Convergence Analysis of MMSE Based Multiuser MIMO Turbo Detector with Linear Precoding Strategies

Juha Karjalainen [1]
[1] Centre for Wireless Communications
University of Oulu
FIN-90014, Oulu, Finland
Email: juha.karjalainen@ee.oulu.fi

Tad Matsumoto [1,2]
[2] Japan Advanced Institute of Science and Technology, 1-1 Asahidai, Nomi
Ishikawa 923-1292, Japan
Email: matumoto@jaist.ac.jp

Wolfgang Utschick [3]
[3] Associate Institute for Signal Processing
Munich University of Technology
Munich, Germany
Email: utschick@tum.de

***Abstract*—This paper studies mutual information transfer properties of iterative multiuser detector with linear precoding schemes for single carrier communications in multipoint-to-point multiple input multiple output (MIMO) channels. Based on multidimensional extrinsic information transfer (EXIT) analysis, we analyze the impact of linear precoding on the convergence properties of the detectors jointly designed with the precoders based on minimum sum mean squared error (MMSE) and maximum sum mutual information rate criteria. Results demonstrate that the use of the linear precoding schemes enhance the separability of the EXIT curves of the simultaneous streams over without precoding; This invokes the idea that different code rate be allocated to the each transmitted streams at the transmitters.**

I. Introduction

In recent years, the design of multiple input multiple output (MIMO) systems based on joint optimization of linear transmitters and receivers has attracted much interest due to its significant flexibility in designing the transmission chain, with the aim of improving system performance and efficiency [1],[2]. However, to fully exploit the advantageous points provided by the joint linear transceiver design perfect channel state information (CSI) has to, in many cases, be available at the both transmitter and receiver sides. In fact, for point-to-point MIMO systems, there has been a lot of research efforts on optimum joint linear transceiver design based on maximum information rate as well as minimum mean square error (MMSE) criteria [2],[3],[4]. As a result of the joint optimization in cyclic prefix-appended multicarrier point-to-point MIMO systems, signal transmission chains in frequency selective channels can be decoupled into independent parallel sub-channels. Hence, the optimization reduces to power allocation problem among the sub-channels. For multipoint-to-point systems, situation becomes more challenging. Recently, Yu et al. [5] has presented an efficient iterative-waterfilling algorithm that maximizes sum capacity of multipoint-to-point MIMO systems by computing optimal transmit covariance matrices for all the users. In [6] Luo et al. presented optimal linear transceiver design in an MMSE sense for multipoint-to-point MIMO systems. References [7],[8] proposed iterative methods to solve the joint linear transceiver design under an MMSE criterion for multipoint-to-point channels.

In contrast to point-to-point cases, in multipoint-to-point MIMO systems the co-channel interference (CCI), due to interference caused by the users sharing the same channel, cannot be handled optimally by the joint linear transceiver design approach. Therefore, it is believed that the non-linear receiver, such as iterative receivers, can achieve to substantial performance improvements.

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Despite the volume of the literatures describing non-iterative joint transceiver design, there is quite limited work considering the combination of iterative equalization and transmitter side processing. Reference [9] proposes a combined use of iterative equalization and transmit beam steering techniques for point-to-point MIMO systems. In Ref. [9]'s proposed scheme signal processing is performed in the time domain per each multipath component without performing the power control by assuming perfect CSI at the both transmitter and receiver sides. Recently, in [10] transmit antenna selection has been proposed for iterative equalization for a point-to-point MIMO system by assuming limited receiver-to-transmitter feedback . However, the previous studies dealing with the combination of transmitter side processing and iterative equalization at the receiver side have neither considered multipoint-to-point scenarios nor the effect of precoding to the convergence property of iterative equalization.

Despite the benefits of linear precoding with linear receivers, a fundamental question associated with iterative receivers arises that how sensitive (or insensitive) the convergence property of iterative receiver is to precoder design criterion. This question has not yet been thoroughly investigated. Therefore, it is set a primary goal of of this paper that we provide deeper insights into these questions by assuming frequency domain soft cancellation minimum mean squared error (FD SC MMSE) iterative (turbo) equalization [11].

