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Space-Efficient Algorithm for Image Rotation

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SUMMARY This paper presents an algorithm for rotating a subimage in place without using any extra working array. Due to this constraint, we have to overwrite pixel values by interpolated values. Key ideas are *local reliability test* which determines whether interpolation at a pixel is carried out correctly without using interpolated values, and *lazy interpolation* which stores interpolated values in a region which is never used for output images and then fills in interpolated values after safety is guaranteed. It is shown that linear interpolation is always safely implemented. An extension to cubic interpolation is also discussed.

key words: *algorithm, in-place algorithm, space-efficient algorithm, linear interpolation, cubic interpolation*

1. Introduction

Demand for high-performance scanners is growing toward paper-less society. There are a number of problems to be resolved in the current scanner technology. One of them is to detect a direction of a document scanned, i.e., which side is the top of the document. One way is to use OCR (Optical Character Recognition) technology to read characters which is now common to scanners. Of course, we want to avoid using OCR since it takes time. Another common problem which we address in this paper is correction of rotated documents. If a document contains only characters, then OCR is definitely a solution. Since it is costly, a geometric algorithm for such correction is required. It consists of two phases. In the first phase we detect rotation angle. Some scanners are equipped with a sensor to detect rotation angle. If no such sensor is available, we could rely on another algorithm called Hough transform [1], [2] for finding line components to detect rotation angle. To simplify the discussion, we assume a hardware sensor to detect rotation angle.

Once rotation angle is obtained, the succeeding process is rather easy if sufficient working storage is provided. Suppose input intensity values are stored in a two-dimensional array $a[.,.]$ and another array $b[.,.]$ of the same size is available. Then, at each lattice point (pixel) in the rotated coordinate system we compute an intensity value using appropriate interpolation (linear or cubic) using intensity values around the lattice point (pixel) in the input array and then store the computed interpolation value at the corresponding element in the array $b[\ ]$. Finally, we output intensity values stored in the array $b[\ ]$. It is quite easy. This method, however, requires too much working storage, which is a serious drawback for devices such as scanners in which saving memory is a serious demand for their built-in softwares and their costs. Is it possible to implement the interpolations without using any extra working storage? This is the question we address in this paper.

We propose a space-efficient algorithm for correcting rotation of a document without using any extra working storage. A simple way of doing this is to compute an interpolation value at each pixel in the rotated coordinate system and store the computed value somewhere in the input array $a[\ ]$ near the point in the original coordinate system. Once we store an interpolation value at some element of the array, the original intensity value is lost and it is replaced by the interpolation value. Thus, if the neighborhood of the pixel in the rotated coordinate system includes interpolated values then the interpolation at that point is not correct or reliable. One of the key observations is that there is an easily-computable condition to determine whether interpolation at a given pixel is reliable or not, that is, whether any interpolated value is included in the neighborhood or not. Using the condition, we first classify pixels in the rotated coordinate system into reliable and unreliable ones. In the first phase we compute interpolation at each unreliable pixel and keep the interpolation value in a queue, which consists of array elements outside the rotated subimage. Then, in the second phase we compute interpolation at every pixel (x, y) in the rotated coordinate system and store the computed value at the (x, y)-element in the array. Finally, in the third phase for each unreliable pixel (x, y) we move its interpolation value stored in the queue back to the (x, y)-element in the array.

This kind of topics may belong to Image Processing or Computer Vision. Unfortunately, as far as the authors know, there are few studies in this direction. As images are growing in size, space-efficient algorithms become more important. The algorithm in this paper may be a good source to other space-efficient algorithms.

There are increasing demands for such space-efficient algorithms. The work in this paper would open a great number of possibilities in applications to computer vision, computer graphics, and build-in software design. Image rotation is one of the most important topics for devices such as scanners. In fact, there are a number of patents such as [3]

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proposing a method for rotating images so that the number of disc accesses is minimized and [4] using JPEG compression.

This paper is organized as follows. In Sect. 2 we give a mathematical description of our problem after preparing necessary notations and definitions. Then, in Sect. 3 we present a condition to determine whether interpolation at a given pixel is reliable or not only using local geometric information. Using the condition, we give an in-place algorithm for correcting a rotated subimage without using any extra working storage. In Sect. 4 we conclude the paper together with some open problems.

2. Problem Definition

In this section we formulate a problem mathematically. An input is an image which contains a subimage rotated by some angle θ. We assume that the rotation angle is a part of input. Furthermore, for simplicity of argument we assume that the document is rotated in a counter-clockwise way. Rotation in the opposite direction can be dealt with in a symmetric manner.

Refer to Fig. 1. It is an image taken by a scanner. A document part in the figure is rotated. Given such a rotated image, we want to correct the rotation. We first execute interpolation at each pixel in the rotated subimage and store those interpolated values over the input image.

