1)$$

where $k = -K, -(K-1), \cdots, 0, \cdots, (K-1), K$, and $X_n^m(S_j)$ denotes the Kansei assessment for product A^m with respect to Kansei attribute X_n given by subject $S_j, j = 1, \cdots, J$.

In the same way, we can obtain a $2K+1$-tuple of probability distributions for product A^m with respect to Kansei attribute X_n,

$$p_n^m = \left[p_n^m(L_n^{-K}), p_n^m(L_n^{-(K-1)}), \cdots, p_n^m(L_n^0), \cdots, p_n^m(L_n^{(K-1)}), p_n^m(L_n^K)\right] \qquad (7.2)$$

and call this tuple as ***Kansei profile*** of A^m with respect to Kansei attribute X_n.

The $2K+1$-tuple of probability distributions, as shown in Table 7.2, can be viewed as a general multi-attribute decision matrix, where each Kansei attribute has $2K+1$ states of nature. For Kansei attribute X_n at the state of nature k, where $k = -K, -(K-1), \cdots, 0, \cdots, (K-1), K$. From Table 7.2 we know that all the products have the same attribute values (fuzzy numbers) for the same indexed linguistic variables. However, the semantics of these linguistic variables are different. Furthermore, the probability distributions (Kansei profiles) with respect to the same indexed linguistic variables are also usually different.

Table 7.2: Kansei profiles of evaluated products: probability distributions of Kansei assessments

Products	Kansei attributes										
	X_1					$\cdots$	X_N				
	$-K$	$\cdots$	0	$\cdots$	K	$\cdots$	$-K$	$\cdots$	0	$\cdots$	K
A^1	$p_1^1(L_1^{-K})$	$\cdots$	$p_1^1(L_1^0)$	$\cdots$	$p_1^1(V_1^K)$	$\cdots$	$p_N^1(L_N^{-K})$	$\cdots$	$p_N^1(L_N^0)$	$\cdots$	$p_N^1(L_N^K)$
A^2	$p_1^2(L_1^{-K})$	$\cdots$	$p_1^2(L_1^0)$	$\cdots$	$p_1^2(V_1^K)$	$\cdots$	$p_N^2(L_N^{-K})$	$\cdots$	$p_N^2(L_N^0)$	$\cdots$	$p_N^2(L_N^K)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
A^M	$p_1^M(L_1^{-K})$	$\cdots$	$p_1^M(L_1^0)$	$\cdots$	$p_1^M(V_1^K)$	$\cdots$	$p_N^M(L_N^{-K})$	$\cdots$	$p_N^M(L_N^0)$	$\cdots$	$p_N^M(L_N^K)$

7.3.3 Specification of consumers' preferences

Having generated Kansei profiles for all evaluated products $A^m \in \mathcal{A}, m = 1, 2, \cdots, M$ as above, we now consider the preferences of consumers. Assume that a potential consumer is interested in a collection of Kansei attributes[2]

$$\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}.$$

As mentioned previously, order relations of Kansei linguistic term sets regarding Kansei attributes vary from person to person according to their character, feeling, aesthetic and so on, a preference function for Kansei attribute $X_n, n = 1, 2, \cdots, N$ is needed.

In the context of multi-attribute decision analysis (MADA), usually there are two types of goal preferences [71, 127].

- Target goal values are adjustable: "more is better" or "less is better";
- Target goal values are fairly fixed and not subject to much change, i.e., too much or too little is not acceptable.

To model consumers' preference order relations on Kansei linguistic term set, we shall define three main types of target preferences[3] as follows:

[2]The number of Kansei attributes selected by consumers may be different from the total number of Kansei attributes. Here for simplicity of denotation we shall use the same number N.

[3]Generally speaking, any target can be defined by consumers. However, as consumers are not so specific about their own personal preference, here we just provide three types of targets.

- *Less is better*: Left Kansei words preferred;
- *More is better*: Right Kansei word preferred;
- *Target goal values are fairly fixed*: Neutral preferred.

Due to the vagueness and uncertainty of Kansei preference values, fuzzy targets are used to represent consumers' preferences. Fig. 7.3 shows the three types of preferences represented by fuzzy targets.

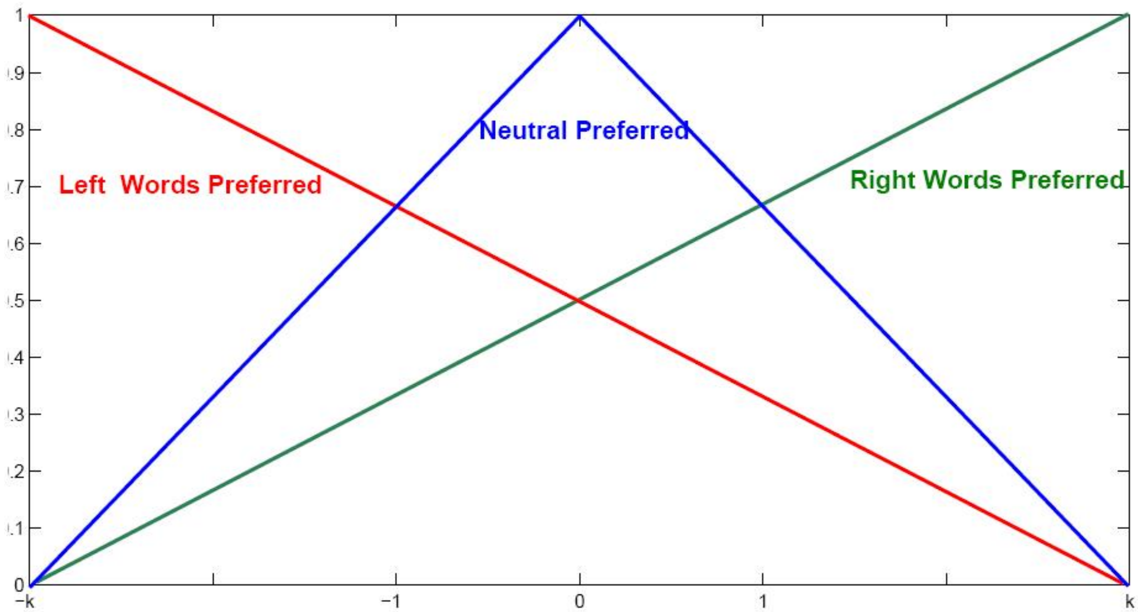

Figure 7.3: Target-oriented preferences

Based on consumer-specified preferences, a collection of fuzzy targets such that $T = (T_1, \cdots, T_n, \cdots, T_N)$, can be obtained with respect to the collection of Kansei attributes $\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}$.

In addition to the preference order relations on Kansei linguistic term sets, consumers may have a priority order of the Kansei attributes. Simply speaking, by saying Kansei attribute X_1 has a higher priority than Kansei attribute X_2, it means that the consumers are not willing to trade off satisfaction to Kansei attribute X_2 until they attain some level of satisfaction of Kansei attribute X_1 [148].

Considering these two types of consumer-specified preferences, we divide the evaluation process into two phases

1. Calculate degree of satisfaction for Kansei attribute X_n;
2. Aggregate partial degrees of satisfaction under the prioritized hierarchy.

Fuzzy target-oriented decision analysis proposed in Chapter 4 has been extended to calculate the degree of satisfaction for Kansei attribute X_n, and then the prioritized OWA aggregation operator proposed in Chapter 6 is used to aggregate the partial degrees of satisfaction. In this regard, we shall view our research problem as ***prioritized multi-attribute fuzzy target-oriented decision analysis***. In the following two subsections, we shall discuss these two steps in further detail.

7.3.4 Calculation of satisfaction degree based on fuzzy target-oriented decision model

According to the principle of target-oriented decision analysis [17], for our general decision matrix as shown in Table 7.2, we can define the probability of product A^m meeting the fuzzy target T_n with respect to Kansei attribute X_n as follows:

$$\Pr(X_n^m \succeq T_n) = \sum_{k=-K}^{K} p_n^m(L_n^k) \cdot \Pr(L_n^k \succeq T_n) \tag{7.3}$$

where L_n^k denotes the k-th linguistic variable for Kansei attribute X_n, where

$$k = -K, -(K-1), \cdots, 0, \cdots, (K-1), K,$$

$p_{mn}(L_n^k)$ denotes the probability distribution of Kansei attribute X_n at linguistic variable L_n^k, and $\Pr(L_n^k \succeq T_n)$ is the probability of L_n^k meeting target T_n.

Central to this problem is how to compute the probability $\Pr(L_n^k \succeq T_n)$ of L_n^k meeting fuzzy target T_n. In Chapter 3 & 4, we already discussed target-oriented decision analysis with different target preferences and hybrid uncertain targets. In this Chapter, for the sake of simplicity, we shall use the cumulative distributive function based approach to calculate $\Pr(L_n^k \succeq T_n)$. In this representation, both L_n^k and T_n are fuzzy numbers. In the following, we first show a generation of target-oriented decision model discussed in Chapter 3 and Chapter 4, and then given the expression of target-oriented decision analysis in Kansei evaluation problems.

General representation of target-oriented decision analysis

Before discussing how to obtain $\Pr(L_n^k \succeq T_n)$, we shall consider the general representation. We firstly assume that, for an attribute X, there exists a fuzzy target, denoted as T. The consequence of attribute X is denoted as X_d. We also assume that $\pi(t)$ is the possibility distribution function of fuzzy target T. We also restrict the consequence of attribute X to a bounded domain $D = [X_{\min}, X_{\max}]$.

If the consequence of attribute X is a crisp number, denoted as x, we have $X_d = x$. Based on the research work in Chapter 4, we can get the following value functions.

- For benefit target

$$\begin{aligned} \Pr(x \succeq T) &= \Pr(x \geq T) \\ &= \frac{\int_{X_{\min}}^{x} \pi(t)dt}{\int_{X_{\min}}^{X_{\max}} \pi(t)dt} \end{aligned} \tag{7.4}$$

- For cost target

$$\begin{aligned} \Pr(x \succeq T) &= \Pr(x \leq T) \\ &= \frac{\int_{x}^{X_{\max}} \pi(t)dt}{\int_{X_{\min}}^{X_{\max}} \pi(t)dt} \end{aligned} \tag{7.5}$$

- For rang level target

$$\Pr(x \succeq T) = \Pr(x \cong T)$$
$$= \begin{cases} \frac{\int_{X_{\min}}^{x} \pi(t)dt}{\int_{X_{\min}}^{T_{ml}} \pi(t)dt}, & \text{if } x < T_{ml}; \\ 1, & \text{else if } x \in [T_{ml}, T_{mu}]; \\ \frac{\int_{x}^{X_{\max}} \pi(t)dt}{\int_{T_{mu}}^{X_{\max}} \pi(t)dt}, & \text{if } x > T_{mu}. \end{cases} \tag{7.6}$$

The equal target preference, such that $T_{ml} = T_{mu}$, is a special case this range level targets.

In the case where the consequence X_d is an interval value over the domain $[X_{\min}, X_{\max}]$, we consider X_d as a random variable with the uniform distribution on $[X_{\min}, X_{\max}]$. Thus we can obtain the probability distribution as

$$p(x) = \frac{1}{X_{\max} - X_{\min}}$$

If X_d is a fuzzy quantity represented by a possibility distribution $\pi(x)$, we have the associated probability distribution of $\pi(x)$ defined by

$$p(x) = \frac{\pi(x)}{\int_{-\infty}^{+\infty} \pi(x)dx}.$$

Having considered X and T as two uncertain variables, we can define the probability of X_d meeting the target T as

$$\Pr(X_d \succeq T) = \int_{-\infty}^{\infty} p(x) \Pr(x \succeq T)dx \tag{7.7}$$

where $\Pr(x \succeq T)$ denotes the target achievement in case of benefit target, cost target, and range/equal target, which can be obtained from Eqs. (7.4)-(7.6).

