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Description	

Self-configurable Mobile Robot Swarms with Hole Repair Capability

Geunho Lee and Nak Young Chong

Abstract—We address the problem of deploying a swarm of autonomous mobile robots toward building an *ad hoc* network of robotic sensors with spatial uniform density. For the purpose, each of the robots configures themselves into an area with geographical constraints through local interactions with two adjacent neighboring robots. The basic idea underlying this work is that robots can be thought of as liquid particles that change their positions conforming to the shape of the container they occupy. The main challenge is how to cope with the accuracy limitations of sensors and possible holes in the configuration. Considering such realistic conditions, the convergence of the proposed method is proved using Lyapunov's theorem. The proposed method is verified to be effective through the simulation for the secure deployments of robotic sensor network.

I. INTRODUCTION

Recently, there has been increasing attention paid to swarm robotics, because it is possible to have a swarm of robots cover an area of interest for such applications as environmental or habitat monitoring, search-and-rescue, and exploration [1]. These applications require that robots adaptively configure themselves into the area controlling the individual robot's motion in a decentralized way [2], and be dispersed in a uniform spatial density without holes. Toward the end, we propose a geometric approach to self-configuration that enables a swarm of robots to adapt its shape to the geographically constrained plane with equilateral triangle lattices. Based on a partially connected mesh topology [14], the proposed approach can take advantage of the redundancy provided by a fully connected topology without the expense and complexity of networking processes.

What is important from the practical point of view is that we consider the problem of limitations in sensor technology and holes in the configuration process and/or node failure. Our main contribution lies in providing an effective approach against the measurement errors employing the Kalman filter [16], and the self-repair capability. By increasing the number of neighboring robots positioned at the uniform distance, the swarm repairs the holes and improves the network connectivity. Regarding the convergence of the proposed algorithms, Lyapunov's theorem is utilized, leading to asymptotic stability of the desired configuration from an arbitrary distribution. Both individual behavior of robots and overall shape of the swarm can be coordinated with scalability in this work.

Decentralized control for robot swarms can be broadly classified into global and local strategies according to whether sensors have range limits. Global strategies [3]

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may provide fast, accurate, and efficient deployment, but are technically non-feasible and lack scalability as the number of robots increases. On the other hand, local strategies are based on interactions between individual robots inspired by colonies of ants or schools of fish, or physical phenomena such as crystallization. Local strategies can further be divided into biological emergence [4][5], behavior-based [6], and virtual physics-based [7]-[13] approaches. Many of the behavior-based and virtual physics-based approaches used such physical phenomena as electric charges [7], gravitational forces [8], spring forces [9][12][13], potential fields [10], van der Waals forces [11], and other virtual models.

Robot swarm configurations achieved by the above-mentioned local interactions may result in lattice-type networks. These configurations offer high level coverage and multiple redundant connections ensuring maximum reliability and flexibility from the standpoint of topology. Depending on whether there are interactions among all robots, the network can be classified into fully and partially connected topologies [14]. The fully connected topologies have each robot interact with all of other robots simultaneously within a certain range. Thus, those approaches might over-constrain individual robots and frequently lead to deadlocks. On the contrary, using the partially connected topology, robots interact selectively with other robots, but are connected to all robots in the formation. For example, robots may choose to exert forces in a certain direction [12], where this selective interaction helps prevent them from being too tightly constrained. Due to similar reason, robots are enabled to achieve faster formation without deadlocks [13]. Using the partially connected topology, our work is to enable robot swarms to construct uniformly spaced equilateral triangles with a minimum number of interacting robots in an area with geographical constraints.

II. PROBLEM STATEMENT

A. Robot Model

We consider a swarm of mobile robots denoted as r_1, \dots, r_n . It is assumed that an initial distribution of all robots is arbitrary and distinct. Each robot autonomously moves on a 2-D plane. They have no leader and no identifiers, and do not share any common coordinate system, and do not retain any memory of past actions [3]. Due to limited sensing range, they can detect the position of other robots only within a certain range. In addition, each robot does not communicate explicitly with other robots. Let the position p_i of a robot r_i be denoted as a state vector

