Reduction Strategies for Term Rewriting Systems

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Abstract

Term rewriting systems have been widely studied as a model for computation. In a term rewriting system, they may exist an infinite reduction sequence starting with a term having normal forms. In order to get a normal form for a given term, we require a normalizing strategy guaranteeing to find a normal form of terms whenever their normal forms exist. Huet and Lévy (1979) showed that a call-by-need strategy is normalizing for every orthogonal (i.e., left-linear and non-overlapping) term rewriting systems. Unfortunately, in general a call-by-need strategy is undecidable. They formalized strong sequentiality guaranteeing a decidable normalizing call-by-need strategy for orthogonal term rewriting system. The work of Huet and Lévy has been extended to several kinds of systems.

In this thesis we first extend the class of left-linear term rewriting systems having a decidable call-by-need strategy. We present the class of NVNF-sequential systems. This class properly includes the class of NV-sequential systems which was introduced by Oyamaguchi (1993). We prove that every orthogonal NVNF-sequential system has a decidable normalizing call-by-need strategy. Then we give growing approximations of term rewriting systems without the assumption of the right-linearity whereas Jacuemard (1993) assumed the right-linearity . We show that our approximations extend the class of orthogonal term rewriting systems having a decidable normalizing call-by-need strategy.

Secondly, we investigate the normalizability of a call-by-need strategy for left-linear overlapping term rewriting systems. We first introduced the notion of stable balanced joinability. It is shown that a call-by-need strategy is normalizing for every stable balanced joinable strongly sequential system. This is a generalization of Toyama's result (1992). We next introduce the notion of NV-stable balanced joinability and prove that every NV-stable balanced joinable NV-sequential system has a decidable normalizing call-by-need strategy.

Finally, we apply the results on call-by-need strategy to the E-strategy adopted by the OBJ algebraic specification languages. The E-strategy chooses a redex according to local strategies which are given to each function symbol. We consider how to give local strategies to make the E-strategy normalizing. For this purpose, we introduced the notion index-transitivity and carefulness. We show that for every index-transitive orthogonal

term rewriting system, if careful local strategies are given to each function symbol then the E-strategy is normalizing.

Key Words: **term rewriting system, reduction strategy, normalizability, sequentiality**