Kansei evaluation based on target-oriented decision analysis

For evaluated product A^m in our general multi-attribute decision matrix, we can get the probability of product A^m meeting fuzzy target T_n with respect to Kansei attribute X_n as follows:

$$\begin{aligned} \Pr(X_n^m \succeq T_n) &= \sum_{k=-K}^{K} p_n^m(L_n^k) \cdot \Pr(L_n^k \succeq T_n) \\ &= \sum_{k=-K}^{K} p_n^m(L_n^k) \cdot \left[\int_{-\infty}^{\infty} p_{L_n^k}(x) \Pr(x \succeq T_n)dx \right] \end{aligned} \tag{7.8}$$

where $p_{L_n^k}(x)$ is the probability density function of Kansei linguistic variable L_n^k and $\Pr(x \succeq T_n)$ can be calculated according to the fuzzy target-oriented decision model mentioned above based on consumers' preference types.

7.3.5 Prioritized aggregation of target achievements

Having computed the probability of meeting consumers' specified fuzzy target-oriented preferences for Kansei attributes selected by consumers, we have to aggregate partial degrees of satisfaction (target achievements) $\Pr(X_n^m \succeq T_n)$. Here for denotational simplicity, we shall use $\Pr_n^m$. One commonly used approach for aggregation is to calculate for product A^m a value $\mathrm{Val}(A^m)$ by using an aggregation function F as

$$F\left(\Pr_1^m, \cdots, \Pr_n^m, \cdots, \Pr_N^m\right)$$

and then order the evaluated products according to these values $\mathrm{Val}(A^m)$.

In many types of applications, people usually associate importance weights with the attributes [23]. A commonly used form for F is a weighted average of the A^m. In this case we calculate

$$\mathrm{Val}(A^m) = \sum_{n=1}^{N} w_n \cdot \Pr_n^m, \text{ where } \sum_{n=1}^{N} w_n = 1.$$

Central to this types of aggregation operators is the ability to trade off between attributes [148].

In some situations, the consumers may not need this kind of tradeoffs between Kansei attributes. In this case, we will have a prioritization hierarchy. Assume that the collection of Kansei attributes $\mathcal{X} = \{X_1, \cdots, X_n, \cdots, X_N\}$ are partitioned into Q distinct priority levels $\mathcal{H} = \{H_1, \cdots, H_q, \cdots, H_Q\}$, such that $H_q = \{X'_{q1}, \cdots, X'_{qi}, \cdots, X'_{qN_q}\}$, where N_q is the Kansei attribute number in priority level H_q, and X_{qi} is the i-th Kansei attribute in category H_q. We also assume a prioritization of these Kansei attributes is $H_1 \succ \cdots > H_q \succ \cdots \succ H_Q$. Table 7.3 shows the priority hierarchy structure of the Kansei attributes.

Table 7.3: Prioritization of Kansei attributes specified by consumers

H_1	$X'_{11}, \cdots, X'_{1i}, \cdots, X'_{XN_1}$
$\vdots$	$\vdots$
H_q	$X'_{q1}, \cdots, X'_{qi}, \cdots, X'_{qN_q}$
$\vdots$	$\vdots$
H_Q	$X'_{Q1}, \cdots, X'_{Qi}, \cdots, X'_{QN_Q}$

In this case, we shall use $\Pr_{qi}^m$ to express the degree of satisfaction for the i-th Kansei attribute in priority level H_q with respect to evaluated product A^m. In Chapter 6, we have already proposed a prioritized OWA aggregation operator based on the assumption that prioritized aggregation can be modeled by using a kind of priority weight in which the weight of a lower priority attribute will be based on its satisfaction to the higher priority attribute. The prioritized aggregation operator suggested using the following steps:

1. For Kansei attributes in priority level H_q regarding product A^m, a degree of satisfaction Sat_q^m is calculated as follows

$$\mathrm{Sat}_q^m = \mathrm{OWA}_\Omega\left(\Pr_{qi}^m, \cdots, \Pr_{qi}^m, \cdots, \Pr_{qi}^m\right) \tag{7.9}$$

where OWA_Ω means the ordered weighted averaging (OWA) aggregation based on consumers' decision attitudinal Ω.

The OWA operator is generally composed of the following three steps [91]:

(a) Reorder the input arguments in descending order,

(b) Determine the weights associated with the OWA operator by using a proper method, and

(c) Utilize the OWA weights to aggregate these reordered arguments.

In this case we would supply a desired level of tolerance Ω and solve the following constrained optimization problem for the i-the element in priority hierarchy level H_q a weight u_{qi}.

$$\text{Maximize} - \sum_{i=1}^{N_q} u_{qi} \cdot \ln u_{qi} \tag{7.10a}$$

$$\text{subject to } \sum_{i=1}^{N_q} \left[\frac{N_q - i}{N_q - 1} \cdot u_{qi} \right] = \Omega, \ 0 \le \Omega \le 1 \tag{7.10b}$$

$$\sum_{i=1}^{N_q} u_{qi} = 1, \ 0 \le u_{qi} \le 1. \tag{7.10c}$$

2. Then a priority weight Z_q^m for priority level H_q is calculated as follows

$$\begin{aligned} Z_q^m &= \prod_{l=1}^{q} \text{Sat}_{l-1}^m \\ &= Z_{q-1}^m \cdot \text{Sat}_{q-1}^m \end{aligned} \tag{7.11}$$

where $Z_0^m = \text{Sat}_0^m = 1$.

3. To calculate the overall degree of satisfaction for product A^m as follows

$$\text{Val}(A^m) = \sum_{q=1}^{Q} Z_q^m \cdot \text{Sat}_q^m \tag{7.12}$$

Once having calculated $\text{Val}(A^m)$ for all the evaluated products, we then select as our optimal choice, the product A^* which satisfy

$$\text{Val}(A^*) = \max_m [\text{Val}(A^m)]. \tag{7.13}$$

7.4 Discussions

Like most studies on Kansei evaluation, one preparatory step in our proposed model is to identify and measure Kansei attributes and then conduct a questionnaire to collect Kansei data. In practical application, the preparatory study is time-consuming, difficult and subjective. This bottleneck lies in most studies of Kansei evaluation. The proposed

Kansei evaluation model can be used for recommendation purpose, as it is a consumer-oriented Kansei evaluation model. It can also be used in Kansei engineering. In product design, Kansei evaluation is an essential step. Once we can obtain consumers' preference information, we can evaluate the products.

In the proposed Kanesi evaluation model, we only consider three types of target preferences. In general, the consumers can define many targets. However, as it is not so easy to capture consumers' preference information, we define three target preferences. Furthermore, in Kansei evaluation problems, usually there are many Kansei attributes. Thus the principle component analysis (PCA) is used to analyze the main factors that impact the evaluation problems. This work is left for the future work.

7.5 Conclusions

Usually consumers purchase or select products according to their functional requirements or psychological needs. In this chapter we concerned ourselves with Kansei evaluation focusing on consumers' psychological needs and feelings according to so-called Kansei attributes, which reflect aesthetic aspects of human perception on products. In particular, a preliminary study is conducted beforehand to obtain Kansei data of products, by means of the semantic differential method and linguistic variables. These Kansei data are then used to generate Kansei profiles for evaluated products by means of the voting statistics. Because consumers' preferences on Kansei attributes of products vary from person to person and target-oriented decision analysis provides a good description of individual preference, the target-oriented decision analysis is used to quantify how well a product meets consumers' Kansei preferences. Due to the vagueness and uncertainty of consumers' preferences, three types of fuzzy targets are defined to represent consumers' preferences. Because consumers usually may prioritize Kansei attributes, i.e., a prioritization hierarchy of Kansei attributes, a prioritized OWA aggregation operator is utilized to aggregate the partial degrees of satisfaction for the evaluated products.

In the next chapter, we shall conduct a case study to illustrate how the proposed Kansei evaluation model works in practice.

Chapter 8

Case Study: Kansei Evaluation of Japanese Traditional Crafts

Abstract: Most research of Kansei evaluation focuses on the product design. However, Kansei evaluations of existing products have generally received less attention. Kansei evaluations of existing products would be of great help for marketing or recommendation purposes, and particularly in the era of e-commence. As the aesthetic aspects (brand image, pattern, personal aesthetics, current trends of fashion etc.) play a crucial role in consumers' perceptions of traditional crafts, Kansei information is essential and necessary for this evaluation problem.

The main focus of this chapter is to conduct a case study for Kansei evaluation for Kanazawa gold leaf, one types of Japanese traditional crafts. To do so, a preparative study is conducted first to gather the Kansei assessments of the thirty products of Kanazawa gold leaf. Secondly, the Kansei profiles of the thirty Kanzawa gold leaf were obtained based on the preparative study. Thirdly, based on two types of preferences (Kansei feeling target and prioritization of these targets), we obtained the Kansei evaluation results. From this case study, the consumers can choose their preferred products of Kanazawa gold leaf according to their Kansei preferences.

8.1 Introduction

In an increasing completive world market, it is more and more important for manufactures to have a customer-oriented approach to improve the attractiveness in development of new products, which should not only satisfy requirements of physical quality, defined objectively, but should also satisfy consumers' psychological needs, by essence subjective. This has actually received much attention since the 1970s from the research community of consumer-oriented design and Kansei engineering. As an essential step in Kansei engineering, Kansei evaluation is today well established methodology in product design and commercial available service in Asia. Kansei evaluation has been applied to product design with successful results, e.g., food industry [5], design of mobile phone models [8, 68, 90], telephones [59], tactile sense on surface roughness [33], material selection and design of Chair [123], machine tool design [102], design of battery drills [118], car design [104], design of baby stroller [129], design of table glasses [112].

Most research of Kansei evaluation focuses on the product design. However, Kansei evaluation of existing products have generally received less attention. Kansei evaluation of existing products would be of great help for marketing or recommendation purposes, and particularly in the era of e-commence, where recommendation systems have become an important research area. Therefore, Llinares and Page [92] have analyzed consumers' emotional response to real estate promotions by using Kansei evaluation techniques. And they conducted a case study by using the urban flats in the city of Valencia (Spain).

According to our knowledge, little research of Kansei evaluation for traditional crafts has been addressed yet. As the aesthetic aspect (brand image, pattern, personal aesthetics, current trends of fashion etc.) plays a crucial role in consumers' perceptions of traditional crafts, Kansei information is essential and necessary for this evaluation problem. In this chapter, we focus on Kansei evaluation of traditional crafts. In Japan, there are many traditional crafts such as fittings, textile, etc. These beautiful, elegant and delicate products are closely related to and have played an important role in Japanese culture and life. Evaluations of these traditional crafts would be of great help for marketing or recommendation purposes. In addition, as consumers' preferences on the Kansei attributes of traditional crafts vary from people to people, we shall use the proposed Kansei evaluation model proposed in Chapter 7 by considering consumers' preference information into account.

Based on the above two reasons, the main objectives of this chapter are twofold.

1. To conduct a case study of Kansei evaluation for traditional crafts in Japan,
2. and to demonstrate the effectiveness of the proposed Kansei evaluation model in Chapter 7.

The rest of this chapter is organized as follows. Section 8.2 introduces some background information of traditional crafts in Japan. In Section 8.3 we conduct a preparative study to gather the Kansei assessments of thirty products of Kanazawa gold leaf. In Section 8.4 we evaluate the thirty products of Kanazawa gold leaf by means of the Kansei evaluation model proposed in Chapter 7. In Section 8.5 we give some discussions of this case study. Finally, some concluding remarks are given in Section 8.6.

8.2 Background of Traditional Crafts in Japan

In Japan there are a large number of traditional craft products which are so closely connected to Japanese traditional culture. As explained in the Web site of *The Association for the Promotion of Traditional Craft Industries* [1], each of the traditional craft products in Japan is "unique fostered through regional differences and loving dedication and provides a continual wealth of pleasure". However, due to the rapid chang of lifestyles of younger generations and the prevalence of modern industrial products with their advantage in cost and usage, the market of traditional crafts in Japan has been shrinking over the last recent decades. In 1974, Japanese government (METI) enacted the so-called Densan Law for the "Promotion of Traditional Craft Industries" as quoted below [2]

> Japan has a great number of items for daily use whose development reflects the country's history, environment and lifestyle. Meanwhile, because of the factors such as changing lifestyle and the development of new raw materials, crafts manufactured with traditional methods and materials are having hard times. Under the circumstances, METI enacted the above law in May 1974 with the objective of promoting the traditional crafts industry in order that traditional crafts bring richness and elegance to people's living and contribute to the development of local economy, consequently, the sound development of nation's economy.