$$p_i = [p_{i,x} \ p_{i,y}]^T. \quad (1)$$

We can also define r_i 's kinematics by $\dot{p}_{i,x} = u_i \cos \theta_i$, $\dot{p}_{i,y} = u_i \sin \theta_i$, where u_i and θ_i are the translational and angular velocity of r_i , respectively. In addition, to provide an optimal estimation of noisy sensor measurements, a Kalman filter is employed [16]. Let's consider the system state vector $p_{i,k}$, $[p_{i,1,k} \ \cdots \ p_{i,n,k}]^T$, defined as the positions of other robots at time k . The Kalman filter model describes the state transition from k to $k+1$ as follows:

$$p_{i,k+1} = A_{i,k} p_{i,k} + \omega_{i,k} \quad (2)$$

where $A_{i,k}$ is the state transition matrix given by an identity matrix and $\omega_{i,k}$ means the system random noise $\omega_{i,k} \sim N(0, W_{i,k})$. The predicted estimate covariance can be obtained as follows:

$$P_{i,k+1|k} = A_{i,k} P_{i,k|k} A_{i,k}^T + W_{i,k} \quad (3)$$

where $W_{i,k} = w_{i,k} I$. (Do not confuse the notation for the r_i 's position p_i in (1) with that for the error covariance P_i .) Then, when r_i detects the position of r_j with respect to r_i 's local coordinate system, the measurement $z_{i,j,k}$ is given by

$$z_{i,j,k} = p_{i,j,k} - p_{i,i,k} + v_{i,j,k} \quad (4)$$

where $v_{i,j,k}$ is zero-mean white Gaussian random process. The estimated measurement would be $\hat{z}_{i,j,k} = \hat{p}_{i,j,k} - \hat{p}_{i,i,k}$. Since $p_{i,i,k}$ and $\hat{p}_{i,i,k}$ becomes zero, the error for the measurement $\tilde{z}_{i,j,k}$ is given by

$$\tilde{z}_{i,j,k} = z_{i,j,k} - \hat{z}_{i,j,k} = \tilde{p}_{i,j,k} + v_{i,j,k} \quad (5)$$

where $\tilde{p}_{i,j,k} = p_{i,j,k} - \hat{p}_{i,j,k}$. Therefore, if r_i measures the position of other robots, the measurement error equations can be written as

$$\tilde{z}_{i,k} = H_{i,k} \tilde{p}_{i,k} + v_{i,k} \quad (6)$$

where $v_{i,k}$ is the measurement random noise $v_{i,k} \sim N(0, M_{i,k})$ and $H_{i,k}$ is the measurement relation function matrix given by an identity matrix. The innovation covariance by the measurement sensor error is given by

$$S_{i,k} = H_{i,k} P_{i,k|k-1} H_{i,k}^T + M_{i,k} \quad (7)$$

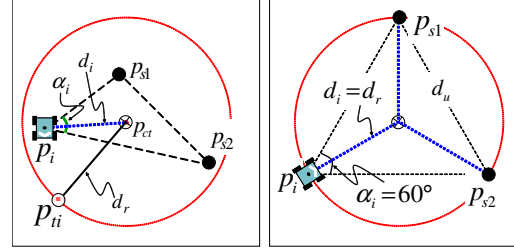
where $M_{i,k}$ is the covariance of the relative state measurements expressed as $m_{i,k} I$. Thus, the filtered state estimate and the error covariance are obtained as follows:

$$\begin{aligned} K_{i,k+1} &= P_{i,k+1|k} H_{i,k+1}^T S_{i,k+1}^{-1} \\ \hat{p}_{i,k+1|k+1} &= \hat{p}_{i,k+1|k} + K_{i,k+1} z_{i,k+1} \\ P_{i,k+1|k+1} &= P_{i,k+1|k} - K_{i,k+1} S_{i,k+1}^{-1} K_{i,k+1}^T \end{aligned} \quad (8)$$

where $K_{i,k+1}$, $\hat{p}_{i,k+1|k+1}$, and $P_{i,k+1|k+1}$ are an optimal Kalman gain, an updated state estimate, and an updated estimate covariance, respectively.