Based upon our recent paper [12] on the converge behaviour of turbo equalizer with minimum sum MSE precoding with *non systematic* repeat accumulate (RA) codes, we extend our previous studies on multiuser uplink MIMO systems to maximum mutual information rate based design with *systematic* RA codes. The optimality definition does not assume any decoder feedback, similarly, as in the case of the minimum sum MSE precoding. Furthermore, in addition to centralized design, we consider de-centralize scenario, where only each user's transmitter side CSI is utilized in the precoder design. Given the system setup, we make a comparison between minimum sum MSE and maximum mutual information rate in centralize and de-centralize based design scenarios.

Since single carrier frequency division multiple access (FDMA) has been recognized as one of the most attractive candidates for uplink transmission scheme in the 3GPP long term evolution scenario making framework [13] we will focus on generic system model that allows the systems to flexibly combine the operation mode from spatial division multiple access (SDMA) and as FDMA.

In this paper, we perform multidimensional extrinsic information transfer (EXIT) analysis by using projection technique [14] that is a straightforward extension of the EXIT chart analysis [15]. The reason for the necessity of the multidimen-

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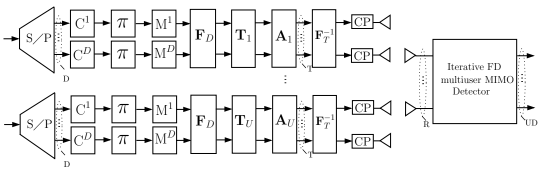


Fig. 1: Linearly precoded multiuser MIMO system for uplink communications.

sional EXIT analysis is because in multipoint-to-point MIMO or point-to-point MIMO with multiple encoders, the mutual information exchange via the turbo loop for a user is affected by the other users/encoders.

II. System Model

In this paper uplink of a single cell system with U synchronous users is considered. The both users and the base station are equipped with multiple antennas, T transmit and R receive antennas, respectively. Each of the simultaneous uplink users multiplexes its fixed number D of data streams through its T transmit antennas. A model of linearly precoded multiuser MIMO uplink system, considered in this paper, is depicted in Fig. 1. After guard period removal,[1] a space-time presentation of the signal vector $\tilde{\mathbf{r}} \in \mathbb{C}^{RK_B\times 1}$ received by the R received antennas is given by

$$\mathbf{r} = \hat{\mathbf{H}}\mathbf{F}_U^{-1}\mathbf{ATFb} + \mathbf{v}, \qquad (1)$$

where $\mathbf{v} \in \mathbb{C}^{RK_B\times 1}$ is a white additive independent identically distributed (i.i.d) Gaussian noise vector with variance σ^2 per dimension, with K_B being the length of discrete Fourier transform (DFT) over entire bandwidth shared by all the users, and $\mathbf{b} \in \mathbb{C}^{UDK_S\times 1}$ is the transmitted multiuser signal vector

$$\mathbf{b} \;\;=\;\; [\mathbf{b}^1, ..., \mathbf{b}^u, ..., \mathbf{b}^U]^\dagger \qquad (2)$$

with $\mathbf{b}^u \in \mathbb{C}^{DK_S\times 1}$, K_S being the length of discrete Fourier transform (DFT) over each user's bandwidth and $u = 1, ..., U$. The sub-vectors $\mathbf{b}^u$ of $\mathbf{b}$ is given by

$$\mathbf{b}^u \;\;=\;\; [\mathbf{b}^{u,1}, ..., \mathbf{b}^{u,d}, ..., \mathbf{b}^{u,D}]^\dagger \qquad (3)$$

denoting the u^{th} user's transmitted streams over the T transmit antennas. $\mathbf{b}^{u,d} \in \mathbb{C}^{K_S\times 1}$ is given by

$$\mathbf{b}^{u,d} \;\;=\;\; [b_1^{u,d}, ..., b_k^{u,d}, ..., b_{K_s}{}^{u,d}]^\dagger, \qquad (4)$$

where $t = 1, ..., T$ and $k = 0, ..., K_S - 1$ index the transmitted symbols of the u^{th} user's t^{th} layer. The precoder matrix $\mathbf{T} \in \mathbb{C}^{UTK_S\times UDK_S}$ is given by