In addition, since 1984 METI has designated the month of November as the "Traditional Crafts Month" as well as conducted publicity and educational programs related to traditional crafts throughout Japan. All of these attempts have been not only important from the economic perspective but also particularly important from the cultural perspective in maintaining a spiritual heritage which makes the country unique.

On the other hand, with the fast growing of e-commerce in today's business, the Internet can be of a great help in traditional craft industries. Manufacturers and retailers via their Web sites can make their marketing better as providing more attractive introduction and, hopefully, personalized recommendation, or even helping bring people back to traditional and cultural values concerning their products. In fact, as reported in a recent Reuters' news article on May 20, 2007 by Mayumi Negishi [3], the kimono (described as one of Japan's oldest works of art) market could impressively improve its situation of shrinking to less than half its size in more than last two decades, helped by a host of Web sites where online tips, for instance, on kimono wear and care or on selecting the right pattern of kimono play a role.

Our main concern here is to conduct Kansei evaluation for Japanese traditional craft products by considering consumers' preferences into account. The consumer-oriented evaluation can provide a personalized recommendation for the consumers. A particular focus is put on the Kansei evaluation of traditional craft products in Ishikawa prefecture, Japan, where Japan Advanced Institute of Science and Technology is located. Fig. 8.1 shows the distribution of tractional craft products in Ishikawa prefecture, Japan.

[1] http://www.kougei.or.jp/english/promotion.html

[2] http://www.kansai.meti.go.jp/english/dentousangyou/top_page.html

[3] http://www.reuters.com/article/technologyNews/idUSN2029563220070520

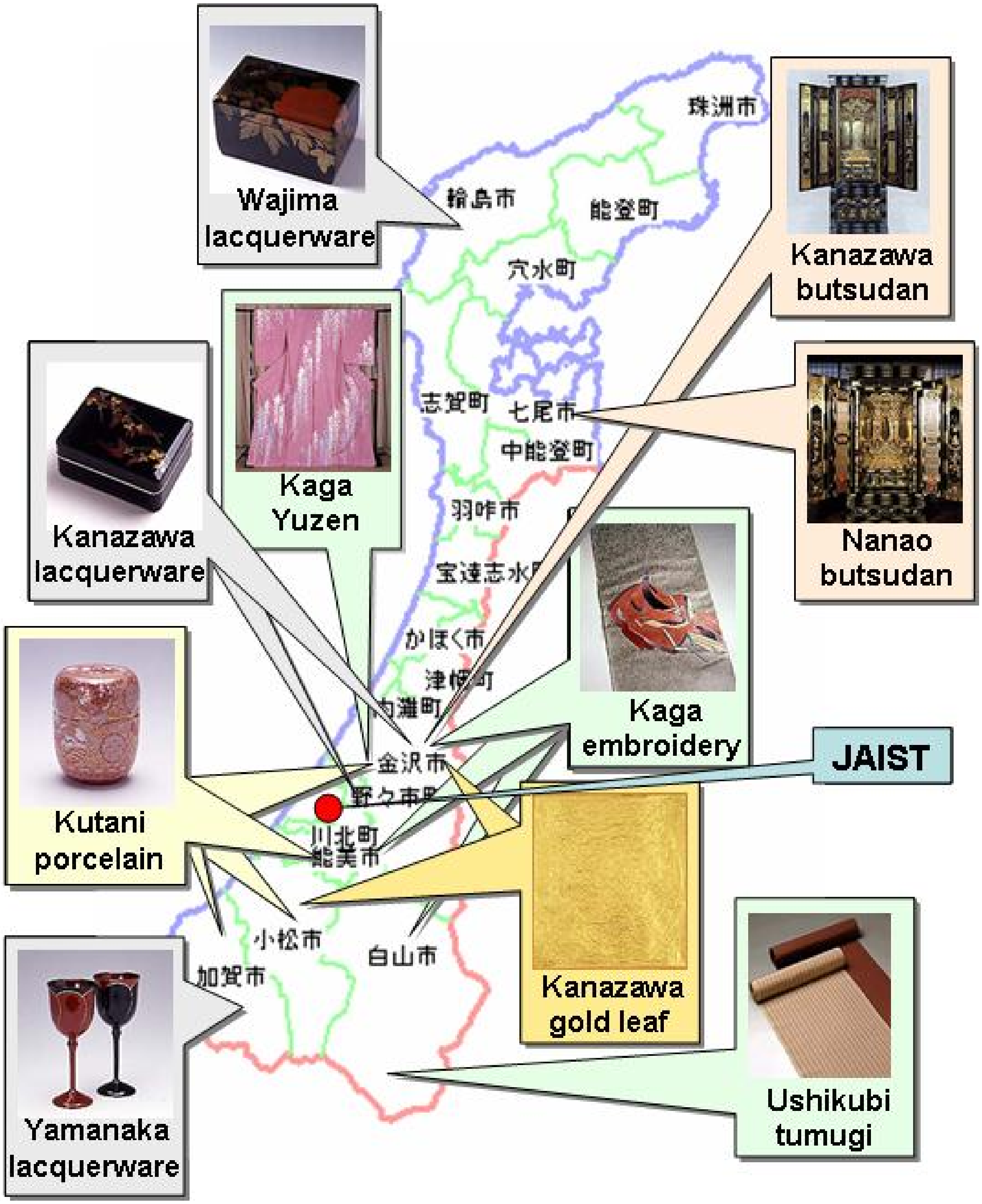

Figure 8.1: Distribution of traditional craft products in Ishikawa, Japan

8.3 A Preparative Study

The aesthetic aspects, such as brand image, pattern, personal aesthetics, current trends of fashion and so on, play a crucial role in consumers' perceptions of traditional crafts, thus Kansei information is essential and necessary for this evaluation problem. In addition, consumers' preference information varies according to their character, feeling, aesthetic and so on.

In this section, we shall conduct a case study for Kansei evaluation of the traditional crafts in Japan to illustrate the Kansei evaluation model proposed in Chapter 7 as well as to provide a personified recommendation for the consumers. The consumers' preferences on Kansei attributes consist of two parts:

1. Consumers' preferences on the relation order on Kansei attributes.
2. Consumers' preferences on the prioritization of Kansei attributes.

8.3.1 Identification of products to be evaluated: Kanazawa gold leaf

We will use the Kanazawa gold leaf, a traditional craft material with a history of over 400 years, as a case study to illustrate the proposed Kansei evaluation model. According to Association for the Promotion of Traditional Craft Industries, the introduction to Tradition Crafts in Japan are as quoted as follows [4]

> The history of Kanazawa gold leaf can be traced back to the latter half of the Sengoku period (1428-1573), when Maeda Toshiie, the feudal lord of the Kaga clan governing the southern part of the area now known as Ishikawa Prefecture, sent a document back to the country from a campaign in Korea, explaining how to produce gold leaf. The Shogunate subsequently set up a gilders' guild and controlled the production and sale of gold leaf throughout the country. After the Meiji Restoration in 1868, however, Kanazawa gold leaf workers took the opportunity on the abolition of governmental control to successfully develop both the techniques and extent of production. Being of such a high quality, Kanazawa maintains its position as the number one center for the production of gold leaf in the country.
>
> The leaf is very thin and in the case of gold leaf is between 0.0001 mm and 0.0002 mm thick. For this reason, it is possible to apply the leaf to materials however complicated the pattern might be. What is more, none of the brilliance of the raw gold is lost at all, and the beauty and splendor of the finished products never cease to captivate the heart of the beholder. It still has a wide range of craft applications in the fields of textiles, lacquer ware, ceramics and on various types of screens, often applied to paper. It is also used on signs and individual carved characters as well as on the mizuhiki decorations for gifts and on the best art mountings. Large amounts of gold leaf in particular are used on household Buddhist altars and on shrine and temple buildings, too. The industry is sustained by 200 firms employing 1,000 staff, among whom they are 26 Master Craftsmen.

Within the framework of a research project supported by the local government, a total of thirty products of Kanazawa gold leaf have been collected for Kansei evaluation, as photographically shown in Fig. 8.2.

[4] http://www.kougei.or.jp/english/crafts/1503/f1503.html

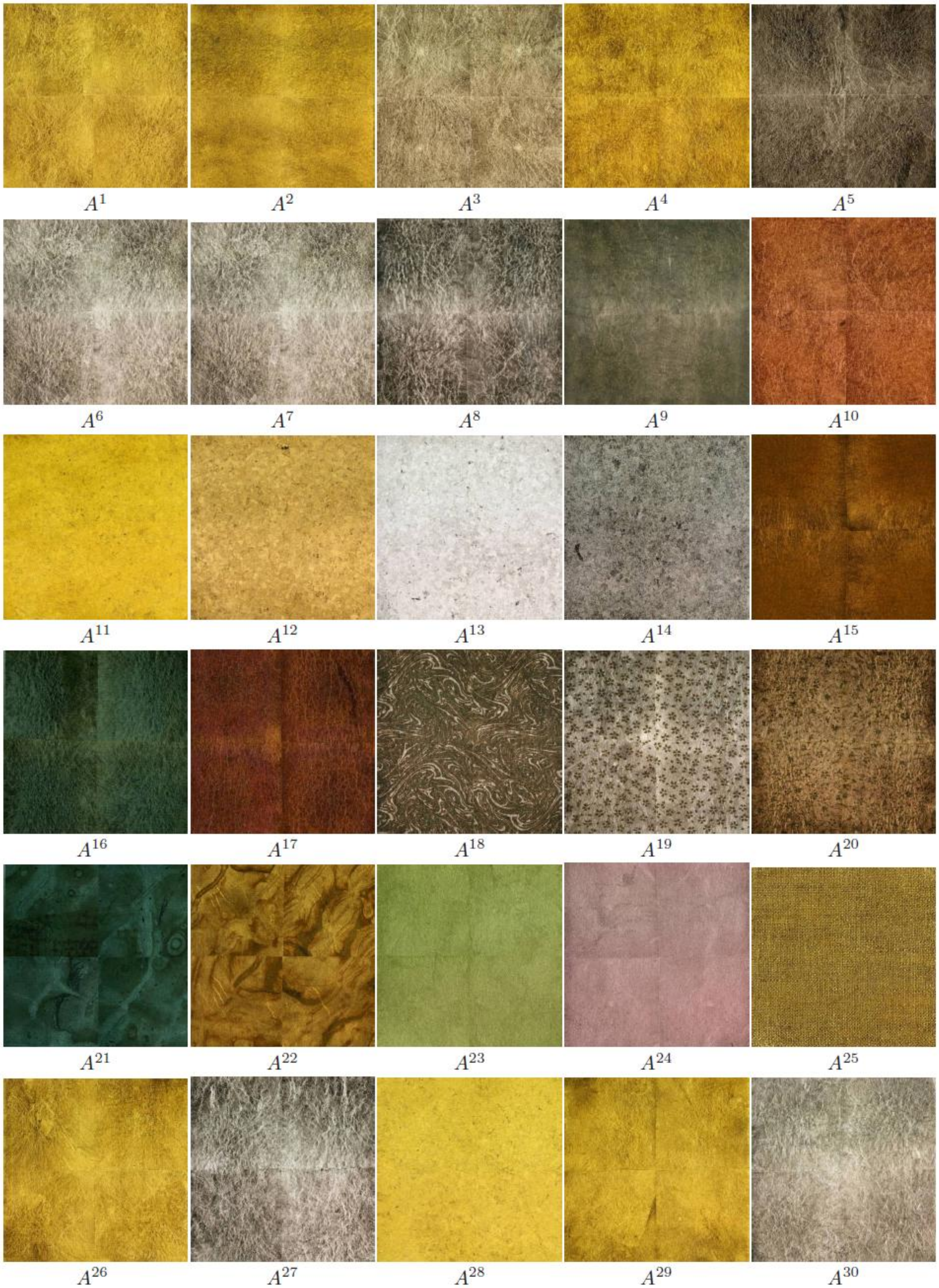

Figure 8.2: Thirty products of Kanazawa gold leaf used for Kansei evaluation

8.3.2 Identification of the subjects

A total of subjects, including relevant researchers of Kansei engineering, senior residents in Kanazawa, and certified masters of traditional crafts, were chosen as subjects. Among these 211 participants, 61.1% (129) were women and 38.9% (82) were men. The distribution of their ages is shown in Table 8.1. The ratio of men and women, as well as the approximate age data of evaluators were considered, trying to match the ratio in the general Japanese population, but the selection of ages from early twenties to latter half of sixties is especially due to those age groups being targeted as potential customers.