B. Notations and Problem Definitions

The distance between the robot r_i 's position p_i and the robot r_j 's position p_j is denoted as $\text{dist}(p_i, p_j)$. We define a uniform interval d_u , the desired distance between each robot in the configuration. r_i detects the position $\{p_1, p_2, \dots\}$ of other robots located within its sensing boundary SB ,



(a) range and bearing (b) equilateral triangle
Fig. 1. Two control parameters in local interaction

yielding a set of the positions O_i with respect to its local coordinates. Next, r_i can select two robots r_{s1} and r_{s2} within r_i 's SB that we call the neighbors of r_i and denote their positions, $\{p_{s1}, p_{s2}\}$, as N_i . Given p_i and N_i , the *Triangular Configuration*, denoted by \mathbb{T}_i , is defined as a set of three distinct positions $\{p_i, p_{s1}, p_{s2}\} = \mathbb{T}_i$, where the internal angle $\angle p_{s1} p_i p_{s2}$ of r_i is denoted by α_i . Now we define the *Equilateral Configuration*, denote by \mathbb{E}_i , as a configuration that all the distance permutations of \mathbb{T}_i are equal to d_u . We need a measure indicating to which degree \mathbb{T}_i is configured into \mathbb{E}_i . Given \mathbb{T}_i , we can express the distance permutations with respect to r_i as the following matrix \mathbf{D}_i .

$$\mathbf{D}_i = \begin{cases} (\text{dist}(p_m, p_n) - d_u)^2 & \text{if } m \neq n \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where $\{\{p_m, p_n\} \mid p_m, p_n \in \mathbb{T}_i = \{p_i, p_{s1}, p_{s2}\}\}$. We will denote $(\text{dist}(p_m, p_n) - d_u)^2$ for simplicity as $(d_k - d_u)^2$. Using \mathbb{T}_i and \mathbb{E}_i , we can formally define the local interaction as follows: Given \mathbb{T}_i , *Local Interaction* is to have r_i maintain d_u with N_i at each time instant toward forming \mathbb{E}_i . Based on the local interaction, we formally address the ADAPTIVE SELF-CONFIGURATION PROBLEM.

Given a swarm of robots with arbitrarily distinct positions in a 2-D plane, how to enable the robots to configure themselves into \mathbb{E}_i while conforming to the plane border.

III. LOCAL INTERACTION

The local interaction algorithm enables three neighboring robots to generate an equilateral triangle of side length d_u , consisting of the function $\varphi_{interaction}$ whose arguments are p_i and N_i at each time. Consider r_i and its two neighbors r_{s1} and r_{s2} located within r_i 's SB . As shown in Fig. 1-(a), three robots are configured into \mathbb{T}_i whose vertices are p_i , p_{s1} , and p_{s2} , respectively. First, r_i finds the centroid p_{ct} of the triangle $\triangle p_i p_{s1} p_{s2}$ with respect to its local coordinates, and measures the angle ϕ between the line connecting two neighbors and r_i 's horizontal axis. Using p_{ct} and ϕ , r_i calculates the target point $p_{ti} = (p_{ti,x}, p_{ti,y})$ by the following equations

$$(p_{ct,x} + d_u \cos(\phi + \pi/2)/\sqrt{3}, p_{ct,x} + d_u \sin(\phi + \pi/2)/\sqrt{3}).$$

r_i attempts to form an isosceles triangle with its two neighbors at each time. By repeatedly doing this, three robots configure into \mathbb{E}_i as illustrated in Fig. 1-(b).

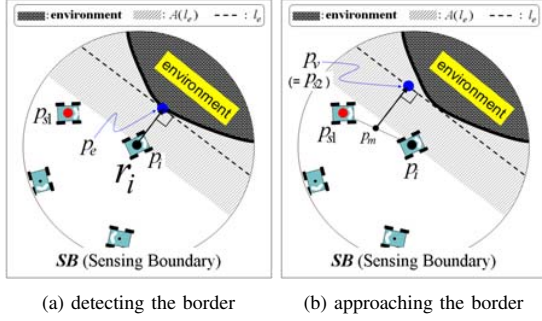


Fig. 2. Robots on the boundary of the swarm interact with the border

Now let's consider the circumscribed circle of an equilateral triangle whose center is p_{ct} of $\triangle p_i p_{s1} p_{s2}$ and radius is d_r . The position of robots can be modeled by the distance from p_{ct} and the internal angle (see Fig. 1-(b)). The distance is controlled by the following equation

$$\dot{d}_i(t) = -a(d_i(t) - d_r), \quad (10)$$

where a is a positive constant and d_r represents $d_u/\sqrt{3}$. Indeed, the solution of (10) is $d_i(t) = |d_i(0)|e^{-at} + d_r$ that converges exponentially to d_r as t approaches infinity. Next, the internal angle is controlled by the following equation