$$\mathbf{T} \;\;=\;\; bdiag\{[\mathbf{T}_1...\mathbf{T}_u...\mathbf{T}_U]^\dagger\}, \qquad (5)$$

where $\mathbf{T}_u \in \mathbb{C}^{TK_S\times DK_S}$ is each user's precoder matrix and the operator $bdiag\{\}$ generates block diagonal matrix from its argument components. The frequency bin allocation matrix $\mathbf{A} \in \mathbb{B}^{UTK_B\times UTK_S}$ for all the users is defined as

$$\mathbf{A} \;\;=\;\; bdiag\{[\mathbf{A}_1...\mathbf{A}_u...\mathbf{A}_U]^\dagger\}, \qquad (6)$$

where $\mathbf{A}_u = bdiag\{[\mathbf{A}_u^1...\mathbf{A}_u^t...\mathbf{A}_u^T]^\dagger\} \in \mathbb{B}^{TK_B\times TK_S}$ denotes each user's frequency bin allocation with $\mathbf{A}_u^t \in \mathbb{B}^{K_B\times K_S}$ being the bin allocation matrix for the t^{th} transmit antenna of the u^{th} user. [2] It should be noticed that depending on the positions of zeros and ones in bin allocation matrix $\mathbf{A}_u$, both SDMA/FDMA based multiple-access methods can be expressed with a unified notation using the matrix $\mathbf{A}$. However, it should be noted that the optimization of frequency bin allocation is out of the scope of this study.

The circulant block channel matrix $\hat{\mathbf{H}} \in \mathbb{C}^{RK_B\times UTK_B}$ is then given as

$$\hat{\mathbf{H}} \;\;=\;\; [\hat{\mathbf{H}}_1, ..., \hat{\mathbf{H}}_u, ..., \hat{\mathbf{H}}_U], \qquad (7)$$

where $\hat{\mathbf{H}}_u \in \mathbb{C}^{RK_B\times TK_B}$ with $u = 1, \cdots, U$ is a circulant block matrix corresponding to the u^{th} user. The circulant block matrix for the u^{th} user is denoted as

$$\hat{\mathbf{H}}_u = \begin{bmatrix} \hat{\mathbf{H}}_u^{1,1} & \cdots & \hat{\mathbf{H}}_u^{1,T} \\ \vdots & \ddots & \vdots \\ \hat{\mathbf{H}}_u^{R,1} & \cdots & \hat{\mathbf{H}}_u^{R,T} \end{bmatrix}, \qquad (8)$$

where the channel submatrices $\hat{\mathbf{H}}_u^{r,t} \in \mathbb{C}^{K_B\times K_B}$ between the t^{th} transmit and the r^{th} receive antennas, $r = 1, ..., R$, are also circulant, as

$$\hat{\mathbf{H}}_u^{r,t} = circ\left\{\left[h_{u,1}^{r,t}, h_{u,2}^{r,t} \ldots h_{u,L}^{r,t}\right]^\dagger\right\}. \qquad (9)$$

The operator $circ\{\ \}$ generates matrix that has a circulant structure of its argument. L denotes the length of the channel, and $h_{u,l}^{r,t}$, $l = 1, ..., L$, the fading gains of multipath channel between the u^{th} user's t^{th} transmit antenna and the r^{th} receive antenna. For the each user's transmit-receive antenna pair the sum of the average power of fading gains is normalized to one. It is well known that the circulant matrices can be diagonalized by the unitary DFT matrix $\mathbf{F_B} \in \mathbb{C}^{K_B\times K_B}$ with the elements $f_{m,k} = \exp^{\frac{j2\pi m_B k_B}{K}}$, where $m_B, k_B = 0, ..., K_B - 1$. Similarly, the circulant block matrices can be block-diagonalized by using block diagonal DFT matrices. The block-diagonalization of $\hat{\mathbf{H}}$ is performed as