2.1 Input Image and Rotated Subimages

Input image G consists of $h \times w$ pixels. Each pixel (x, y) is associated with an intensity level. The set of all those pixels (or lattice points in the xy-coordinate system) is denoted by $G^{\#}_{wh}$ and its bounding rectangle by G_{wh}.

Rotated subimage R consists of $H \times W$ pixels, which form a set $R^{\#}_{WH}$ of pixels (or lattice points in the XY-coordinate system). An intensity level at each pixel (X, Y) is calculated by interpolation using intensity levels in the neighborhood.

We have two coordinate systems, one for the original input and the other for the rotated document. They are denoted by xy and XY, respectively. The rectangle corresponding to the input image is denoted by G_{wh} where w and h are horizontal and vertical dimensions of the rectangle, respectively. By $G^{\#}_{wh}$ we denote a set of lattice points in the rectangle. More precisely, they are defined by

$$G_{wh} = \{(x, y) \mid 0 \le x < w \text{ and } 0 \le y < h\}, \text{ and}$$
$$G^{\#}_{wh} = \{(x, y) \mid x = 0, 1, \ldots, w-1, \text{ and } y = 0, 1, \ldots, h-1\}.$$

We implicitly assume that intensity values are stored at array elements corresponding to lattice points in the set $G^{\#}_{wh}$. Now, we have another rectangle, which is a bounding box of a rotated image. We denote it by R_{WH}, where W and H are width and height of the rectangle, respectively. The set of lattice points in R_{WH} is denoted by $R^{\#}_{WH}$. More precise definitions are given by

$$R_{WH} = \{(X, Y) | 0 \le X < W \text{ and } 0 \le Y < H\}, \text{ and}$$
$$R^{\#}_{WH} = \{(X, Y) | X = 0, 1, \ldots, W-1, \text{ and } Y = 0, 1, \ldots, H-1\}.$$

Figure 2 illustrates two rectangles, G_{wh} as $ABCD$ and R_{WH} as $PQRS$.

2.2 Output Image and Location Function

An interpolation value calculated at a pixel $(X, Y) \in R^{\#}_{WH}$ in a rotated subimage is stored (or overwritten) at some pixel $s(X, Y) \in G^{\#}_{wh}$ in the original input image. The function $s(\,)$

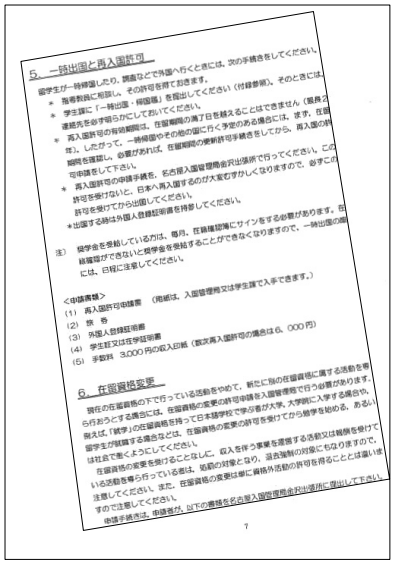

Fig. 1 An image containing a rotated subimage.

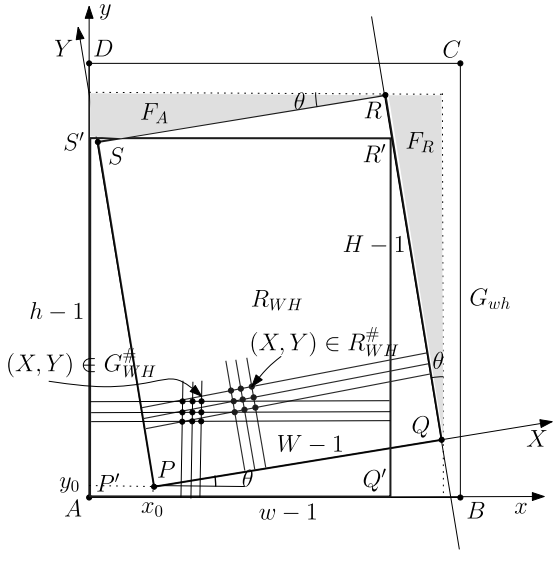

Fig. 2 Two rectangles G_{wh} and R_{WH}.

determining the location is referred to as a *location function*. A simple function is $s(X, Y) = (X, Y)$ which maps a pixel (X, Y) in $R^{\#}_{WH}$ to a pixel (X, Y) in $G^{\#}_{wh}$. We may use different location functions, but this simple function seems best for row-major and column-major raster scans. So, we implicitly fix the function.