Table 8.1: Age distributions of subjects participating in the evaluation process

Age	Number of subject	Percentage
20 to 29	56	26.5%
30 to 39	51	24.2%
40 to 49	51	24.2%
50 to 59	4	1.9%
≥ 60	49	23.2%
Total	211	100%

It should be noted that, in many studies of Kansei engineering, the number of subjects involved in experimental studies usually ranges from 10 to 35 [27, 85, 90, 91]. For purposes of our Kansei evaluation, such a small number of subjects may not provide enough information from various points of view, and may bring a statistical bias. To possibly reduce the subjectiveness of the assessments, a number of subjects, with a larger size, 211, were selected.

8.3.3 Identification and measurement of Kansei attributes

Before gathering Kansei assessment data of the 30 products of Kanazawa gold leaf by the 211 subjects, preliminary research was carried out to select Kansei attributes, by consulting with local manufactures and selling shops. Finally, 26 opposite pairs of Kansei words were selected through a brainstorming process. The bipolar Kansei words for the n-the Kansei attribute are represented as $\mathrm{KW}_n = < \mathrm{KW}_n^-, \mathrm{KW}_n^+ >$, where $n = 1, \cdots, 26$. A 7 point scale was used to put a value each Kanazawa gold leaf with respect to each one of the Kansei attributes. In the literature, the point scale can be 5 point [92, 106], 7 point [102] or 9 point [53]. Choice of the odd point scale depends on specific problems. The smaller the point scale is, the less differential semantics the Kansei attributes have.

As human perceptions are subjective and not objective, therefore the assessment provided by the individuals are vague and uncertain. In such a case, linguistic descriptors are straight direct provided by the subjects to assess Kanazawa gold leaf. The fuzzy linguistic approach [163, 167] provides a systematic way to represent linguistic variables in a natural assessment procedure. It does not require a subject to provide a precise value at which an uncertain factor exists. Thus each Kansei attribute X_n is represented by a 7-scale linguistic term set L_n and triangular fuzzy numbers are used to denote the linguistic variable, such that

$$\begin{aligned} L_n &= \{L_n^{-3}, L_n^{-2}, L_n^{-1}, L_n^0, L_n^1, L_n^2, L_n^3\} \\ &= \{\text{ Very KW}_n^-, \text{KW}_n^-, \text{ Fairly KW}_n^-, \text{ Neutral }, \text{ Fairly KW}_n^+, \text{KW}_n^+, \text{ Very KW}_n^+\} \\ &= \{(-3,-3,-2),(-3,-2,-1),(-2,-1,0),(-1,0,1),(0,1,2),(1,2,3),(2,3,3)\} \end{aligned}$$

Table 8.2 shows the Kansei attributes with linguistic variables and triangular fuzzy numbers, where Kansei words were used in Japanese at first and approximately translated into English in this study.

Table 8.2: Kansei attributes of traditional crafts, shown using linguistic variables and triangular fuzzy numbers.

X_n	Left Kansei word KW_n^-	7-scale Kansei linguistic variable							Right Kansei word KW_n^+
		L_n^{-3}	L_n^{-2}	L_n^{-1}	L_n^0	L_n^{+1}	L_n^{+2}	L_n^{+3}	
X_1	conventional	☐	☐	☐	☐	☐	☐	☐	unconventional
X_2	simple	☐	☐	☐	☐	☐	☐	☐	compound
X_3	solemn	☐	☐	☐	☐	☐	☐	☐	funny
X_4	formal	☐	☐	☐	☐	☐	☐	☐	casual
X_5	serene	☐	☐	☐	☐	☐	☐	☐	forceful
X_6	still	☐	☐	☐	☐	☐	☐	☐	moving
X_7	pretty	☐	☐	☐	☐	☐	☐	☐	austere
X_8	friendly	☐	☐	☐	☐	☐	☐	☐	unfriendly
X_9	soft	☐	☐	☐	☐	☐	☐	☐	hard
X_{10}	blase	☐	☐	☐	☐	☐	☐	☐	attractive
X_{11}	flowery	☐	☐	☐	☐	☐	☐	☐	quiet
X_{12}	happy	☐	☐	☐	☐	☐	☐	☐	normal
X_{13}	elegant	☐	☐	☐	☐	☐	☐	☐	loose
X_{14}	delicate	☐	☐	☐	☐	☐	☐	☐	large-hearted
X_{15}	luxurious	☐	☐	☐	☐	☐	☐	☐	frugal
X_{16}	gentle	☐	☐	☐	☐	☐	☐	☐	pithy
X_{17}	bright	☐	☐	☐	☐	☐	☐	☐	dark
X_{18}	reserved	☐	☐	☐	☐	☐	☐	☐	imperious
X_{19}	free	☐	☐	☐	☐	☐	☐	☐	regular
X_{20}	level	☐	☐	☐	☐	☐	☐	☐	indented
X_{21}	lustered	☐	☐	☐	☐	☐	☐	☐	matte
X_{22}	transpicuous	☐	☐	☐	☐	☐	☐	☐	dim
X_{23}	warm	☐	☐	☐	☐	☐	☐	☐	cool
X_{24}	moist	☐	☐	☐	☐	☐	☐	☐	arid
X_{25}	colorful	☐	☐	☐	☐	☐	☐	☐	sober
X_{26}	plain	☐	☐	☐	☐	☐	☐	☐	gaudy, loud

8.3.4 Gathering Kansei assessments

Having identified the subjects and Kansei attributes, the next step is to gather the Kansei profiles of the thirty products of Kanazawa gold leaf. A total 211 population of subjects were invited to assess the thirty products of Kanazawa gold leaf in a simultaneous way. Each subject was given an answer sheet to rate the Kansei data for each Kansei attribute regarding each product. A sample answer sheet is given in Fig. 8.3 and the simultaneous process is shown in Fig. 8.4.


まず最初に素材のサンプル番号をご記入ください。 は－2

氏名記入欄 92.

No.		3	2	1	0	1	2	3	
1.	定番の								斬新な
2.	シンプルな								複雑な
3.	おごそかな								楽しげな
4.	フォーマルな								カジュアルな
5.	穏やかな								勢いのある
6.	静かな								にぎやかな
7.	かわいらしい								渋い
8.	親しみやすい								よそよそしい
9.	やわらかい								かたい
10.	飽きやすい								飽きのこない
11.	華やかな								落ち着いた
12.	めでたい								日常の
13.	品のある								品のない
14.	繊細な								豪快な
15.	豪華な								質素な
16.	やさしい								力強い
17.	あかるい								くらい
18.	なごやかな								凛とした
19.	自由な								規則的な
20.	なめらかな								凹凸のある
21.	つやのある								つやのない
22.	透明感のある								くすんでいる
23.	あたたかな								ひんやりした
24.	みずみずしい								乾いた
25.	派手な								地味な
26.	あっさりした								こってりした

★素材の番号は入っていますか？記入漏れはありませんか？

Figure 8.3: A sample of the answer sheet in Japanese

Figure 8.4: Gathering data for evaluation of Kanazawa gold leaf

8.4 Consumer-Oriented Kansei Evaluation of Kanazawa Gold Leaf

From now on, we shall evaluate the thirty products of Kanazawa gold leaf based on the Kansei evaluation model proposed in Chapter 7.

8.4.1 Generation of Kansei profiles

Having obtained the Kansei assessments given by the subjects, we can obtain Kansei profiles as follows. For evaluated product $A^m, m = 1, 2, \cdots, 30$, we define for Kansei attribute X_n, $n = 1, 2, \cdots, 26$, a probability distribution function $p_n^m : L_n \rightarrow [0, 1]$ as follows

$$p_n^m(L_n^k) = \frac{|\{S_j \in \mathcal{S} : X_n^m(S_j) = L_n^k\}|}{|\mathcal{S}|}$$

where $k = -3, -2, -1, 0, 1, 2, 3$, and $X_n^m(S_j)$ denotes the Kansei assessment for product A^m with respect to Kansei attribute X_n given by subject $S_j, j = 1, \cdots, 211$. In the same way, we can obtain a 7-tuple of probability distributions for product A^m with respect to Kansei attribute X_n, such that

$$p_n^m = \left[p_n^m(L_n^{-3}), p_n^m(L_n^{-2}), p_n^m(L_n^{-1}), p_n^m(L_n^{0}), p_n^m(L_n^{+1}), p_n^m(L_n^{+2}), p_n^m(L_n^{+3})\right].$$

For example, the Kansei profile of Kanazawa gold leaf A^1 is graphically shown in Fig. 8.5.

8.4.2 Specification of consumers' preferences

Now let us consider the consumer-oriented evaluation for this problem. Assume that a consumer has selected seven Kansei attributes she/he cares about, such that $\mathcal{X} = \{X_4, X_{10}, X_{11}, X_{15}, X_{21}, X_{25}, X_{26}\}$. The seven Kansei attributes selected by the consumer and their corresponded bipolar Kansei words are shown from Column 1 to 2 in Table 8.3.

We further assume that the consumer specifies seven targets according to the seven selected Kansei attributes, denoted as $T = (T_4, T_{10}, T_{11}, T_{15}, T_{21}, T_{25}, T_{26})$. The targets specified by the consumer consist of two parts: semantics and fuzzy values. There are three types of target preferences in this study: left Kansei word preferred, neutral preferred, and right Kansei word preferred. Triangular fuzzy numbers are used to represent these three targets. Table 8.3 shows the seven targets, their associated semantics and fuzzy values, respectively.

In addition, the consumer may have a prioritization of the seven targets/Kansei attributes. We also assume that the seven Kansei attributes are partitioned into 3 distinct priority levels H_1, H_2, H_3, where $H_1 \succ H_2 \succ H_3$, as shown in Table 8.4. The attributes are re-denoted according to their priority levels. For example, attribute X_4 is the first one in priority level H_1, thus we use $X_{11}^{'}$ to denote X_4. In this prioritization hierarchy structure, attributes X_4 and X_{26} have the highest priority level, i.e., the consumer is not willing to trade off satisfaction with the Kansei attributes in hierarchy level H_2 and H_3, until she/he has attained some level of satisfaction regarding X_4 and X_{26}.

Based on these two types of preferences, we divide our evaluation procedure into two steps: calculation of target achievements regarding each Kansei attribute and prioritized aggregation. In the following, we shall discuss these two steps in great detail.