$$\hat{\mathbf{H}} = \mathbf{F}_R^{-1}\mathbf{\Gamma}\mathbf{F}_U, \qquad (10)$$

where $\mathbf{\Gamma} \in \mathbb{C}^{RK_B\times UTK_B}$ is the corresponding diagonal block matrix, and $\mathbf{F}_R^{-1} = \frac{1}{K_B}\mathbf{F}_R^\ddagger \in \mathbb{C}^{RK_B\times RK_B}$ is the unitary block inverse discrete fourier transform (IDFT) matrix. ‡ indicates the Hermitian transpose, and $\mathbf{F}_R \in \mathbb{C}^{RK_B\times RK_B}$ is block-diagonal DFT matrix given by $\mathbf{F}_R = \mathbf{I}_R \otimes \mathbf{F}_B$ for the R received antennas, where $\mathbf{F}_B \in \mathbb{C}^{K_B\times K_B}$ is the unitary DFT matrix with $\mathbf{I}_R \in \mathbb{R}^{R\times R}$ being an identity matrix and the symbol $\otimes$ indicates the Kronecker product. $\mathbf{F}_U \in \mathbb{C}^{UTK_B\times UTK_B}$ is given by $\mathbf{F}_U = \mathbf{I}_U \otimes \mathbf{F}_T$ for the transmit antennas of all users, where $\mathbf{I}_U \in \mathbb{R}^{U\times U}$ is an identity matrix and $\mathbf{F}_T = \mathbf{I}_T \otimes \mathbf{F}_B$ with $\mathbf{I}_T \in \mathbb{R}^{T\times T}$ being an identity matrix. Correspondingly, the block-diagonal DFT matrices $\mathbf{F}_D \in \mathbb{C}^{DK_S\times DK_S}$ and $\mathbf{F} \in \mathbb{C}^{UDK_S\times UDK_S}$ are defined as $\mathbf{F}_D = \mathbf{I}_D \otimes \mathbf{F}_S$ and $\mathbf{F} = \mathbf{I}_U \otimes \mathbf{F}_D$, respectively. The matrix $\mathbf{F}_S \in \mathbb{C}^{K_S\times K_S}$ is the unitary DFT matrix with $\mathbf{I}_D \in \mathbb{R}^{D\times D}$ being an identity matrix. Average signal-to-noise ratio per receiver antenna is defined as ratio of information bit power and noise power, as $SNR = \frac{\tilde{P}_u}{2\sigma^2}$, where, $\tilde{P}_u$ is the average transmitted symbol energy per user.

III. The Joint MMSE Based Transceiver Design

Our goal here is to design transmitter-receiver pairs for all the users that minimize the total Mean Square Error (MSE) $\mathbb{E}_{tot}$ of the system subject to transmit power constraint for the each user. Let the MMSE optimization problem for the joint

[1] We restrict ourselves to the case where the length of guard period is larger than or as large as the channel memory length.

[2] All the frequency bin allocation matrices of u^{th} user, $\mathbf{A}_u^t$, are assumed to be equivalent for each transmit antenna.

precoder and equalizer design be expressed as follows [6]

$$[\mathbf{T}_1, \mathbf{\Omega}_1, ..., \mathbf{T}_u, \mathbf{\Omega}_u, ..., \mathbf{T}_U, \mathbf{\Omega}_U] = \min_{[\mathbf{T}_1, \mathbf{\Omega}_1, ..., \mathbf{T}_u, \mathbf{\Omega}_u, ..., \mathbf{T}_U, \mathbf{\Omega}_U]} \mathbb{E}_{tot}$$

$$\mathbb{E}_{tot} = \sum_{u=1}^{U} \mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u) \quad (11)$$

$$s.t: \quad Tr(\mathbf{T}_u \mathbf{T}_u^{\ddagger}) \leq p_u$$

where $\mathbf{\Omega}_u \in \mathbb{C}^{RK_S \times DK_S}$ corresponds to the u^{th} user's receive MMSE filter and p_u is the transmission power for u^{th} user. $\mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u)$ is the MSE of the u^{th} user given by

$$\mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u) = Tr\{E\{\mathbf{e}_u \mathbf{e}_u^{\ddagger}\}\} \quad (12)$$