$$\dot{\alpha}_i(t) = k(\beta_i(t) + \gamma_i(t) - 2\alpha_i(t)), \quad (11)$$

where k is a positive constant. Because the total internal angle of a triangle is 180° , (11) can be re-written as

$$\dot{\alpha}_i(t) = k'(60^\circ - \alpha_i(t)), \quad (12)$$

where k' is $3k$. The solution of (12) is $\alpha_i(t) = |\alpha_i(0)|e^{-k't} + 60^\circ$ that converges exponentially to 60° as t approaches infinity.

Note that (10) and (12) imply that three robots eventually form an equilateral triangle of side length d_u . In order to show the convergence, we will take advantage of Lyapunov stability theory [15]. Consider the following scalar function

$$f_{l,i} = \frac{1}{2}(d_i - d_r)^2 + \frac{1}{2}(60^\circ - \alpha_i)^2 \quad (13)$$

that is always positive definite except $d_i \neq d_r$ and $\alpha_i \neq 60$. The derivative of the scalar function is given by

$$\dot{f}_{l,i} = -(d_i - d_r)^2 - (60^\circ - \alpha_i)^2, \quad (14)$$

which is obtained using (10) and (12). Eq. (14) is negative definite. The scalar function $f_{l,i}$ is radially unbounded since it tends to infinity as $\|\mathbf{x}\| \rightarrow \infty$. Therefore, the equilibrium state is asymptotically stable, implying that r_i reaches a vertex of \mathbb{E}_i .

IV. ADAPTIVE SELF-CONFIGURATION

Adaptive self-configuration is decomposed into self-configuration and uniform conformation, each of which is solved based on the local interaction. If detecting the plane border within its SB as illustrated in Fig. 2-(a), r_i defines a point p_e projected from p_i onto the surface with the minimum distance d_e and then computes the tangent $e'(t)$ to the surface

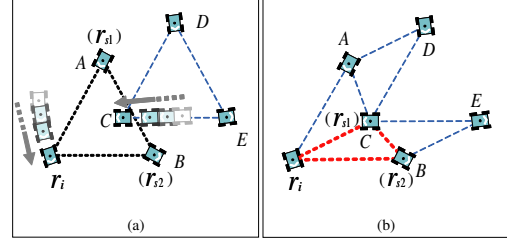


Fig. 3. Dynamically changing neighbors (a) r_i moves toward p_{ti} , (b) r_i selects new neighbors according to the minimum perimeter condition)

at p_e . (For convenience, l_e will be used instead of $e'(t)$.) It is obvious that l_e is perpendicular to the vector $\overrightarrow{p_i p_e}$, termed the surface direction. Let $A(l_e)$ denote the area between the border and the line passing through p_i and parallel to l_e within SB . In order to determine whether r_i needs to interact with the surface, r_i checks if no neighbors exist in $A(l_e)$ or if $d_e \leq \frac{\sqrt{3}d_u}{2}$. If the condition is satisfied, r_i executes the uniform conformation algorithm, otherwise, executes the self-configuration algorithm.

A. Self-configuration

Self-configuration is related to deploying a swarm of robots into equilateral triangle lattices based on the local interaction. In order to form a triangle, r_i selects the first neighbor r_{s1} located the shortest distance from itself. The second neighbor r_{s2} is selected such that the total distance from p_{s1} to p_i passing through p_{s2} is minimized. Then, r_i forms \mathbb{T}_i with N_i , and computes the target point p_{ti} by $\varphi_{interaction}$. Self-configuration enables a robot swarm to have a multitude of equilateral triangular lattices, denoted by $\sum_{i=1}^n \mathbb{E}_i$. Specifically, r_i dynamically changes the neighbors within SB at each time, enabling the robots to configure themselves without having adjacent triangles partly overlapping each other (see Fig. 3).