2.3 Correspondence between Two Coordinate Systems

Let (x_0, y_0) be the xy-coordinates of the lower left corner of a rotated document (more exactly, the lower left corner of the bounding box of the rotated subimage). Now, a pixel (X, Y) in $R^{\#}_{WH}$ is a point (x, y) in the rectangle G_{wh} with

$$x = x_0 + X\cos\theta - Y\sin\theta, \text{ and}$$
$$y = y_0 + X\sin\theta + Y\cos\theta.$$

The corresponding point (x, y) defined above is denoted by $p(X, Y)$.

2.4 Scan Order $\sigma(X, Y)$

Let σ be a scanning order over the pixels in $R^{\#}_{WH}$. It is a mapping from $R^{\#}_{WH}$ to a set of integers $\{0, 1, \ldots, WH - 1\}$, that is, $\sigma(X, Y) = i$ means that the pixel (X, Y) is scanned in the i-th order. If σ is a row-major raster scan, $\sigma(X, Y) = X + Y \times W$ where $X = 0, \ldots, W - 1$ and $Y = 0, \ldots, H - 1$. A column-major raster order is symmetrically characterized by $\sigma(X, Y) = Y + X \times H$.

We could also use some angle to scan pixles. More precisely, we move a line of a specified angle from bottom to top. Pixels are reported in the order when the line hits them. If we use a line of a tiny angle counterclockwisely from the positive x-axis, then the pixels are reported in the row-major raster order.

2.5 Window $N_d(x, y)$ for Interpolation

Following a scan order σ, we take pixels in a rotated image and for each pixel (X, Y) we compute an intensity value at (X, Y) by interpolation using intensity values of pixels in the neighborhood of the corresponding point $(x, y) = p(X, Y)$ in the input image. There are a number of algorithms for interpolation. The simplest one called the nearest neighbor algorithm copies an intensity level from the nearest pixel. Linear interpolation performs interpolation by linear combination of intensity values at four immediate neighbors. An algorithm using cubic polynomials for interpolation is called a cubic interpolation. Window used for the interpolation is denoted by $N_d(x, y)$, where d is a parameter to determine the size of the window. The value of d is 1 for linear interpolation and 2 for cubic one. The window $N_d(x, y)$ for interpolation is defined by

$$N_d(x, y) = \{(x', y') \in G^{\#}_{wh} \mid \lfloor x \rfloor - d + 1 \le x' \le \lfloor x \rfloor + d, \lfloor y \rfloor - d + 1 \le y' \le \lfloor y \rfloor + d\}.$$

The set $N_d(x, y)$ consists of at most $4d^2$ elements. We do not describe how linear or cubic interpolation is calculated.

2.6 Basic Interpolation Algorithm

The following is a basic algorithm for interpolation with a scan order σ and location function $s(\,)$.

Basic interpolation algorithm
Phase 1:Scan rotated subimage
for each $(X, Y) \in R^{\#}_{WH}$ in a scan order σ do
- · Calculate a location $p(X, Y) = (x, y)$ in the xy-coordinate system.
- · Execute interpolation at (x, y) using intensity levels in the window $N_d(x, y)$.
- · Store the interpolation value at a pixel $s(X, Y) \in G^{\#}_{wh}$ specified by the location function.

Phase 2: Clear the margin
for each $(x, y) \in G^{\#}_{wh}$ do
if no interpolation value is stored at (x, y)
then the intensity level at (x, y) is set to *white*.

The basic algorithm above is simple and efficient. Unfortunately, it may lead to incorrect interpolations since when we calculate an interpolation value at some pixel we may reuse intensity levels resulting from past interpolations. More precise description follows:

Suppose we scan pixels in a rotated subimage $R^{\#}_{WH}$ and an interpolation value computed at each point (X, Y) is stored at the pixel specified by the location function $s(X, Y)$. We say interpolation at $(X, Y) \in R^{\#}_{WH}$ is *reliable* if and only if none of the pixels in the window $N_d(x, y)$ keeps interpolation value. Otherwise, the interpolation is *unreliable*. "Unreliable" does not mean that the interpolation value at the point is incorrect. Consider an image of the same intensity level. Then, interpolation does not cause any change in the intensity value anywhere. Otherwise, if we use interpolated values for interpolation, the computed value is different from the true interpolation value. We use the terminology "*unreliable*" in this sense. A pixel (X, Y) is called *reliable* if interpolation at (X, Y) is reliable and *unreliable* otherwise.

Figure 3 shows how frequently and where unreliable interpolations occur. The figure (a) is the result when rotation angle is 5 degrees in a counter-clockwise direction with window of size 1. When we increase the rotation angle to 10 degrees, we have more unreliable pixels as shown in (b). In the same setting, if we increase the window size from 1 to 2, then the number of unreliable pixels increases further as shown in (c). All these results are obtained when there is no left or bottom margin. If we have 3-pixel-wide bottom margin, i.e., $y_0 = 3$, then all unreliable pixels are gone as shown in (d).

