ABSTRACT

On Negation-Limited Circuit Complexity

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A central problem of the theory of computing is to understand the inherent complexity of computational tasks in terms of resources required. We study the complexity of Boolean functions over a model, *circuits*. While proving a superlinear lower bound on general circuits remain a hard open problem, we investigate a restricted versions of circuit, called *negation-limited circuit*.

In this thesis, we first consider the negation-limited circuits over basis $\{\wedge, \vee, \neg\}$. We show some properties of negation-limited circuits. From such properties, we show lower bounds on size and depth of negation-limited circuits computing Boolean functions. In particular, we obtain a $5.33n$ lower bound on size negation-limited circuits computing parity functions. Then, we consider the complexity of negation-limited inverters. By inverters, The complexity of general circuits is related to that of negation-limited circuits. Thus, it is important to understand the negation-limited inverters. We show an upper bound on depth of negation-limited inverters. Such an upper bound possibly matches the lower bound. We also show a $7.33n$ lower bound on size. Under a natural assumption, we obtain an $\Omega(n \log n)$ lower bound on size for negation-limited inverters, while the best upper bound on size is $O(n \log n)$.

Next, we consider the negation-limited threshold circuits. We first show a lower bound on the minimum number of negation gates in threshold circuits computing any given Boolean function. We also show an upper bound on the minimum number of negation gates in threshold circuits computing some given Boolean functions which matches the lower bound.

For inverters and parity functions, we show the negation-limited threshold circuits computing such functions which consists of minimum number of negation gates. As an application of negation-limited circuit complexity, we obtain a lower bound on size of threshold circuits computing any given Boolean function. In particular, such a lower bound for parity function matches our upper bound.

Keywords: computational complexity, negation-limited circuits, threshold circuits, inverter, parity function.