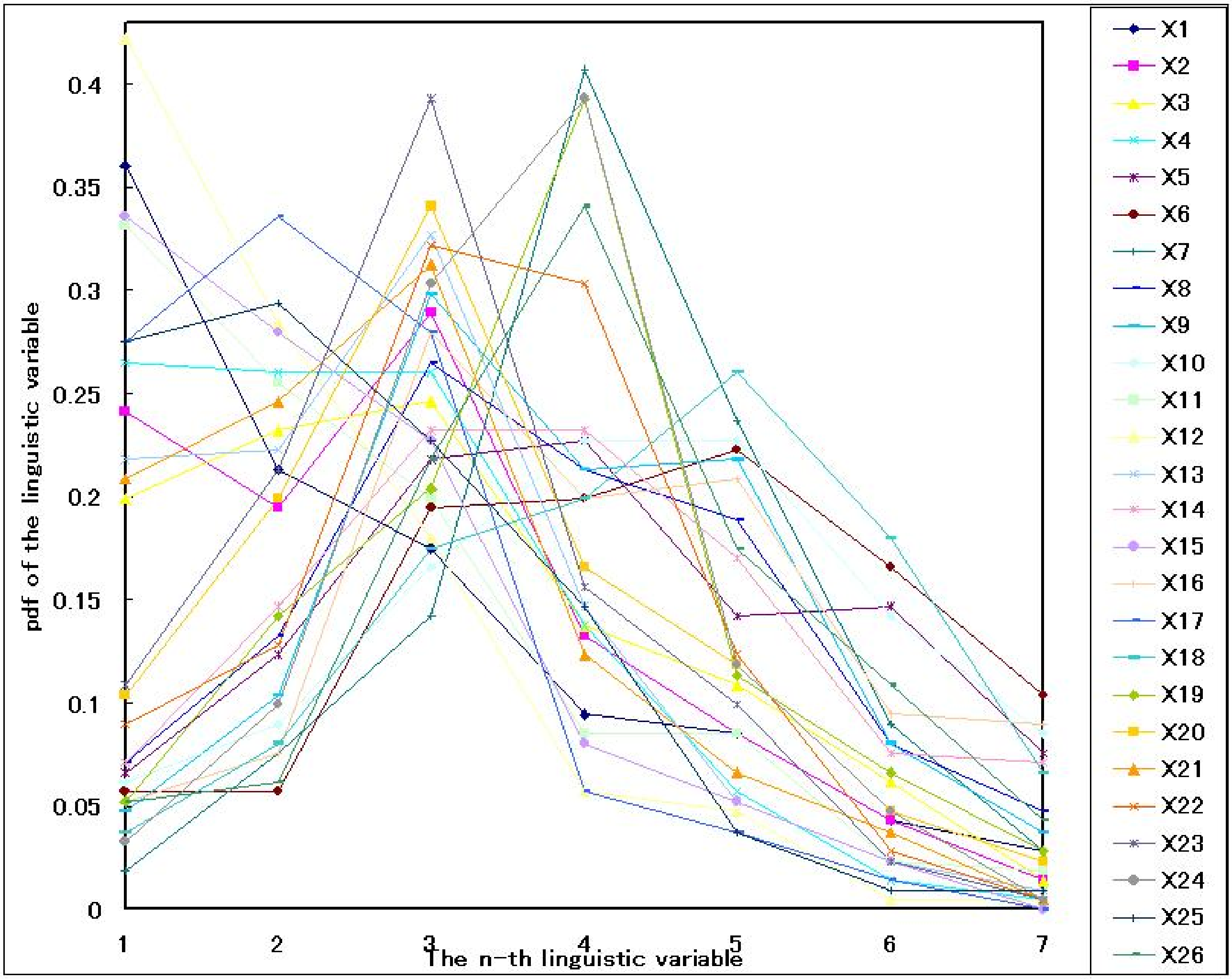

Figure 8.5: Kansei profile of Kanazawa gold leaf A^1

Table 8.3: The preferred seven Kansei attributes and their corresponded targets

Kansei attributes	Bipolar Kansei words	Target	Semantics of targets	Fuzzy values of targets
X_4	<formal, casual>	T_4	Neutral preferred	(-3,0,3)
X_{10}	<blase, attractive>	T_{10}	Attractive preferred	(-3,3,3)
X_{11}	<flowery, quiet>	T_{11}	Flowery preferred	(-3,-3,3)
X_{15}	<luxurious, frugal>	T_{15}	Luxurious preferred	(-3,-3,3)
X_{21}	<lustered, matte>	T_{21}	Matte preferred	(-3,3,3)
X_{25}	<colorful, sober>	T_{25}	Colorful preferred	(-3,-3,3)
X_{26}	<plain, gaudy>	T_{26}	Neutral preferred	(-3,0,3)

Table 8.4: Prioritization of the seven Kansei attributes/feeling targets

H_1	$X_4(X'_{11}), X_{26}(X'_{12})$
H_2	$X_{11}(X'_{21}), X_{15}(X'_{22}), X_{21}(X'_{23})$
H_3	$X_{10}(T'_{31}), X_{25}(X'_{32})$

8.4.3 Calculation of target achievements

According to Chapter 7, we can obtain the probability of product A^m meeting fuzzy feeling target T_n with respect to Kansei attribute X_n by meas of the following function

$$\Pr_n^m = \Pr(X_n^m \succeq T_n) = \sum_{k=-3}^{3} p_n^m(L_n^k) \cdot \left[\Pr(L_n^k \succeq T_n)\right] \tag{8.1}$$

where $p_n^m(L_n^k)$ is the probability distribution function of Kansei linguistic variable L_n^k.

Taking product A^1 and Kansei attribute X_4 as an example. According to Kansei profile of product A^1 in Fig. 8.5, we know that the probability distribution of each Kansei linguistic variable with respect to Kansei attribute X_4 is as follows.

$$\begin{aligned} p_4^1 &= \left[p_4^1(L_4^{-3}), p_4^1(L_4^{-2}), p_4^1(L_4^{-1}), p_4^1(L_4^{0}), p_4^1(L_4^{+1}), p_4^1(L_4^{+2}), p_4^1(L_4^{+3})\right] \\ &= [0.2654, 0.2607, 0.2607, 0.1374, 0.0569, 0.0142, 0.0047] \end{aligned} \tag{8.2}$$

We also know that the linguistic variables for Kansei attribute X_4 are

$$\begin{aligned} L_4 &= \left[L_4^{-3}, L_4^{-2}, L_4^{-1}, L_4^{0}, L_4^{1}, L_4^{2}, L_4^{3})\right] \\ &= \{\text{Very formal}, \text{Formal}, \text{Fairly formal}, \text{Neutral}, \text{Fairly casual}, \text{Casual}, \text{Very casual}\} \\ &= \{(-3,-3,-2), (-3,-2,-1), (-2,-1,0), (-1,0,1), (0,1,2), (1,2,3), (2,3,3)\} \end{aligned} \tag{8.3}$$

The feeling target T_4 specified by the consumer is a neutral preferred target, denoted as $(-3, 0, 3)$. $\Pr(L_4^k \succeq T_4)$ can be obtained according to the extended target-oriented decision model with respect to equal target preference discussed in Chapter 7 as follows.

$\Pr(L_4^{-3} \succeq T_4)$	$\Pr(L_4^{-2} \succeq T_4)$	$\Pr(L_4^{-1} \succeq T_4)$	$\Pr(L_4^{0} \succeq T_4)$	$\Pr(L_4^{1} \succeq T_4)$	$\Pr(L_4^{2} \succeq T_4)$	$\Pr(L_4^{3} \succeq T_4)$
0.0185	0.1296	0.4630	0.7963	0.4630	0.1296	0.0185

According to Eq. 8.1, we can obtain the probability $\Pr_n^m$ of Kansei attribute X_4 meeting target T_4 regarding product A^1 as

$$\begin{aligned} \Pr_4^1 &= \sum_{k=-3}^{3} p_4^1(L_4^k) \cdot \left[\Pr(L_4^k \succeq T_4)\right] \\ &= 0.2654 * 0.0185 + 0.2607 * 0.1296 + 0.2607 * 0.4630 + \\ &\quad 0.1374 * 0.7963 + 0.0569 * 0.4630 + 0.0142 * 0.1296 + 0.0047 * 0.0185 \\ &= 0.2971 \end{aligned}$$

Similarly, we can obtain the probabilities of other products meeting the seven targets, as shown in Table 8.5.

Table 8.5: Probabilities of each Kansei attribute meeting the target with respect to the thirty products of Kanazawa gold leaf

Products	Probability of each Kansei attribute meeting target						
	T_4	T_{10}	T_{11}	T_{15}	T_{21}	T_{25}	T_{26}
	X_4	X_{10}	X_{11}	X_{15}	X_{21}	X_{25}	X_{26}
A^1	0.2971	0.3579	0.5965	0.6249	0.1371	0.5948	0.4777
A^2	0.2750	0.3915	0.5728	0.6052	0.1259	0.5690	0.4761
A^3	0.3724	0.3943	0.3714	0.4328	0.1350	0.3924	0.4703
A^4	0.2823	0.3702	0.6327	0.6848	0.1111	0.6514	0.4603
A^5	0.3687	0.3409	0.3194	0.3800	0.2113	0.2881	0.4919
A^6	0.3239	0.4107	0.4519	0.5487	0.1440	0.4647	0.4798
A^7	0.3629	0.3795	0.3243	0.3824	0.1839	0.3018	0.4451
A^8	0.3666	0.3484	0.3497	0.4072	0.1898	0.3508	0.4619
A^9	0.3603	0.4140	0.2218	0.3727	0.2168	0.2606	0.4719
A^{10}	0.3877	0.3247	0.3901	0.4322	0.1584	0.4268	0.5056
A^{11}	0.3345	0.3854	0.5815	0.5904	0.1507	0.5631	0.4803
A^{12}	0.3555	0.3857	0.4800	0.5293	0.1789	0.4937	0.5135
A^{13}	0.3492	0.3854	0.2562	0.3369	0.2829	0.2947	0.4482
A^{14}	0.3808	0.3819	0.2928	0.4034	0.2185	0.3292	0.4761
A^{15}	0.3866	0.3215	0.3977	0.4516	0.1767	0.4443	0.4640
A^{16}	0.3703	0.3526	0.1613	0.2392	0.1980	0.2088	0.4735
A^{17}	0.3871	0.2737	0.3051	0.3458	0.2535	0.3420	0.4735
A^{18}	0.3329	0.1944	0.2882	0.3497	0.3081	0.3246	0.3455
A^{19}	0.2881	0.3441	0.4014	0.3241	0.2010	0.3352	0.5161
A^{20}	0.2529	0.3509	0.3597	0.3054	0.2256	0.3276	0.5146
A^{21}	0.3582	0.2455	0.2890	0.3684	0.2115	0.3337	0.4314
A^{22}	0.3382	0.2260	0.4740	0.5587	0.1598	0.5821	0.3166
A^{23}	0.3050	0.2948	0.4366	0.4096	0.1503	0.4772	0.4661
A^{24}	0.2529	0.3486	0.5639	0.4425	0.1335	0.5162	0.4798
A^{25}	0.2876	0.3158	0.3592	0.3887	0.5311	0.3654	0.4635
A^{26}	0.2834	0.3771	0.5974	0.6304	0.1178	0.6117	0.4840
A^{27}	0.3476	0.4248	0.3424	0.4201	0.1618	0.3609	0.4509
A^{28}	0.3224	0.3740	0.5682	0.6067	0.1447	0.5637	0.4703
A^{29}	0.2792	0.3967	0.6098	0.6462	0.1189	0.6140	0.4593
A^{30}	0.3260	0.3953	0.3058	0.4268	0.1763	0.3529	0.4645

8.4.4 Prioritized aggregation of target achievements

To illustrate the prioritized aggregation process, we shall take product A^1 as an example. According to prioritization of the seven targets in Table 8.4 and the partial satisfaction degrees in Table 8.5, we can obtain the target achievements with respect to the seven predefined feeling targets regarding product A^1 as follows

For priority level H_1: $\mathrm{Pr}_4^1 = 0.2971, \mathrm{Pr}_{26}^1 = 0.4777$

For priority level H_2: $\mathrm{Pr}_{11}^1 = 0.5965, \mathrm{Pr}_{15}^1 = 0.6249, \mathrm{Pr}_{21}^1 = 0.1371$

For priority level H_3: $\mathrm{Pr}_{10}^1 = 0.3579, \mathrm{Pr}_{25}^1 = 0.5948$

Assume that the consumer specifies her/his attitudinal character $\Omega = 0.5$. In this case, the consumer prefers an averaging tradeoffs between different target achievements of targets. Here, the attitudinal character can be related to the linguistic quantifier for determining the weights. The OWA weighting vectors for priority hierarchy $H_q, q = 1, 2, 3$ are

For priority level H_1: $U_1 = (0.5, 0.5)$

For priority level H_2: $U_2 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

For priority level H_3: $U_3 = (0.5, 0.5)$

Then the prioritized OWA aggregation is proceeded as follows:

1. To calculate a degree of satisfaction for each priority hierarchy level:
   $\mathtt{Sat}_1^1 = \mathrm{OWA}_{0.5}(0.2971, 0.4777) = 0.3874$

   $\mathtt{Sat}_2^1 = \mathrm{OWA}_{0.5}(0.5965, 0.6249, 0.1371) = 0.4528$

   $\mathtt{Sat}_3^1 = \mathrm{OWA}_{0.5}(0.3579, 0.5948) = 0.4764$

2. To calculate the induced priority weight for each priority hierarchy level by means of product t-norm:
   $Z_1^1 = Z_0^1 * \mathtt{Sat}_0^1 = 1.0;$

   $Z_2^1 = Z_1^1 * \mathtt{Sat}_1^1 = 0.3874;$

   $Z_3^1 = Z_2^1 * \mathtt{Sat}_2^1 = 0.1754$

3. To calculate the global value of satisfaction for product A^1:

$$\mathrm{Val}(A^1) = Z_1^1 * \mathtt{Sat}_1^1 + Z_2^1 * \mathtt{Sat}_2^1 + Z_3^1 * \mathtt{Sat}_3^1 = 0.6464$$

Similarly, we can obtain the aggregation values for other products with different attitudinal characters. In this study, we used 11 Ω values such that

$$\Omega = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0].$$

Table 8.6 shows the ranking list of the top 5 products with 11 attitudinal characters.