Figure 4 shows effects of other scan orders. The ordinary raster order is characterized as left-to-right while bottom-to-top, that is, it first scans the bottom row from

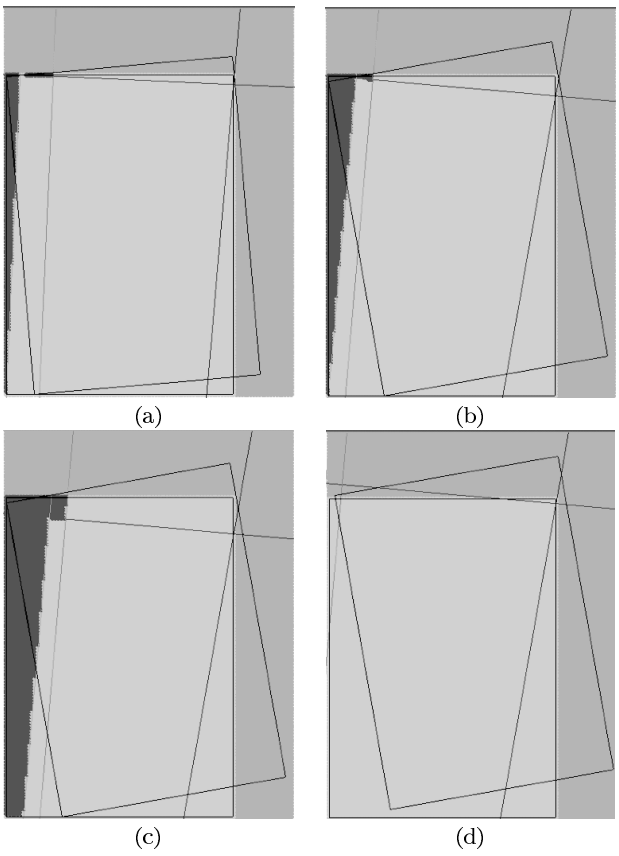

Fig. 3 Distribution of unreliable pixels. In the figure pixels the region painted red (or darkly painted part if no color is available) are unreliable. Image size is 234×170, and rotation is counterclockwisely. (a) Row-major raster with $d = 1$ with rotation angle = 5 degrees, (b) same but with angle = 10 degrees, (c) same but with window size = 2, and (d) same but with $y_0 = 3$.

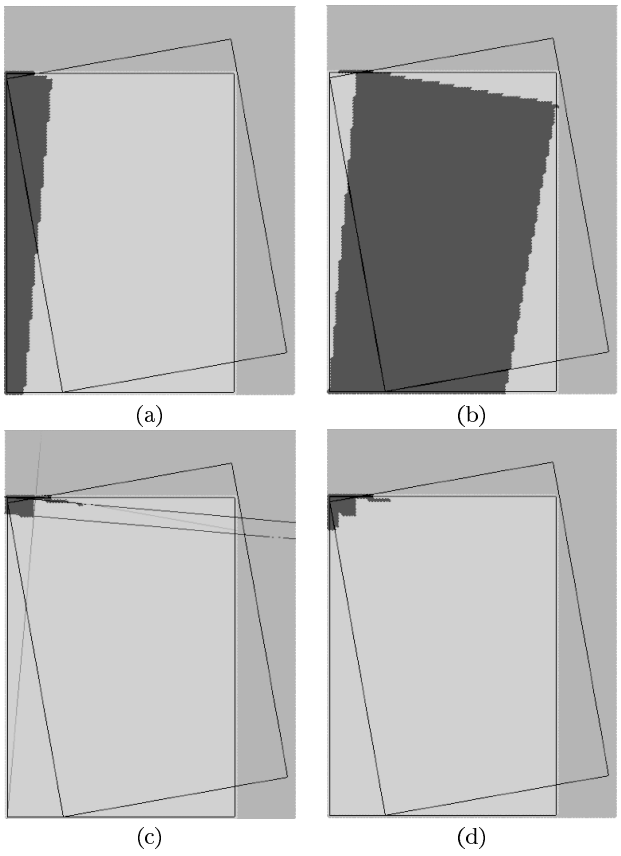

Fig. 4 Distribution of unreliable pixels. In the figure pixels the region painted red (or darkly painted part if no color is available) are unreliable. Image size is 234×170 and rotation angle is 10 degrees counterclockwisely, (a) Row-major raster (left-to-right while bottom-to-top), (b) row-major reverse raster (right-to-left while bottom-to-top), (c) column-major raster, and (d) row-major raster along 45-degree lines.

left to right and then moves to its upper row. The figure (a) shows the result of the row-major raster characterized as left-to-right while bottom-to-top. Similarly, (b) is the result for the row-major reverse raster order characterized as right-to-left while top-to-bottom, and (c) as that for the column-major raster order characterizied as bottom-to-top while left-to-right. In the figure (d), pixels are scanned in the row-major raster order along 45-degree lines characterized as left-to-right while bottom-to-top along 45-degree lines.