Now we examine the effect of changing neighbors in configuring \mathbb{E}_i . We use Lyapunov's theory with a scalar function given by

$$f_{sc,i} = \sum_{\mathbb{T}_i} (d_k - d_u)^2 + f_{l,i} \quad (15)$$

where $f_{l,i}$ is given by (13) and $\sum_{\mathbb{T}_i} (d_k - d_u)^2$ is defined as the constant value associated with \mathbb{T}_i at each time (see (9)). Thus, from (13), the scalar function of (15) is always positive definite except $d_i \neq d_r$ and $\alpha_i \neq 60$. (If \mathbb{T}_i is equal to \mathbb{E}_i , it is easily seen that $\sum_{\mathbb{T}_i} (d_k - d_u)^2$ reaches 0, resulted from $d_r = d_u/\sqrt{3}$.) The derivative of the scalar function is given by

$$\dot{f}_{sc,i} = \dot{f}_{l,i} = -(d_i - d_r)^2 - (60^\circ - \alpha_i)^2. \quad (16)$$

Eq. (16) is negative definite. Finally, the scalar function $f_{sc,i}$ is radially unbounded since it tends to infinity as $\|\mathbf{x}\| \rightarrow \infty$. Therefore, the equilibrium state is asymptotically stable, implying that r_i reaches a vertex of \mathbb{E}_i from an arbitrary \mathbb{T}_i .

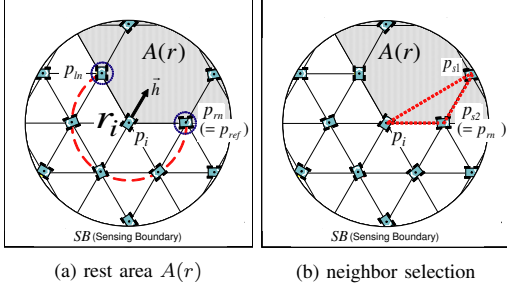


Fig. 4. Illustration of the self-reparation algorithm

Now we show the convergence property for a swarm of n robots. The n -order scalar function \mathbf{F}_{sc} is defined as

$$\mathbf{F}_{sc} = \sum_{i=1}^n f_{sc,i} = \sum_{i=1}^n \sum_{\mathbb{T}_i} (d_k - d_u)^2 + \sum_{i=1}^n f_{l,i}. \quad (17)$$

It is straightforward to verify that \mathbf{F}_{sc} is positive definite and $\dot{\mathbf{F}}_{sc}$ is negative definite. \mathbf{F}_{sc} is radially unbounded since it tends to infinity as t approaches infinity. Consequently, a swarm of n robots converges into \mathbb{E}_i for their N_i .

B. Uniform Conformation

Now we describe how to enable a swarm of robots to conform to the plane border. This means that the positions on the border (or the virtual static robots) need to be incorporated into \mathbb{T}_i . If the border is detected by r_i , it defines r_{s1} located at the shortest distance and check the conditions whether no other neighbors exist in $A(l_e)$ or $d_e \leq \frac{\sqrt{3}d_u}{2}$. If the condition is satisfied, r_i computes the midpoint p_m of $\overline{p_i p_{s1}}$ that is projected onto l_e and defined as p_v (see Fig. 2-(b)). Now p_v is considered as p_{s2} , and N_i is defined as $\{p_{s1}, p_v\}$. It is readily evident that r_i can compute p_{ti} by $\varphi_{interaction}$.

Note that p_v is a virtual, static point. Furthermore, it is difficult to identify the border since each robot has their SB . Therefore, it is almost impossible for them to exactly form \mathbb{E}_i . To take this effect into account, we introduce another measure for \mathbb{T}_i given by

$$\mathbf{D}_{i,e} = \begin{bmatrix} 0 & (dist(p_i, p_{s1}) - d_u)^2 \\ (dist(p_i, p_{s2}) - d_u)^2 & 0 \end{bmatrix}. \quad (18)$$

$\mathbf{D}_{i,e}$ is applied only when any robots located in close to the plane border interact with p_v .