Table 8.6: Top 5 products under prioritized aggregation with 11 attitudinal characters Ω

Attitudinal character Ω	Ranking order of top 5 Kanazawa gold leaf
$\Omega = 0.0$	$A^{17} \succ A^{14} \succ A^{15} \succ A^{5} \succ A^{10}$
$\Omega = 0.1$	$A^{17} \succ A^{14} \succ A^{15} \succ A^{10} \succ A^{5}$
$\Omega = 0.2$	$A^{17} \succ A^{10} \succ A^{12} \succ A^{15} \succ A^{14}$
$\Omega = 0.3$	$A^{12} \succ A^{10} \succ A^{15} \succ A^{11} \succ A^{17}$
$\Omega = 0.4$	$A^{12} \succ A^{11} \succ A^{10} \succ A^{28} \succ A^{15}$
$\Omega = 0.5$	$A^{12} \succ A^{11} \succ A^{28} \succ A^{10} \succ A^{1}$
$\Omega = 0.6$	$A^{12} \succ A^{11} \succ A^{1} \succ A^{26} \succ A^{4}$
$\Omega = 0.7$	$A^{12} \succ A^{11} \succ A^{26} \succ A^{4} \succ A^{1}$
$\Omega = 0.8$	$A^{4} \succ A^{26} \succ A^{1} \succ A^{11} \succ A^{12}$
$\Omega = 0.9$	$A^{4} \succ A^{26} \succ A^{1} \succ A^{29} \succ A^{11}$
$\Omega = 1.0$	$A^{4} \succ A^{26} \succ A^{1} \succ A^{29} \succ A^{2}$

8.5 Analysis of Obtained Results and Discussions

In this section, we shall analyze the obtained results from two aspects: calculation of satisfaction degree for each Kansei attribute and prioritized aggregation. And then give some discussions on how to link the model with practice.

8.5.1 On probability of each Kansei attribute meeting target

In this case study, we used the following function to obtain the probability of Kansei attribute X_n meeting consumer's fuzzy feeling target T_n with respect to product A^m

$$\mathrm{Pr}_n^m = \mathrm{Pr}(X_n^m \succeq T_n) = \sum_{k=-3}^{3} p_n^m(L_n^k) \cdot \left[\mathrm{Pr}(L_n^k \succeq T_n)\right]$$

In this function, $p_n^m(L_n^k)$ is the probability distribution function of Kansei linguistic variable L_n^k, L_n^k is used to represent the uncertain assessments provided by the subjects, and T_n is used to represent consumer's uncertain feeling target according to her/his preference on Kansei attribute X_n. The current function can capture consumers' preference information and uncertainty of assessments. Once the consumer has specified her/his feeling targets, the probabilities of meeting these targets depend on the provability distribution $p_n^m(L_n^k)$ of the seven linguistic variables. In addition, the target achievements are obtain by the target-oriented decision model, which focuses on whether the attribute consequence meets consumer' feeling target.

Compared with related research of Kansei evacuation, our Kansei evaluation model can not only capture the uncertain assessments by the population of subjects via voting statistics, but also can capture the uncertain assessments by each subject via fuzzy linguistic variables. Some researchers try to use linguistic 2-tuple model [56] to do Kansei evaluation problems, e.g. Martínez [98]. However, as we already mentioned in Chapter 7, the consumers may have inconsistent preference order relations, thus the sensory evaluation model based on linguistic 2-tuple model is limited. Some researcher also used the utility theory to obtain the satisfaction degrees for different Kansei attributes, e.g. [19, 67, 69, 80, 100, 115, 169]. In this case, the consumer must define a utility function for each Kansei attribute, which is very complex and difficult in real applications. In this regard, consumers' feeling target toward each Kansei attribute is a straight way for the consumer to express her/his preference information.

In real applications, we can do some questionnaire to collect consumers' feeling targets toward different Kansei attributes. Another possible solution is to use the rating techniques in recommender systems.

8.5.2 On prioritized aggregation

To discuss the aggregation phase, we first analyze the current prioritization structure and then consider two special cases.

Analysis of current prioritization hierarchy

Under current prioritization hierarchy as shown in Table 8.4, Kansei attributes X_4 and X_{26} have the highest priority. In this case, the consumer prefers meeting her/his feeling targets T_4 and T_{26} first. If these two targets are achieved in some level, then she/he will consider other targets. From the prioritized aggregation phase in Subsection 8.4.4, we know that lower satisfaction degree of higher priority Kansei attributes will induce lower priority weights for the attributes in lower priority level. The induced priority weights are product dependent. This is the fundamental feature of prioritized OWA aggregation operator discussed in Chapter 6.

As attributes X_4 and X_{26} are in priority level H_1, there exists some tradeoffs between the target achievement of their associated targets. In general, importance weight plays an important role in overseeing the tradeoffs. As specify importance weights for different attribute/targets is time-consuming, the OWA operator is used to aggregate the target achievements of Kansei attributes in the same priority level. With different attitudinal character Ω, we can obtain different aggregated values. Table 8.7 shows the prioritized aggregation with eleven attitudinal characters for the thirty Kanazawa gold leaf.

Assume that the consumer specifies $\Omega = 0$. In this case, the consumer prefers the most pessimistic attitudinal, i.e. she/he prefers that "all" type of aggregation in the same priority level. When $\Omega = 1.0$, the consumer prefers "exist one" type of aggregation.

It is clearly that the prioritized OWA aggregation allows the tradeoffs between Kansei attributes in the same priority level and does not allow tradeoffs between Kansei attributes in different priority levels. In the literature of Kansei evaluation, usually the importance weighted aggregation is used. In this sense, the prioritized OWA operator is a better solution for the aggregation in Kansei evaluation problems.

Table 8.7: Prioritized aggregation with 11 attitudinal characters for Kanazawa gold leaf

Products	Prioritized aggregations with 11 attitudinal characters Ω										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A^1	0.3524	0.4099	0.4669	0.5251	0.5850	0.6464	0.7092	0.7730	0.8367	0.8987	0.9538
A^2	0.3231	0.3801	0.4370	0.4953	0.5554	0.6172	0.6805	0.7447	0.8091	0.8719	0.9283
A^3	0.4424	0.4767	0.5095	0.5416	0.5735	0.6052	0.6366	0.6677	0.6982	0.7276	0.7542
A^4	0.3253	0.3854	0.4455	0.5075	0.5718	0.6385	0.7073	0.7776	0.8485	0.9181	0.9809
A^5	0.4691	0.4952	0.5214	0.5479	0.5748	0.6020	0.6297	0.6577	0.6860	0.7144	0.7426
A^6	0.3898	0.4357	0.4813	0.5274	0.5744	0.6223	0.6711	0.7204	0.7700	0.8191	0.8654
A^7	0.4498	0.4731	0.4959	0.5188	0.5417	0.5648	0.5880	0.6114	0.6347	0.6577	0.6799
A^8	0.4604	0.4873	0.5133	0.5391	0.5649	0.5907	0.6164	0.6421	0.6676	0.6925	0.7160
A^9	0.4587	0.4765	0.4966	0.5184	0.5417	0.5666	0.5930	0.6211	0.6513	0.6840	0.7206
A^{10}	0.4690	0.5060	0.5415	0.5768	0.6121	0.6475	0.6829	0.7182	0.7531	0.7868	0.8174
A^{11}	0.4043	0.4597	0.5127	0.5656	0.6187	0.6722	0.7259	0.7794	0.8318	0.8814	0.9236
A^{12}	0.4437	0.4919	0.5390	0.5862	0.6340	0.6823	0.7311	0.7801	0.8288	0.8762	0.9195
A^{13}	0.4651	0.4819	0.4994	0.5174	0.5358	0.5548	0.5741	0.5940	0.6144	0.6355	0.6574
A^{14}	0.4914	0.5124	0.5345	0.5574	0.5811	0.6056	0.6309	0.6570	0.6841	0.7121	0.7416
A^{15}	0.4769	0.5082	0.5382	0.5677	0.5971	0.6265	0.6558	0.6850	0.7137	0.7415	0.7667
A^{16}	0.4425	0.4591	0.4762	0.4937	0.5115	0.5297	0.5482	0.5672	0.5866	0.6064	0.6267
A^{17}	0.5121	0.5293	0.5467	0.5642	0.5820	0.6000	0.6182	0.6367	0.6554	0.6743	0.6933
A^{18}	0.4475	0.4523	0.4574	0.4627	0.4682	0.4739	0.4798	0.4859	0.4921	0.4987	0.5056
A^{19}	0.3655	0.4042	0.4437	0.4843	0.5259	0.5685	0.6122	0.6567	0.7022	0.7483	0.7946
A^{20}	0.3286	0.3684	0.4091	0.4506	0.4930	0.5363	0.5805	0.6255	0.6713	0.7177	0.7646
A^{21}	0.4525	0.4696	0.4871	0.5050	0.5234	0.5422	0.5615	0.5812	0.6015	0.6221	0.6433
A^{22}	0.3786	0.4058	0.4319	0.4578	0.4839	0.5101	0.5365	0.5629	0.5891	0.6144	0.6371
A^{23}	0.3644	0.4038	0.4427	0.4821	0.5222	0.5631	0.6045	0.6464	0.6882	0.7291	0.7668
A^{24}	0.2984	0.3472	0.3979	0.4512	0.5071	0.5657	0.6270	0.6907	0.7565	0.8236	0.8901
A^{25}	0.4236	0.4536	0.4855	0.5190	0.5539	0.5902	0.6281	0.6676	0.7090	0.7526	0.7996
A^{26}	0.3294	0.3893	0.4493	0.5111	0.5749	0.6409	0.7087	0.7778	0.8472	0.9150	0.9758
A^{27}	0.4242	0.4535	0.4824	0.5115	0.5409	0.5706	0.6006	0.6310	0.6614	0.6916	0.7207
A^{28}	0.3865	0.4401	0.4924	0.5449	0.5983	0.6524	0.7073	0.7625	0.8172	0.8699	0.9165
A^{29}	0.3255	0.3837	0.4414	0.5005	0.5614	0.6240	0.6881	0.7532	0.8184	0.8818	0.9383
A^{30}	0.4038	0.4331	0.4633	0.4944	0.5266	0.5599	0.5941	0.6294	0.6657	0.7030	0.7412

Analysis of other prioritization cases

In previous examples, we consider a prioritization hierarchy as shown in Table 8.4. Now let us consider another two special cases.