These experimental results suggest that the number of unreliable pixels heavily depend on a scan order we choose. It must be related to rotation direction (left or right turn) and also on rotation angle.

3. Lazy Interpolation and Local Reliability Test

An idea to avoid such incorrect interpolation is to find all unreliable pixels and keep their interpolation values somewhere in a region which is not used for output image. In the following algorithm we use a queue to keep such interpolation values.

[Lazy Interpolation]
Q: a queue to keep interpolation values at unreliable pixels.
for each pixel $(X, Y) \in R^{\#}_{WH}$ in a scan order σ do
  if (X, Y) is unreliable
    then push the interpolation value at (X, Y) into the queue Q.
for each pixel $(X, Y) \in R^{\#}_{WH}$ in the order σ do
  if (X, Y) is unreliable
    then pop a value up from the queue Q and store the value at the pixel $s(X, Y)$.
    else calculate the interpolation value at (X, Y) and store the value at the pixel $s(X, Y) \in G^{\#}_{wh}$.

Here are two problems. One is how to implement the queue. The other is how to check unreliability of a pixel. It should be remarked that both of them must be done without using any extra working storage.

Suppose we scan pixels in a rotated subimage $R^{\#}_{WH}$ according to a scan order σ and interpolation using a window of size d around each point (X, Y) is calculated and stored at an array element $s(X, Y)$ specified by the location function. Now we can define another sequence τ to determine an or-

der of all pixels in $G^{\#}_{wh}$ to receive interpolated values. That is, the function τ is defined so that

$$\tau(s(X,Y)) = \sigma(X,Y)$$

holds for any $(X,Y) \in R^{\#}_{WH}$. Since a rotated subimage is smaller than the original image, some pixels in the original image are not used for output image. That is, there are pixels (x,y) in $G^{\#}_{wh}$ such that there is no (X,Y) in $R^{\#}_{WH}$ with $(x,y) = s(X,Y)$. For such pixels (x,y) we define $\tau(x,y) = WH$. More precisely, τ is a mapping from $G^{\#}_{wh}$ to $\{0,1,\ldots,WH\}$ such that

$\tau(x,y) = i < WH$ if i-th computed interpolation value is stored at (x,y) in $G^{\#}_{wh}$,
$\tau(x,y) = WH$ if no interpolation value is stored at (x,y).

Then, interpolation at (X,Y) is reliable in the sense defined in the previous section if none of the pixels in its associated window keeps interpolated value, that is,

$$\tau(x,y) \geq \sigma(X,Y) \text{ for each } (x,y) \in N_d(p(X,Y)).$$

This condition is referred to as the reliability condition.

3.1 Row-Major Raster Scan

Consider a simple case where σ is a row-major raster scan. Let $(x,y) = p(X,Y)$, that is,

$$x = x_0 + X\cos\theta - Y\sin\theta,$$
$$y = y_0 + X\sin\theta + Y\cos\theta.$$

If we order those pixels in the interpolation window of size d around (x,y) in the order of receiving interpolation values, then the first point is $(\lfloor x\rfloor - d + 1, \lfloor y\rfloor - d + 1)$ because interpolation values are also filled in $G^{\#}_{wh}$ in the same row-major raster order (restricted to the part $0 \leq x < W$ and $0 \leq y < H$). If the first part has not received any interpolation value, that is, if $\tau(\lfloor x\rfloor - d + 1, \lfloor y\rfloor - d + 1) \geq \sigma(X,Y)$, then the pixel (X,Y) is reliable. Otherwise, it is unreliable. By the definitions of σ and τ, we have a simpler expression of the condition.

Lemma 1: [Local Reliability Condition] Assuming a row-major raster order for σ and τ, pixel $(X,Y) \in R^{\#}_{WH}$ is unreliable if and only if

(1) $x_0 + X\cos\theta - Y\sin\theta - d + 1 < X$ and $y_0 + X\sin\theta + Y\cos\theta - d < Y$, or
(2) $x_0 + X\cos\theta - Y\sin\theta - d + 1 < W$ and $y_0 + X\sin\theta + Y\cos\theta - d + 1 < Y$.

Proof By the condition stated above, a pixel (X,Y) is unreliable if and only if

(1) $\lfloor x_0 + X\cos\theta - Y\sin\theta\rfloor - d + 1 \leq X - 1$ and $\lfloor y_0 + X\sin\theta + Y\cos\theta\rfloor - d + 1 \leq Y$, or
(2) $\lfloor x_0 + X\cos\theta - Y\sin\theta\rfloor - d + 1 \leq W - 1$ and $\lfloor y_0 + X\sin\theta + Y\cos\theta\rfloor - d + 1 \leq Y - 1$.