V. SELF-REPARATION

Robots attempt to reach a uniform spatial density, but probably holes remain in a converged distribution. This is because each robot determines their direction of movement based on the current position of neighbors. To repair the holes, r_i changes its neighbors. Let P_u denote the set of robot positions located within the range of d_u . r_i defines its heading \vec{h} with respect to the local coordinates. Let $ang(\vec{m}, \vec{n})$ denote the angle between two arbitrary vectors \vec{m} and \vec{n} . As shown in Fig. 4-(a), r_i selects the reference neighbor p_{ref} in P_u such that the value of $ang(\vec{h}, \overline{p_i p_{ref}})$

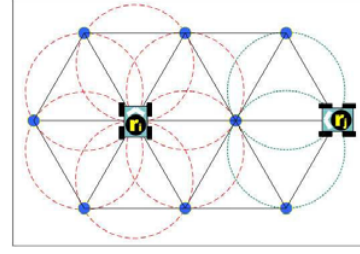


Fig. 5. r_i and r_j form $\sum_{m=1}^6 (\mathbb{E}_i)_m$ and $\sum_{m=1}^2 (\mathbb{E}_j)_m$, respectively.

is minimized. r_i then checks if any neighbor exists in the area obtained by rotating $\overline{p_i p_{ref}}$ 60 degrees clockwise. If there exists one, r_i checks the next neighbor by sweeping another 60 degree clockwise. r_i continues to check until it finds a hole, then the last neighbor is defined as p_{ln} . Similarly, r_i attempts to find neighbors by rotating $\overline{p_i p_{ref}}$ counterclockwise and locate the last neighbor p_{rn} . The reparation area $A(r)$ is defined as the area between $\overline{p_i p_{rn}}$ and $\overline{p_i p_{ln}}$ in SB , where no element of P_u exists. As illustrated in Fig. 4-(b), r_i selects the first neighbor located the shortest distance away from p_i in $A(r)$ as p_{s1} . The second position is defined such that the total distance from p_{s1} to p_i can be minimized through either p_{rn} or p_{ln} . As a result, p_{ti} can be determined by $\varphi_{interaction}$.

Like the surface tension of liquids caused by intermolecular forces, as illustrated in Fig. 5, the self-reparation will have each robot attempt to reach the maximum possible number of desired configurations \mathbb{E}_i within SB given by

$$\max \left[\sum_{m=1}^s (\mathbb{E}_i)_m \right] \quad (19)$$

where s is greater than or equal to 1 and less than or equal to 6, as the desired configuration is a hexagon composed of 6 equilateral triangle lattices. Therefore, a collective configuration reaches a swarm of robots with $\max[\sum (\mathbb{E}_i)]$ while filling up holes.

To obtain $\max[\sum_{m=1}^s (\mathbb{E}_i)_m]$, our algorithm changes the neighbors according to the condition whether r_i forms \mathbb{E}_i at any time. Therefore, we can modify (15) as follows:

$$f_{sr,i} = \begin{cases} \sum_{m=1}^c (f_{l,i})_m + f_{sc,i} & \text{if } \mathbb{T}_i = \mathbb{E}_i \\ f_{sc,i} & \text{otherwise} \end{cases} \quad (20)$$

where $f_{sc,i}$ is given in (15) and c is less than $\max[s]$. Using (20), (19) can be re-written as

$$f_{sr,i} = \min \left[\sum_{m=1}^s (f_{l,i})_m \right]. \quad (21)$$

Here, (21) enables r_i to reach the minimum energy level by maximizing the number of \mathbb{E}_i .

Next, let the internal energy, increasing or decreasing during self-configuration, be denoted as q_i given by

$$q_i = \sum_{m=1}^c (f_{l,i})_m. \quad (22)$$

When \mathbb{T}_i is equal to \mathbb{E}_i , q_i forces r_i to locally interact toward forming another equilateral triangular lattice by changing neighbors. We assume that q_i starts with any nonnegative value $q_i(0)$ and evolves according to the following equation:

$$\dot{q}_i = \sum_{m=1}^c (\dot{f}_{l,i})_m. \quad (23)$$

Note that q_i forces r_i to minimize $f_{sc,i}$. If q_i decreases, we can predict that r_i moves toward $\min[\sum_{m=1}^s (f_{l,i})_m]$. By doing this repeatedly, the holes will get eliminated.