1. Only one priority level
   If the consumer has specified the same seven Kansei attributes and the does not specify the priority hierarchy of the seven Kansei attributes, i.e. all the Kansei attributes are in the same priority level such that

$$H_1 = \{X_4, X_{10}, X_{11}, X_{15}, X_{21}, X_{25}, X_{26}\}.$$

   It means that the consumer allows a tradeoffs between the seven Kansei attributes/targets. In this case, the prioritized OWA aggregation reduces to the original OWA aggregation operator. The OWA aggregated values with different attitudinal characters are shown in Table 8.8. Assume that the consumer specifies $\Omega = 0$, then the aggregation mode is in fact

$$\text{Val}(A^m) = \min[\text{Pr}_4^m, \text{Pr}_{10}^m, \text{Pr}_{11}^m, \text{Pr}_{15}^m, \text{Pr}_{21}^m, \text{Pr}_{25}^m, \text{Pr}_{26}^m]$$

Table 8.8: Top 5 Kanazawa gold leaf under OWA aggregation with 11 attitudinal characters

Ω	Weighting vector	Top 5 Kanazawa gold leaf
$\Omega = 0.0$	(0,0,0,0,0,0,1)	$A^{25} \succ A^{13} \succ A^{17} \succ A^{20} \succ A^{14}$
$\Omega = 0.1$	(0.0018,0.0048,0.0127,0.0336,0.0890,0.2354,0.6227)	$A^{25} \succ A^{13} \succ A^{17} \succ A^{12} \succ A^{14}$
$\Omega = 0.2$	(0.0155,0.0271,0.0472,0.0822,0.1433,0.2497,0.4351)	$A^{25} \succ A^{12} \succ A^{11} \succ A^{28} \succ A^{13}$
$\Omega = 0.3$	(0.0439,0.0608,0.0842,0.1166,0.1614,0.2236,0.3096)	$A^{12} \succ A^{11} \succ A^{25} \succ A^{28} \succ A^{1}$
$\Omega = 0.4$	(0.0862,0.1005,0.1171,0.1364,0.1589,0.1852,0.2158)	$A^{11} \succ A^{4} \succ A^{29} \succ A^{1} \succ A^{28}$
$\Omega = 0.5$	(0.1429,0.1429,0.1429,0.1429,0.1429,0.1429,0.1429)	$A^{4} \succ A^{29} \succ A^{26} \succ A^{1} \succ A^{11}$
$\Omega = 0.6$	(0.2158,0.1852,0.1589,0.1364,0.1171,0.1005,0.0862)	$A^{4} \succ A^{29} \succ A^{26} \succ A^{1} \succ A^{11}$
$\Omega = 0.7$	(0.3096,0.2236,0.1614,0.1166,0.0842,0.0608,0.04399	$A^{4} \succ A^{29} \succ A^{26} \succ A^{1} \succ A^{11}$
$\Omega = 0.8$	(0.4351,0.2497,0.1433,0.0822,0.0472,0.0271,0.0155)	$A^{4} \succ A^{29} \succ A^{26} \succ A^{1} \succ A^{2}$
$\Omega = 0.9$	(0.6227,0.2354,0.0890,0.0336,0.0127,0.0048,0.0018)	$A^{4} \succ A^{29} \succ A^{26} \succ A^{1} \succ A^{2}$
$\Omega = 1.0$	(1,0,0,0,0,0,0)	$A^{4} \succ A^{29} \succ A^{26} \succ A^{1} \succ A^{28}$

2. Strict priority case
   There exists another special case where only one Kansei attribute/feeling target in each priority level is considered, i.e. the consumer does not need the tradeoffs

between the target achievements with respect to different Kansei attributes. In this case, the attitudinal character Ω will not affect the aggregation value for each priority level, thus the aggregation results depend only upon priority hierarchy of the seven Kansei attributes. For purposes of simplicity, we assume that the prioritization has been made in order of the index of Kansei attributes, denoted as

$$X_4 \succ X_{10} \succ X_{11} \succ X_{15} \succ X_{21} \succ X_{25} \succ X_{26}.$$

Then the ranking list of the top 5 Kanazawa gold leaf that best meet consumer's preferences is:

$$A^{12} \succeq A^3 \succeq A^{11} \succeq A^{14} \succeq A^{10}.$$

8.5.3 Discussions

In this study, a preparative study is conducted first to gather the Kansei assessments for the thirty products of Kanazawa gold leaf. Although it is difficult and time-consuming to gather the Kansei assessments, the preparative study is a common step in Kansei engineering. In Kansei engineering, Kansei evaluation is an essential step. As Kansei engineering can be used by designers as a design aid to develop products that are able to match consumers' Kansei. In this sense, this case study provides a possible solution to consumer-oriented design in Kansei engineering research for the Kanzawa gold leaf.

On the other hand, as Kansei engineering can be used by consumers to select products based on their Kansei requirements, this case study also provides a possible solution for the recommender systems. Japanese traditional crafts usually have the following properties: low purchasing frequency, high price, high involvement in selection of preferred crafts. As the aesthetic aspect (brand image, pattern, personal aesthetics, current trends of fashion etc.) plays a crucial role in consumers' perceptions of traditional crafts, Kansei information is essential and necessary for the consumers. Thus a preparative study is quite necessary in choice of traditional crafts. In fact, in recommender systems, product rating techniques have been well developed. We can use the rating techniques in recommender systems to fulfil the preparative study step. In addition, it is not so easy to find consumers' preference information on different Kansei attributes of the traditional crafts, consumers' feeling target provides a good solution to find the preferred crafts.

8.6 Conclusions

In this chapter, a case study of Kansei evaluation for thirty products of Kanazawa gold leaf, one type of Japanese traditional crafts, was conducted to help the consumers to choose their preferred crafts. To do so, a preparative study is conducted first to gather the Kansei assessments of the thirty products of Kanazawa gold leaf. Secondly, the Kansei profiles of the thirty Kanzawa gold leaf were obtained based on the preparative study. Thirdly, based on two types of preferences (Kansei feeling target and prioritization of these targets), we obtained the Kansei evaluation results. From this case study, the consumers can choose their preferred products of Kanazawa gold leaf according to their Kansei preferences.

Chapter 9

Contributions and Future Work

9.1 Main Contributions of the Thesis

In this thesis, we have presented a study of multi-attribute target-oriented decision analysis and its applications to Kansei evaluation problems. Among the nine chapters of the thesis, the main chapters are Chapter 3, Chapter 4, Chapter 5, Chapter 6, Chapter 7, and Chapter 8. The main contributions of this thesis are as follows.

1. **The first main contribution is that *we propose two methods to target-oriented decision model with different target preferences and extend those two methods to fuzzy target-oriented decision analysis.***

   (a) *The first sub-contribution in this part is that we develop two methods for target-oriented decision analysis with different target preferences.*
   In most studies on target-oriented decision making, monotonic assumptions are given in advance to simplify the problems, e.g., the attribute/criteria wealth. However, there are three types of target preferences. Two methods have been proposed to model the different target preference types: cumulative distribution function (cdf) based method and level set based method. No matter which method is selected, both of these two methods can induce four shaped value functions: S-shaped, inverse S-shaped, convex, and concave, which represents decision maker's psychological preference. The main difference between these two methods is that the level set based model induces a stricter value function than the cdf based model.

   (b) *The second sub-contribution in this part is that we extend those two random target-oriented decision analysis to fuzzy uncertain targets.*
   Target-oriented decision model assumes that target has a random probability distribution. Fuzzy uncertainty is considered by decision makers to linguistically specify their uncertain targets. In this thesis, two methods of fuzzy target-oriented decision analysis with respect to different target preferences have been proposed. Firstly, a thorough analysis of possibility-probability transformations is given, and then the proportional approach is properly used to transform a possibility distribution into the probability distribution. Secondly, two methods of fuzzy target-oriented decision analysis have been obtained from the random target-oriented decision model. Finally, some widely used fuzzy

targets used in the pioneering work by Bellman and Zadeh [12] are selected to illustrate the fuzzy target-oriented model. Our research outperforms better in terms of three aspects.

2. **The second main contribution is that *we develop a non-additive multi-attribute target-oriented decision model based on fuzzy measure and fuzzy integral, and put forward a prioritized aggregation operator to model the prioritization between targets/attributes.***

   (a) *The first sub-contribution in this part is that we model the interdependence between different targets based on λ-fuzzy measure and Choquet fuzzy integral.*

   The use of fuzzy measures and fuzzy integral in MADA enables us to model some interaction phenomena existing among different attributes. As we shall see, multi-attribute target-oriented function has a similar structure with fuzzy measure, and fuzzy integral does not assume the independence. The fact that fuzzy integral model does not need to assume the independence of each target, means it can be used in non-linear situations. Thus we use fuzzy measure and fuzzy integral to model the interaction among targets. Furthermore, even if, in an objective sense any two targets are independent, they are not necessarily considered to be independent from the DM's subjective viewpoint. This explains why a fuzzy integral is more appropriate. Since the specification for fuzzy measures requires the values of a fuzzy measure for all subsets, the λ-fuzzy measure is used in order to reduce the difficulty of collecting information and the Choquet fuzzy integral is used to model the dependence in multi-attribute target-oriented decision analysis. A bisection search algorithm is also designed to identify the fuzzy measures of individual attributes group with a given λ value.

   (b) *The second sub-contribution in this part is that we put forward a prioritized OWA aggregation operator to model the prioritization between different targets.*

   Firstly the OWA operator is used to obtain the satisfaction degree for each priority level. To preserve the tradeoffs among the attributes in the same priority level, the degree of satisfaction for each priority level is viewed as a pseudo attribute. Secondly, we suggest that roughly speaking any t-norm can be used to model the priority relationships between the attributes in different priority levels. To keep the slight change of priority weight, strict Archimedean t-norms perform better in inducing priority weight. As Hamacher family of t-norms provide a wide class of strict Archimedean t-norms ranging from the product to weakest t-norm, Hamacher t-norms are selected to induce the priority weight for each priority level. Thirdly, considering DM's requirement toward the higher priority levels, a benchmark based approach is proposed to induce priority weight for each priority level, i.e., "the satisfactions of the higher priority attributes are larger than or equal to the DM's requirements". We suggest that the weights of lower priority level should depend on the benchmark achievement of all the higher priority levels.

   To illustrate the effectiveness and advantages of the prioritized OWA operator mentioned above, we conduct several comparative analysis with previous work on prioritized aggregation.

3. **The third contribution is that *we develop a Kansei evaluation model based on prioritized multi-attribute fuzzy target-oriented decision analysis. A case study is also conducted to illustrate the proposed Kansei evaluation model.***

   To overcome the those two above-mentioned problems in current research on Kansei evaluation, we put forward a Kansei evaluation model based on fuzzy target-oriented decision analysis and prioritized OWA aggregation operator. Firstly, like the traditional Kansei evaluation method, a preparatory experiment study is conducted in advance to select Kansei attributes by means of semantic differential (SD) method. In order to obtain Kansei data of products, a number of people are selected to assess products regarding these Kansei attributes. Secondly, these Kansei data are used to generate Kansei profiles for evaluated products by making use of the voting statistics. Thirdly, according to consumer-specified preferences on Kansei attributes, three main types of fuzzy targets are defined, to represent the consumers' preferences. Based on the principle of target-oriented decision analysis, we can obtain the satisfaction degrees (probabilities of meeting targets) regarding the Kansei attributes selected by consumers for all the evaluated products. Finally, considering prioritization of the Kansei attributes, the prioritized OWA aggregation operator is used to aggregate the partial satisfaction degrees for the evaluated products.

   Kansei evaluation has been applied to consumer products with successful results, e.g., table glasses, housing assessment, telephones, cars, and mobile phones. However, Kansei evaluation of traditional crafts has not been addressed yet In Japan, there are many traditional crafts such as fittings, textile, etc. These beautiful, elegant and delicate products are closely related to and have played an important role in Japanese culture and life. Evaluations of these traditional crafts would be of great help for marketing or recommendation purposes. Thus the Japanese traditional crafts are used as a case study to illustrate the proposed Kansei evaluation model. By using our model, consumers can choose their preferred crafts according to their preferences.