and $\mathbf{e}_u = \mathbf{b}_u - \hat{\mathbf{b}}_u \in \mathbb{C}^{DK_S \times 1}$ is error vector with $\hat{\mathbf{b}}_\mathbf{u} \in \mathbb{C}^{DK_S \times 1}$ being the estimate of transmitted streams at the output of MMSE filter, given by

$$\hat{\mathbf{b}}_u = \mathbf{F}_D^{-1} \mathbf{\Omega}_u^{\ddagger} \hat{\mathbf{r}}_u \quad (13)$$

where the vector $\hat{\mathbf{r}}_u \in \mathbb{C}^{RK_B \times 1}$ combines the soft-cancellation outputs for the linearly precoded transmitted streams, as

$$\hat{\mathbf{r}}_u = \hat{\mathbf{r}} + \tilde{\mathbf{\Gamma}}_u \mathbf{T}_u \mathbf{F}_D \mathbf{S}(n) \tilde{\mathbf{b}}^u. \quad (14)$$

Here, the matrix $\tilde{\mathbf{\Gamma}}_u = \widehat{\mathbf{A}}_u^{\ddagger} \mathbf{\Gamma}_u \mathbf{A}_u \in \mathbb{C}^{RK_S \times TK_S}$ is the effective channel matrix corresponding to the frequency bins of the channel allocated to the u^{th} user with $\mathbf{\Gamma}_u \in \mathbb{C}^{RK_B \times TK_B}$ being the u^{th} desired user's frequency domain channel matrix. The frequency bin allocation matrix $\widehat{\mathbf{A}}_u \in \mathbb{R}^{RK_B \times RK_B}$ at the receiver side is given as

$$\widehat{\mathbf{A}}_u = bdiag\{[\widehat{\mathbf{A}}_u^1 ... \widehat{\mathbf{A}}_u^r ... \widehat{\mathbf{A}}_u^R]^{\dagger}\}, \quad (15)$$

where $\widehat{\mathbf{A}}_u^r = \mathbf{A}_u^t \in \mathbb{B}^{K_B \times K_S}$. The output $\hat{\mathbf{r}} \in \mathbb{C}^{RK_B \times 1}$ of the soft cancellation and the soft estimate $\tilde{\mathbf{b}}^u \in \mathbb{C}^{DK_S \times 1}$ of the u^{th} user's transmitted streams user are described more in detailed in the Appendix A.

A. Iterative Equalization

Due to the lack of space, the full derivation of the receive filter matrices is omitted in this paper. Instead, we only present the result and part of the derivation in Appendix B.

B. Linear Precoding

1) MinSum-MSE: In this subsection the design of a set of precoders for the U users is considered. In the precoder design it is assumed that a-priori information provided by each user's channel decoder is not utilized. Now, let us rewrite the total MSE of the system using (31), (29) and (11) as:

$$\mathbb{E}_{tot} = \sum_{u=1}^{U} Tr\{\mathbf{\Sigma}_{b_u}\} - Tr\{\mathbf{\Sigma}_{b_u} \mathbf{S}(n) \mathbf{F}_D^{-1} \mathbf{T}_u^{\ddagger} \tilde{\mathbf{\Gamma}}_u^{\ddagger} \mathbf{\Sigma}_{\hat{\mathbf{r}}_u}^{-1} \tilde{\mathbf{\Gamma}}_u \mathbf{T}_u \mathbf{F}_D \mathbf{S}(n) \mathbf{\Sigma}_{b_u}^{\ddagger}\}. \quad (16)$$

Recall that *a*-priori information is not utilized, which results in $\mathbf{\Sigma}_{b_u} = \mathbf{I}$, $\mathbf{\Sigma}_{\hat{\mathbf{r}}_u} = \mathbf{\Sigma}_{\hat{\mathbf{r}}}$ and $\mathbf{\Delta} = \mathbf{I}$. Moreover, the sampling matrices $\mathbf{S}(n)$ can also be eliminated, due to the time-invariance assumption of the residual interference over the frame. Now, the total MSE in (16) can be re-written as follows

$$\mathbb{E}_{tot} = UDK_s - RK_S + \sigma^2 Tr\{\mathbf{\Sigma}_{\hat{r}}^{-1}\}. \quad (17)$$