To show that each robot converges into $\max[\sum_{m=1}^s (\mathbb{E}_i)_m]$ while increasing the number of neighboring robots after a finite number of activation steps, we use Lyapunov's theory and show the convergence of r_i using (20) and (22), with a scalar function defined as:

$$f_{sr,i} = f_{sc,i} + q_i. \quad (24)$$

Recall that $q_i(0)$ is initialized to a nonnegative value and evolves according to (23). Moreover, q_i is defined in such a way that it increases when $f_{l,i}$ lacks. Whenever $\mathbb{T}_i = \mathbb{E}_i$, q_i is set to $q_i(0)$. On the other hand, $f_{sc,i}$ is positive definite by (15). Since $f_{sc,i} > 0$ and $q_i > 0$, it is clear that $f_{sr,i} > 0$. Next, differentiating $f_{sr,i}$ gives

$$\dot{f}_{sr,i} = \dot{f}_{sc,i} + \dot{q}_i. \quad (25)$$

$$(\dot{f}_{sr,i} = \dot{f}_{sc,i} + \sum_{m=1}^c (\dot{f}_{l,i})_m = \dot{f}_{l,i} + \sum_{m=1}^c (\dot{f}_{l,i})_m)$$

which can be simplified to

$$\dot{f}_{sr,i} = \sum_{m=1}^s (\dot{f}_{l,i})_m. \quad (26)$$

It is easy to see that $\dot{f}_{sr,i}$ is negative definite. Moreover, the scalar function $f_{sr,i}$ is radially unbounded since it tends to infinity as $\|\mathbf{x}\| \rightarrow \infty$ even though q_i remains a positive constant. Therefore, based on Lyapunov's theory, the position of r_i converges into $\sum_{m=1}^s (\mathbb{E}_i)_m$.

Now we show the convergence property for a swarm of n robots. The n -order scalar function \mathbf{F}_{sr} is defined as

$$\mathbf{F}_{sr} = \sum_{i=1}^n f_{sr,i} = \sum_{i=1}^n f_{sc,i} + \sum_{i=1}^n q_i. \quad (27)$$

It is straightforward to verify that \mathbf{F}_{sc} is positive definite. Next, differentiating \mathbf{F}_{sc} gives

$$\dot{\mathbf{F}}_{sr} = \sum_{i=1}^n \dot{f}_{sc,i} + \sum_{i=1}^n \dot{q}_i. \quad (28)$$

$\dot{\mathbf{F}}_{sr}$ is negative definite and radially unbounded since it tends to infinity as t approaches infinity. Consequently, a swarm of n robots converges into $\sum_{i=1}^n (\max[\sum_{m=1}^s (\mathbb{E}_i)_m])$.

VI. SIMULATION RESULTS

We performed simulations to investigate the convergence and robustness properties of the proposed algorithm. The standard deviation of the error in the sonar readings is assumed to be 10%. Our algorithm terminates when all robots converge into the distance $d_u \pm 1\%$ with their neighbors.

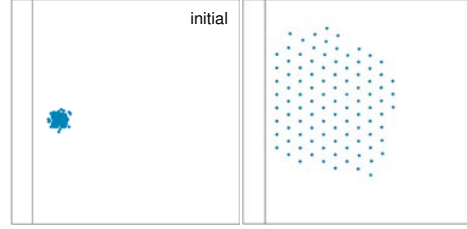


Fig. 6. Dispersion of 100 robots over a flat surface

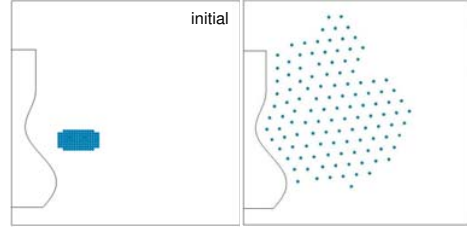


Fig. 7. Dispersion of 120 robots over a curved surface

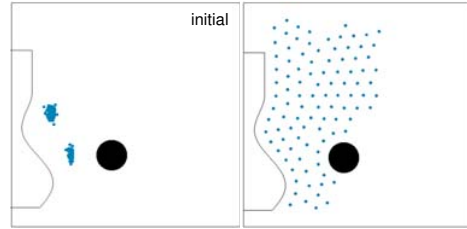
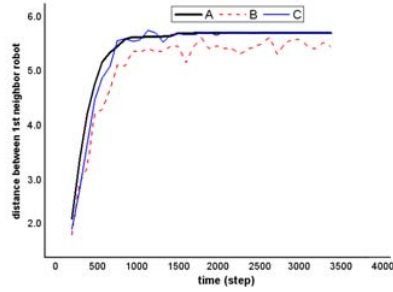