Let a and b be two arbitrary positive real numbers. Then, $\lfloor a\rfloor \geq \lfloor b\rfloor$ holds if and only if $a \geq \lfloor b\rfloor$. Also, $\lfloor a\rfloor \leq \lfloor b\rfloor$ holds if and only if $a < \lfloor b\rfloor + 1$. (If $\lfloor a\rfloor \leq b$ then $a - 1 < \lfloor a\rfloor \leq b$, and so $a < b + 1$. If $a < b + 1$ then $\lfloor a\rfloor \leq a < b + 1$. Because of integrality, $\lfloor a\rfloor \leq b + 1 - 1 = b$.) Using these inequalities, the above condition can be restated as in the lemma.

An importance of Lemma 1 is that it suggests a way of testing reliability of interpolation at each pixel without using any working array. That is, it suffices to check the two conditions in the lemma.

By Lemma 1, a pixel (X,Y) is unreliable if and only if

(1) $Y > -\frac{1-\cos\theta}{\sin\theta}X + \frac{x_0-d+1}{\sin\theta}$ and $Y > \frac{\sin\theta}{1-\cos\theta}X + \frac{y_0-d}{1-\cos\theta}$ or
(2) $Y > \frac{\cos\theta}{\sin\theta}X - \frac{W-x_0+d-1}{\sin\theta}$ and $Y > \frac{\sin\theta}{1-\cos\theta}X + \frac{y_0-d+1}{1-\cos\theta}$.

By L_1, L_2, L_3 and L_4 we denote the four lines associated with the unreliability condition above. They are defined by

L_1: $Y = -\frac{1-\cos\theta}{\sin\theta}X + \frac{x_0-d+1}{\sin\theta}$,
L_2: $Y = \frac{\sin\theta}{1-\cos\theta}X + \frac{y_0-d}{1-\cos\theta}$,
L_3: $Y = \frac{\cos\theta}{\sin\theta}X - \frac{W-x_0+d-1}{\sin\theta}$,
L_4: $Y = \frac{\sin\theta}{1-\cos\theta}X + \frac{y_0-d+1}{1-\cos\theta}$.

Then, a pixel (X,Y) is unreliable if and only if the point (X,Y) is above the two lines L_1 and L_2 or above the two lines L_3 and L_4.

3.2 Column-Major Raster Scan

What happens if we use a column-major raster order instead of row-major order? By similar arguments we have a similar observation.

Lemma 2: Assuming a column-major raster order for σ and τ, a pixel $(X,Y) \in R^{\#}_{WH}$ is unreliable if and only if

(1′) $x_0 + X\cos\theta - Y\sin\theta - d < X$ and $y_0 + X\sin\theta + Y\cos\theta - d + 1 < Y$, or
(2′) $x_0 + X\cos\theta - Y\sin\theta - d + 1 < X$ and $H > y_0 + X\sin\theta + Y\cos\theta - d + 1 \geq Y$.

By Lemma 2, a pixel (X,Y) is unreliable if and only if

(1′) $Y > -\frac{1-\cos\theta}{\sin\theta}X + \frac{x_0-d}{\sin\theta}$ and $Y > \frac{\sin\theta}{1-\cos\theta}X + \frac{y_0-d+1}{1-\cos\theta}$ or
(2′) $Y > -\frac{1-\cos\theta}{\sin\theta}X + \frac{x_0-d+1}{\sin\theta}$ and $Y < -\frac{\sin\theta}{\cos\theta}X + \frac{H-y_0+d-1}{\cos\theta}$.

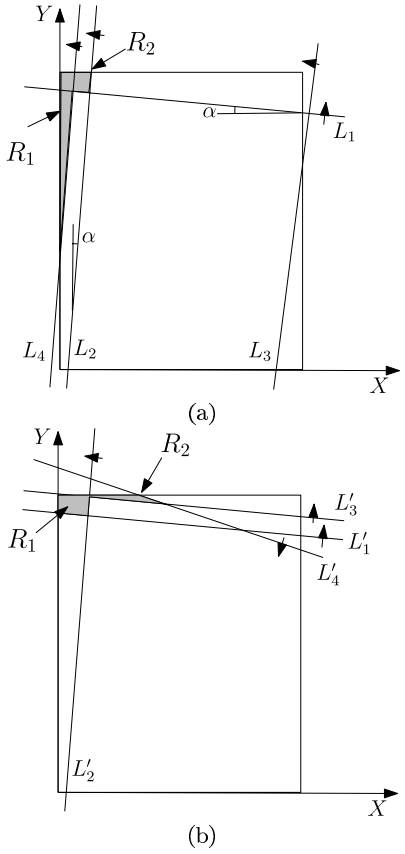

Fig. 5 Regions of unreliable pixels, (a) for row-major raster order, and (b) for column-major raster order.