It can be also observed that the total MSE in (17) is not jointly convex with respect to $\mathbf{T}_u$. Therefore, the problem has to be reformulate into convex to find the global optimum. In this paper, we follow closely the technique presented in [6] to reformulate the problem. First of all, an auxiliary matrix $\mathbf{U}_u \in \mathbb{C}^{TK_S \times TK_S}$ is introduced as

$$\mathbf{U}_u = \mathbf{T}_u \mathbf{T}_u^{\ddagger}. \quad (18)$$

Now, by using (18) the total MSE in (17) can be re-written as

$$\mathbb{E}_{tot} = UDK_s - RK_B + \sigma^2 Tr\{\mathbf{E}\} \quad (19)$$

where the MSE matrix $\mathbf{E} \in \mathbb{C}^{RK_S \times RK_S}$ is given as

$$\mathbf{E} = (\sigma^2 \mathbf{I} + \tilde{\mathbf{\Gamma}} \mathbf{U} \tilde{\mathbf{\Gamma}}^{\ddagger})^{-1}. \quad (20)$$

As a results of this, our objective function in (19) becomes convex with respect to $\mathbf{U}$ and constraints are convex as well after the Schur's complement computation. Therefore, convex optimization methods, e.g. semidefinite programming (SDP) can be used to find the global optimum. SDP problems, in general, can be solved efficiently using e.g standard convex optimization package [16]. Finally, by using (19) and via Schur's complement the joint transmitter-receiver MMSE design problem can be stated as an SDP problem, as

$$\begin{aligned} \min_{\mathbf{E}, \mathbf{U}_1, ..., \mathbf{U}_u, ..., \mathbf{U}_U} \quad & Tr\{\mathbf{E}\} \\ s.t. \quad & Tr\{\mathbf{U}_u\} \leq p_u \\ & \mathbf{E}\, satisfies (22) \\ & \mathbf{U}_u \succeq \mathbf{0}, \qquad u = 1, ..., U \end{aligned} \quad (21)$$

with

$$\begin{bmatrix} \mathbf{E} & \mathbf{I} \\ \mathbf{I} & \sigma^2 \mathbf{I} + \sum_{u=1}^{U} \tilde{\mathbf{\Gamma}}_u \mathbf{U}_u \tilde{\mathbf{\Gamma}}_u^{\ddagger} \end{bmatrix} \succeq \mathbf{0}. \quad (22)$$

Now, the set of optimal transmit covariance matrices $\mathbf{U}_u$ for all users can be found by using (21). Therefore, in the MMSE sense the optimal linear precoders $\mathbf{T}_u$ of all the users are obtained by applying the singular value decomposition separately to each $\mathbf{U}_u$, resulting in

$$\mathbf{T}_u = \mathbf{V}_u \mathbf{P}_u^{\frac{1}{2}}. \quad (23)$$

The diagonal matrix $\mathbf{P}_u^{\frac{1}{2}} \in \mathbb{R}^{DK_S \times DK_S}$ is the power allocation matrix of the u^{th} user with diagonal elements corresponding to the square root of power allocated on each frequency bin. Correspondingly, $\mathbf{V}_u \in \mathbb{C}^{TK_S \times DK_S}$ is the beamformer matrix of the u^{th} user.

2) Max-Rate: In this subsection we consider precoder design based on maximization of mutual information. Correspondingly, as in the case of MinSum-MSE, it is assumed that a-priori information provided by each user's channel decoder is not utilized. Reference [2] shows that the link between mutual information and MSE matrix can be written as

$$I = -\log_2 |\mathbf{E}|. \quad (24)$$

As a results of this, the maximization of I is equivalent to the minimization of $|\mathbf{E}|$ [2]. The minimization problem of $|\mathbf{E}|$ belongs to a class of convex optimization problems and can be solved e.g standard convex optimization package [16]. However, in this paper we utilize very well known Yu et all's efficient iterative waterfilling algorithm [5] to find the optimal transmit covariance matrices $\mathbf{U}_u$ for each user that maximizes the sum capacity of MIMO multiple access channel. Since iterative waterfilling algorithm is very well known details are not given here and can be found in [5].

