Numerical Methods for Solving Optimal Control Problems Using Chebyshev Polynomials

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Abstract

Many computational methods have been proposed to solve optimal control problems. These methods are classified as indirect methods and direct methods. This thesis is based on solving optimal control problems using direct methods in which an optimal control problem is converted into a mathematical programming problem. The direct methods can be employed by using the parameterization technique which can be applied in three different ways: Control parameterization, control-state parameterization and state parameterization. The control parameterization and the control-state parameterization have been used extensively to solve general optimal control problems. However, the use of the state parameterization was limited to very special cases. In this thesis, we solve general optimal control problems by using the state parameterization.

This thesis presents numerical methods to solve unconstrained and constrained optimal control problems. The solution method is based on using the second method of the quasilinearization to replace the nonlinear optimal control problem by a sequence of time-varying linear quadratic optimal control problems. Each of these problems is solved by converting it into quadratic programming problem. To this end, the state parameterization technique is employed by using the Chebyshev polynomials of the first type to approximate the system state variables by a finite length Chebyshev series of unknown parameters.

In addition, in this thesis we describe a method to determine the optimal feedback control of nonlinear optimal control problems. To facilitate the computation of the optimal feedback control law, a new property of Chebyshev polynomials called differentiation operational matrix is derived.

The proposed methods have been applied on several examples and we find that the proposed methods give better or comparable results compared with some other methods. Additionally, to make sure that the proposed methods can handle practical problems, we applied these methods on two practical problems, F8 fighter aircraft and container crane problems.

Key Words: **Optimal control problem, constrained optimal control problem, state parameterization, Chebyshev polynomials, quadratic programming, optimal feedback control.**