By L'_1, L'_2, L'_3 and L'_4 we denote the four lines above:

$$L'_1\colon Y = -\tfrac{1-\cos\theta}{\sin\theta}X + \tfrac{x_0-d}{\sin\theta},$$
$$L'_2\colon Y = \tfrac{\sin\theta}{1-\cos\theta}X + \tfrac{y_0-d+1}{1-\cos\theta},$$
$$L'_3\colon Y = -\tfrac{1-\cos\theta}{\sin\theta}X + \tfrac{x_0-d+1}{\sin\theta},$$
$$L'_4\colon Y = -\tfrac{\sin\theta}{\cos\theta}X + \tfrac{H-y_0+d-1}{\cos\theta}.$$

Figures 5(a) and (b) depict the four lines and the region of unreliable pixels bounded by them for each of the row-major and column-major raster orders.

3.3 Lazy Interpolation for $d = 1$

Now we know how to detect possibility of unreliable pixel each in constant time. If each pixel is reliable, we just perform interpolation. Actually, if the bottom margin y_0 is large enough, then the location $s(X, Y)$ keeping interpolation value is far from a point (X, Y) and thus it does not affect interpolation around the point. Of course, if the window size d is large, then interpolations become more frequently unreliable.

Here we present an in-place algorithm for correcting rotation. For the time being we shall concentrate ourselves in the simpler case $d = 1$. A key to our algorithm is the local test on reliability. In our algorithm we scan $R^{\#}_{WH}$ twice. In the first scan, we check whether (X, Y) is a reliable pixel or not each in constant time. If it is not reliable, we calculate an interpolation value and store it somewhere in $G^{\#}_{wh}$ using a pixel outside the rectangle determining the output image. We call such a region a *refuge*.

In-place algorithm for correcting rotation

Phase 1: For each $(X, Y) \in R^{\#}_{WH}$ check whether a pixel (X, Y) is reliable or not. If it is not, then calculate interpolation there and store the value in the refuge F.

Phase 2: For each $(X, Y) \in R^{\#}_{WH}$ check whether a pixel (X, Y) is reliable or not. If it is not, then update the value at $(X, Y) \in G^{\#}_{wh}$ by the interpolation value stored in the refuge F.
Otherwise calculate interpolation there and store the value at $(X, Y) \in G^{\#}_{wh}$.

The algorithm above works correctly when $d = 1$. The most important is that the total area of refuge available is always greater than the total number of unreliable pixels.

Theorem 3: The algorithm above correctly computes interpolations for row-major and column-major raster scans with the location function $s(X, Y) = (X, Y)$.

Proof We do not prove correctness of the algorithm since it is almost trivial. We only prove that we can always find a sufficiently large refuge F. Because of similarity we only prove the theorem for the row-major raster scan.

As described earlier, the region of unreliable pixels is divided into two regions, one bounded by the two lines L_1 and L_2, and the other by L_4 and the left boundary of R_{WH}. The two regions are denoted by R_1 and R_2 in this order, as shown in Fig. 5.

We have two rectangles G_{wh} corresponding to an input image and R_{WH} to a rotated subimage. With the location function $s(X, Y) = (X, Y)$, the output image is determined by rotating R_{WH} clockwisely by the angle θ and translating it so that the lower left corner coincides with the lower left corner of G_{wh}. Drawing the horizontal line through the upper right corner and vertical line through the lower right corner of R_{WH}, we have two regions F_R and F_A, as shown in Fig. 2, which can be used as refuge. In other words, we can store any values there without affecting correct interpolations to be output.

To ease the proof we assume that there is no margin between the two rectangles G_{wh} and R_{WH}, that is, the four corners of R_{WH} all lie on the boundary of G_{wh}. In this case we have $x_0 = (H - 1)\sin\theta$ and $y_0 = 0$. Since $d = 1$, the line L_1 passes through $(0, H - 1)$ and L_4 does $(0, 0)$.

$$L_1\colon Y = -\tfrac{1-\cos\theta}{\sin\theta}X + \tfrac{x_0}{\sin\theta},$$
$$L_2\colon Y = \tfrac{\sin\theta}{1-\cos\theta}X + \tfrac{y_0-1}{1-\cos\theta},$$
$$L_4\colon Y = \tfrac{\sin\theta}{1-\cos\theta}X + \tfrac{y_0}{1-\cos\theta}.$$

The angle α between the line L_4 and the vertical line is smaller than θ because

$$\tan(\alpha) = \frac{1-\cos\theta}{\sin\theta} < \tan\theta.$$

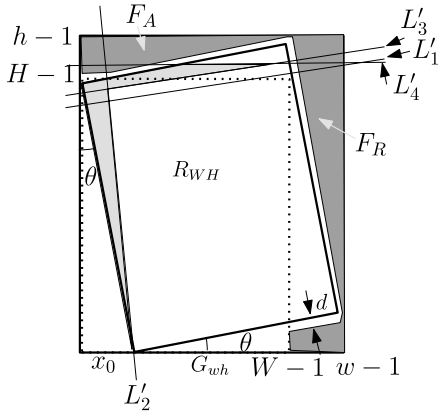

Fig. 6 The region of unreliable pixels and right and top refuges F_R and F_A.