IV. Convergence Property Evaluation

Simulation parameters are summarized as follows; The number of users $U = 1, 2, 3$, receiver antennas at base station $R = 2, 4$, transmit antennas per user $T = 2, 4$, streams per user $D = 2, 4$, FFT sizes are $F_B = F_S = 512$, QPSK ($M = 4$) with Gray mapping, and a rate 1/3 systematic RA channel code [17] for all streams in the system. The decoding is performed with the sum-product algorithm. The number of decoding iterations is set at 6. A quasistatic Rayleigh fading channel with $L = 5$ is assumed where each path has equal average gain. In the case of multiuser transmission, SDMA is assumed as a multiple access method. The number of iterative waterfilling algorithm iterations depends on the number of users: 8 iterations with $U = 2$ and 12 iterations with $U = 3$.

In this paper, we use the EXIT chart [15] as well as its projection [14] techniques to analyze convergence properties of the proposed iterative multiuser detector with uplink precoding. The results of analysis were then averaged over channel realizations. Let us now define the following multidimensional EXIT functions that describe the convergence properties of the iterative multiuser detector. The equalizer mutual information is measured at the output of demapper. The extrinsic information at the output of equalizer output for the d^{th} stream of u^{th} user is given by

$$I(u)^E_{E_d} = f(\mathbf{r}, \mathbf{I}(1)^E_A, ..., \mathbf{I}(u)^E_A, ..., \mathbf{I}(U)^E_A), \quad (25)$$

where the equalizer *a* priori mutual information vector of the u^{th} user is defined as $\mathbf{I}(u)^E_A = [I(u)^E_{A_1} ... I(u)^E_{A_d} ... I(u)^E_{A_D}]$ with $I(u)^E_{A_d}$ being *a* priori information of the equalizer for the d^{th} stream of the u^{th} user. By contrast, the extrinsic information at the output of decoder for the d^{th} stream of the u^{th} user is given by

$$I(u)^D_{E_d} = f(I(u)^D_{A_d}), \quad (26)$$

where $I(u)^D_{A_d}$ is *a* priori information for the d^{th} stream of the u^{th} user's decoder.

A. Point-to-Point

First of all, a scenario with $T = D = R = 2$ is considered. Figure 2a depicts the EXIT chart obtained by the projection technique. It is worth noting that the starting and ending points of the EXIT curves with the both precoding schemes are at different levels. This is due to fact that the power allocation introduces stream-wise inter symbol interference (ISI) even though beamformer matrix, $\mathbf{V}_u$, can perfectly decouple streams from each other in the spatial domain. In contrast to traditional multi-carrier transmission with joint linear transceiver approach, this is a significant difference. In order to illuminate the fact that the ISI is due to the power allocation, we apply SVD to the point-to-point channel matrix. Furthermore we assume that the left-hand eigenvectors of the channel matrix, $\mathbf{D}_u \in \mathbb{C}^{RK_S \times RK_S}$, are used as a receiver matrix.[3] It turns out that the received signal for u^{th} user can be rewritten as

$$\mathbf{r} = \underbrace{\mathbf{F}_s^{-1}\mathbf{P}_u^{\frac{1}{2}}\mathbf{F}_s}_{\text{blockwise circulant}}\mathbf{b}_u + \mathbf{F}_s^{-1}\mathbf{D}_u^{\ddagger}\widehat{\mathbf{A}}_u^{\dagger}\mathbf{F}_R\mathbf{v}. \quad (27)$$

Now, we realize that depending on the power allocation over the sub-carriers, the strength of ISI changes. Particularly, with low SNR values the MaxRate strategy aims to allocate all the power to the strongest eigenmodes of the channel, according to the maximum mutual information rate-based water filling principle. As a consequence, severer ISI is caused, resulting in larger difference between the starting and ending points of the EXIT curve. Similar observation can be made also with MinSumMSE. However, the power allocation with MinSumMSE is performed according to minimum sum MSE-based water filling, which results in different starting and ending points of the EXIT curves streamwise. However, in the case of no precoding it is seen that the starting and ending points are nearly the same.