9.2 Suggestions for Future Research

The development of this thesis not only provides approaches to modeling and analyzing decision problems with target while considering decision makers' behavioral preferences, but it also opens up new avenues to further research in multi-attribute decision making. Below are some possible directions for future research.

9.2.1 Bipolar scale aggregation in multi-attribute target-oriented decision model

Target-oriented decision model assumes that the target divides the outcomes into gains and loss, thus the outcomes below or exceeding the reference point should have different impacts on the aggregation of partial target achievements. In this case, from the point view of aggregation, multi-attribute target-oriented decision model satisfies the conditions that the values to be aggregated lie on different bipolar scales, where 0 is the worst score, 1 is the best score, and there exists different reference points, denoted as e. For different attributes, the values e are probably different as different attributes may have different target distributions. The resulting continuous piecewise linear aggregation function has the ability to represent decisional behaviors that depend on the "positive" or "negative" satisfaction of some of the attributes.

The motivation for such a work may be only mathematical. However, there are psychological evidence that in many cases, scores or utilities manipulated by humans lie on a bipolar scale, that is to say, a scale with a neutral value making the frontier between good or satisfactory scores, and bad or unsatisfactory scores. With our convention, good scores are positive ones, while negative scores reflect bad scores. Most of the time, our behavior with positive scores is not the same than with negative ones: for example, a conjunctive attitude may be turned into a disjunctive attitude when changing the sign of the scores. So, it becomes important to define aggregation operators being able to reflect the variety of aggregation behaviors on bipolar scales.

Recently, Grabisch et al. [48] have proposed a bipolar fuzzy integral to model this type of aggregation within a bounded domain $[-1, 1]$ based on prospect theory [70, 133], where the reference point is zero. Target-oriented decision model represents the S-shaped function, thus we believe the target-oriented model can also satisfy this point. However, there are some differences between multi-attribute target-oriented model and the bipolar aggregation operator proposed by Grabisch et al.

1. The bounded domains are different. In our research the bounded domain is $[0, 1]$, whereas the bipolar aggregation operator assumes $[-1, 1]$. This difference may be only mathematical. However, in decision theory it represents different semantics.

2. The reference points are different. In our research the reference points can be different, whereas the bipolar aggregation operator assumes a constant reference point 0.

3. In fact, the bipolar aggregation operator proposed by Grabisch et al. [48] is based on the prospect theory. Although target-oriented model can represents the S-shaped value function, it is different from the prospect theory and have some advantages than prospect theory.

Another possible research is to consider different importances and priorities into the proposed aggregation operator as importance and priority information have different semantics and meanings.

9.2.2 Continuous multi-attribute decision making based on target-oriented decision model

Multi-criteria decision analysis (MCDA) problems can be categorized into two classes: discrete and continuous MCDA [127], also known as multi-attribute decision analysis (MADA). In this study we only focused on MADA. In the context of continuous MCDA (multi-objective decision analysis), utility theory is one of widely used techniques. As there are some drawbacks in utility theory, thus it is possible and necessary to apply the target-oriented decision model into multi-objective decision making. Another reason for this work is the behavioral preferences. Distance-based approaches are quite broadly used in goal programming. As different distances should have different impacts on decision makers' preferences, thus it is better to consider the target-oriented model in goal programming. In fact, as pointed out by Todorov et al. [124], probability can be viewed as some kind of distance while considering decision makers' psychological preferences.

Furthermore, in the literature several several subfields are developed rather independently, such as Goal Programming and multi-objective Decision Analysis. Target-oriented decision model will provide opportunities for collaboration with MCDM/MAUT researchers, leading to synergistic advances and less fragmentation of these fields. In this sense, this research will provide a future research direction to collaborate with MCDM/MAUT researchers [135]. Fig. 9.1 graphically depicts the review of MODM approaches by Sen and Yang [119].

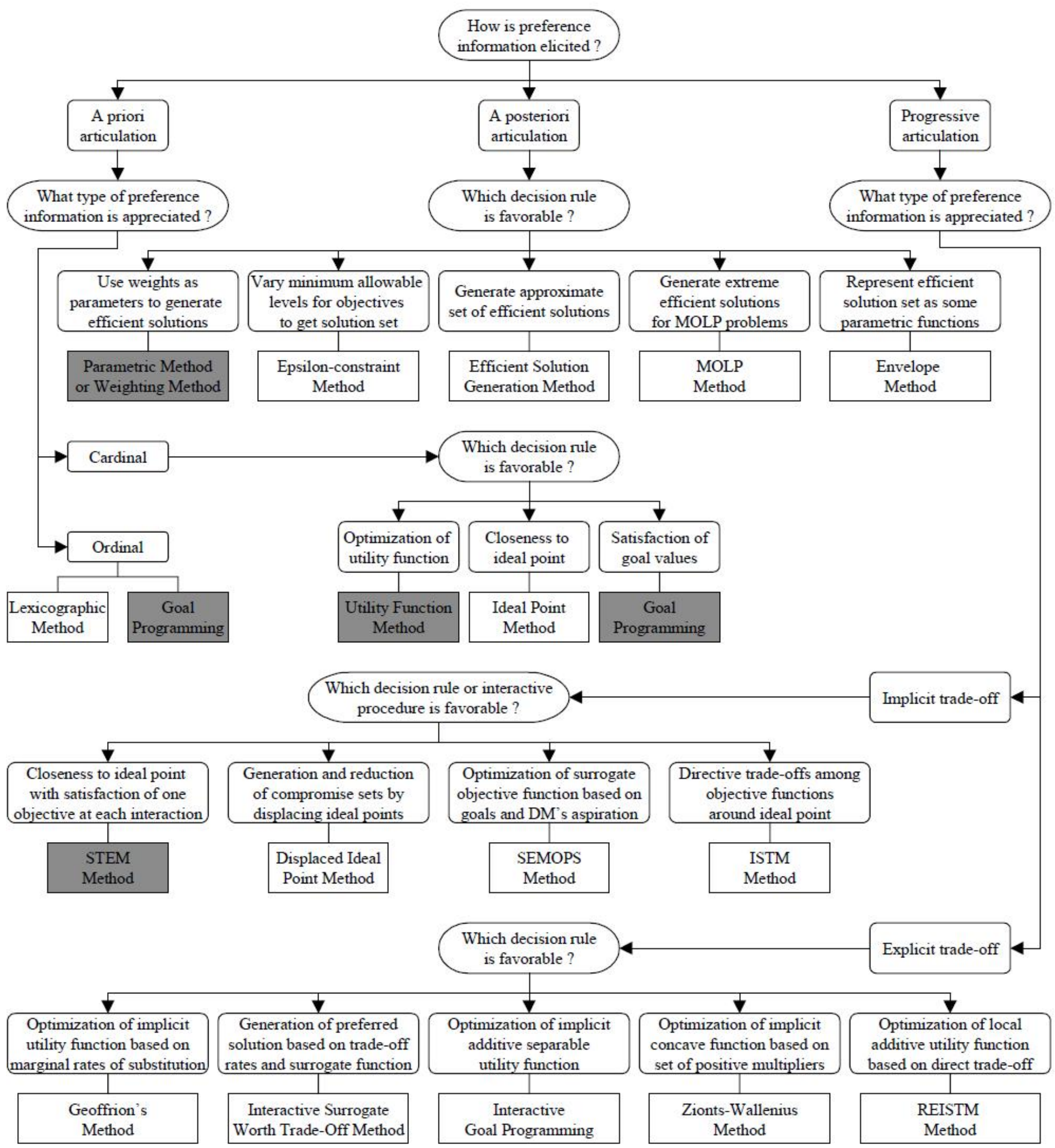

Figure 9.1: Decision tree for MODM technique, adapted from Sen & Yang [119]

9.2.3 Applications in recommender systems

Last but not at least, another future research is try to apply this research into the recommender systems. The motivation for this research theme comes from the Kansei evaluation model and case study in Chapter 7 & 8.

In the context of purchase decision making, a typical two-stage process may unfold as follows. First, the consumer screens a large set of relevant products, without examining any of them in great depth, and identifies a subset that includes the most promising alternatives. Particularly, for the kinds of products of **low purchasing frequency, high involvement, and high price**, e.g., traditional crafts in Japan, the consumer usually has no sufficient knowledge to evaluate the products. Therefore, in addition to the ability to interact with the consumer to acquire and analyze his requirements, the system needs to have specific domain knowledge to evaluate different products and to suggest the optimal ones satisfying the consumers' requirements. Fig. 9.2 shows the interactive recommendation strategy.

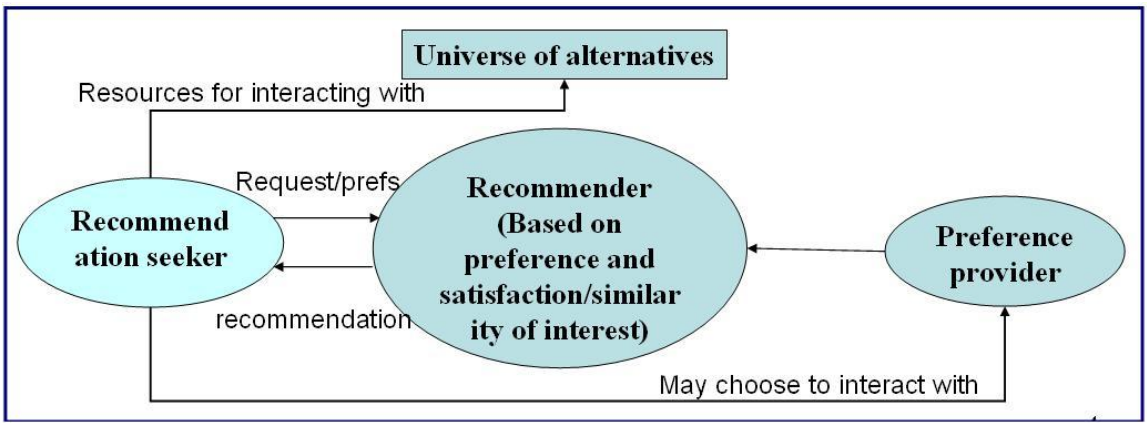

Figure 9.2: A recommendation strategy

1. **Applying target-oriented decision model into critique-based recommender systems**
   Many highly interactive recommender systems engage users in a conversational dialog in order to learn their preferences and use their feedback to improve the systems recommendation accuracy. Such interaction models have been referred as conversational recommenders or critiquing-based recommender systems [4]. The main component of the interaction is that of example and critique. The system simulates an artificial salesperson that recommends example options based on a user's current preferences and then elicits her feedback in the form of critiques such as "I would like something cheaper" or with "faster processor speed". These critiques form the critical feedback mechanism to help the system improve its accuracy in predicting the users needs in the next recommendation cycle. In many critique-based recommender systems, different comparison methods are used to revise consumers' preference/requirements. Utility theory is one of the most widely used method to compare and evaluate consumers' critiques. Thus our first research objective is to view the critiques as targets.

2. **Considering both subjective and objective information in recommender systems**
   In interactive recommender systems, most researchers focus on objective information such as size, weight of the product. In non-interactive recommendations systems, most researchers focus on using some algorithm to predict the utility value of the product to be recommended. The first point is to consider both subjective and objective information in interactive recommender systems. Another point is to consider Kansei information (a kind of subjective information). In non-interactive recommender systems, most work tries to use consistent preference order relation to rate the product, such as linguistic word "good". However, as discussed in Chapter 7, Kansei words usually have different preference relations. We believe that this research is missed in the literature of recommender systems.

3. A software-based decision support system (`DSS`) could help a consumers implement this approach easily and expeditiously. Hence, a computer-based `DSS` should be developed to integrate the recommendation methodology and assist in practical applications.

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Journal Papers

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