Fig. 8. Dispersion of 100 robots over a curved surface with an obstacle

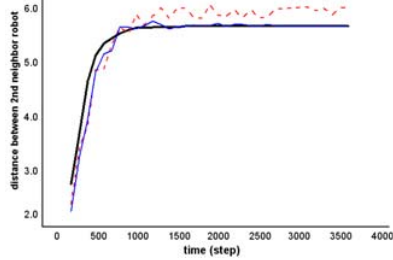
Fig. 6 shows that 100 robots configure themselves into a uniform density pattern over a flat surface. It is observed that the robots could converge into \mathbb{E}_i conforming to the surface. Fig. 7 demonstrates that 120 robots configured themselves into a curved surface. Most of the robots that do not detect the surface conformed to it through the local interaction. Fig. 8 presents a more complex geographic condition with a circular obstacle. It is also observed that the robots could converge into \mathbb{E}_i , conforming to the surfaces of the obstacle as well as the border.

Fig. 9 shows the mean values of distance between 120 robots and their N_i in Fig. 7 according to the iteration step. Here, the black bold line, the blue line, and the red dashed line indicate the case of no sensor error, the Kalman filtered sensor readings, and non-filtered sensor readings, respectively. As can be seen from the figure, each robot could converge into \mathbb{E}_i in spite of measurement errors.

Finally, robustness is verified against robot failures under the same surface condition as Fig. 7. Five robots unexpectedly failed in Fig. 10-(a), and the same number of holes appeared in the deployment. Each robot checks the existence of holes within SB . If there are holes around it, the algorithm allows each of the robots change their position to fill the holes. Fig. 10-(b) presents the results of redeployment with 95 robots. In addition, the lost robots were replaced by



(a) distance variation between r_i and r_{s1}



(b) distance variation between r_i and r_{s2}

Fig. 9. Distance variations between r_i and N_i during self-configuration ('A'-line: no sensor error, 'B'-line: non-filtered, 'C'-line: Kalman filtered)

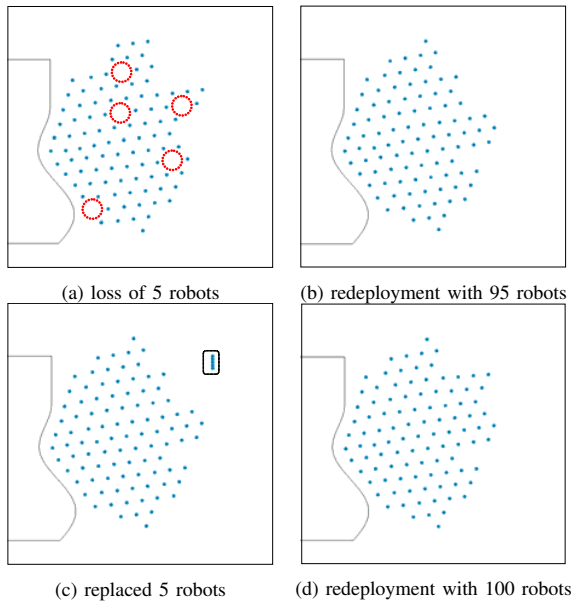


Fig. 10. Robustness against loss of 5 robots over a curved surface

the same number of new robots in Fig. 10-(c). Fig. 10-(d) shows the results of redeployment with 100 robots. From the results, the adaptive self-configuration algorithm has proven effective in improving the robustness of uniform coverage over a designated area.

VII. CONCLUSION

The adaptive self-configuration problem was addressed to disperse a swarm of robots in a geographically constrained plane. From the practical point of view, we did not use

robot identifiers, common coordinates, global orientation, and direct communication. Robots computed their position without requiring memories of past actions or states, helping cope with transient errors. Also, sensors were subject to strict range and accuracy limitations. Under such conditions, we proposed distributed algorithms that enable robots to configure themselves into triangular lattices adapting to the environment. Among all the possible types of regular polygons, the equilateral triangle lattice can reduce the computational burden and is highly scalable, and less influenced by neighboring robots. The proposed local interaction, where robots were allowed to interact with only two dynamically selected neighbors, is computationally efficient, as they utilize only position information of other robots. By collecting such local behavior of each robot, a uniformly spaced swarm of robots was organized to fill in the environment. Moreover, the proposed algorithm improved the robustness of uniform coverage control over a designated area against robot failures. The convergence of the algorithm was proven mathematically, and also verified through extensive simulations.

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