Thus, the area of the region (R_1 in Fig. 5(a)) bounded by L_4 and the left boundary is smaller than the refuge F_R bounded by the line RQ and the right boundary of G_{wh} (see Fig. 2).

By the same reason we can also prove that the area of the region R_2 bounded by L_1 and L_2 is smaller than that of the region F_A above the line SR in Fig. 2. This completes the proof.

3.4 Lazy Interpolation for $d = 2$

With a larger window of size $d \geq 2$ the algorithm above does not work due to insufficient area of the refuge. Fortunately, if the lower margin, y_0, is at least $d-1$, then the lazy interpolation for the column-major raster works correctly. When $y_0 = d-1$ and $d \geq 2$, the unreliable region is the union of the two regions R_1 above L'_1 and L'_2 and R_2 above L'_3 and below L'_4. The line L'_2 passes through the origin, we can use the right refuge F_R as before for R_1.

What about the region R_2 bounded by L'_3 and L'_4? The line L'_4 is parallel to the horizontal side of the rectangle G_{wh} and the line L'_3 has smaller slope than the upper side of the rotated rectangle. Hence, the angle between L'_3 and L'_4 is smaller than θ. This implies that the region R_2 bounded by L'_3 and L'_4 has smaller area than the upper refuge F_A. See Fig. 6 for illustration.

In case of insufficient bottom margin, that is, if $y_0 < d-1$, unfortunately, we cannot use the algorithm above for a larger window, $d \geq 2$ since we may have so many unreliable pixels even in the case. The idea here is to use a queue to store interpolation values at unreliable pixels and pop them up whenever storing them does not cause any harm for interpolations. The region outside the rotated image and the output image, shown in Fig. 6, can be used for the purpose.

Assume a row-major raster order. Suppose we are going to calculate interpolation at pixels in a row Y. Then, the pixel values below the row $\lfloor y_0 + Y\cos\theta \rfloor - d$ (including the row) are never used for interpolations. Let us call the row the high limit for Y. If it is greater than the previous high limit, i.e., $\lfloor y_0 + (Y-1)\cos\theta \rfloor - d$, then we can safely store interpolation values at the row. This observation leads to the following algorithm.

In-place algorithm 2 for correcting rotation

Q = a queue containing interpolated values, using the region in the refuge.
for each row $Y = 0$ to $H-1$ do
  for each $X = 0$ to $W-1$
    Compute interpolation at (X, Y).
    if (X, Y) is unreliable
      then push the interpolation value at (X, Y) into the queue Q.
  if $\lfloor y_0 + Y\cos\theta \rfloor - 2 > \lfloor y_0 + (Y-1)\cos\theta \rfloor - 2$
    then $Y' = \lfloor y_0 + (Y-1)\cos\theta \rfloor - 2$.
    for each $X = 0$ to $W-1$
      if (X, Y') is unreliable
        then store the value popped from Q at $s(X, Y')$.
        else calculate interpolation value at (X, Y') and store it at $s(X, Y')$.

Unfortunately, no formal proof has not been obtained for correctness of the algorithm above. However, it has caused no problem for practical applications.

4. Concluding Remarks and Future Works

In this paper we have presented in-place algorithms for correcting rotation of a subimage contained in an image using interpolation. We have shown that as long as interpolation is implemented by linear interpolation algorithm we can always correct any rotation without using any extra working array. Correctness proof for a larger window used for cubic interpolation has been left as an open problem.

In this paper we considered two scan orders, row-major and column-major raster orders. Many other scan orders are possible. In addition to row- and column major raster scans we could scan an image at any angle. One of promising scans is the following: First, find a rotation angle θ. Then, round it to an angle θ' defined by two pixels in a rotated subimage. Using this approximate angle, we can scan all of pixels in the rotated subimage without any extra working storage.

It is interesting to evaluate and compare those scan orders by the number of unreliable pixels. The best scan order may depend on margins. In our experience, if the bottom margin is greater than the left margin then the row-major raster is better than the column-major one. If the left margin is larger than the bottom margin, the column-major raster outperforms row-major raster. But there is no formal proof.

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