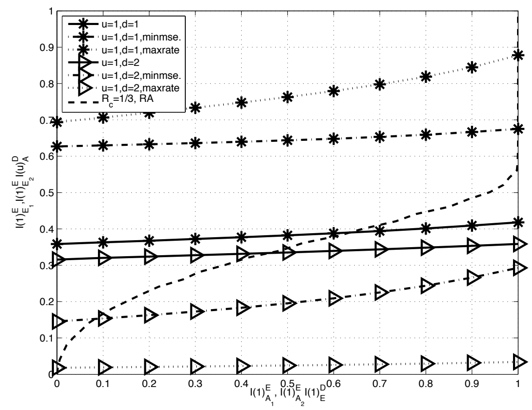
Figure 2b shows the EXIT curves obtained by projection with and without precoding for $T = D = R = 4$ and SNR$= -2$dB. It can be seen that the both precoding methods result in large separation between the stream-wise equalizer EXIT curves and in different intersection points with the RA code. However, without precoding the equalizer's EXIT curves of the streams intersect the RA code nearly at the same point. Similarly, as in the case of $T = D = R = 2$, with low SNR values MaxRate precoding aims to allocate the most of the power to the strongest eigenmodes of the channel. As a result of this, MaxRate waterfilling allocates nearly zero powers to the streams, $d = 3, 4$. Therefore, it is clear that the equalizer can not converge at all with the streams $d = 3, 4$. Correspondingly, MaxRate allocates most of the transmission power to one of the streams, $d = 1$, also resulting in severe ISI. However, for this particular stream, $d = 1$, iterative equalizer EXIT curve reaches very close to the point, $I(1)^E_{E_1} = 1$, in which infinitesimally low bit error rate can be achieved. The observations described above invokes the idea that in order to minimize the rateloss of the transmission, especially with precoding strategies, different coding rates, instead of fixed coding rates, should be employed at the transmitter.

B. Multipoint-to-Point

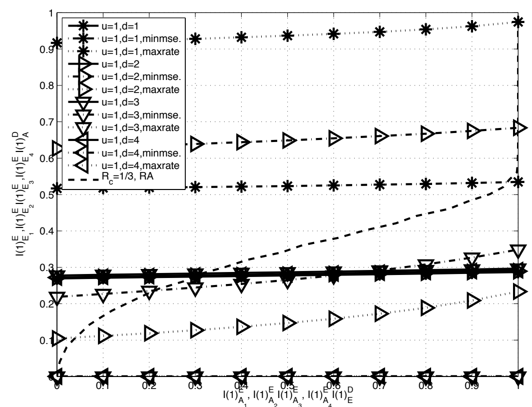
Figure 3a presents the EXIT curves of the equalizer and the decoder for $U = 2$ $T = 2, R = 4$ and SNR$= -2$dB in the centralizes design scenario. As can be seen, the both starting and ending points of EXIT curves have significantly larger difference in the multipoint-to-point case compared to in the point-to-point case, both with and without precoding. As expected, this implies that iterative equalizer has a bigger role in determining the performance of multipoint-to-point MIMO than in point-to-point systems. Particularly, more significant differences between stream-wise equalizer EXIT curves can be observed with precoding schemes than without precoding. Again as seen in figures 2a and 2b, at the low SNR values iterative waterfilling allocates most of the power available only to the strongest eigenmodes of the channel in the both users cases. Thus, there are two EXIT curves of the equalized streams, $I(1)^E_{E_1}$ and $I(2)^E_{E_1}$, that intersect with the code curve relative close to top right corner of the EXIT chart. Correspondingly, due to the power allocation the other two EXIT curves of the equalized streams with MaxRate precoding, $I(1)^E_{E_2}$ and $I(2)^E_{E_2}$, intersect with the code curve at low left corner of the EXIT chart.

Comparing the decentralized design figure 3b with the centralized design figure 3a , it is found that the performance of MinSumMSE degrades more significantly than MaxRate in decentralized design for $U = 2$, $T = 2, R = 4$ and SNR$= -2$dB. In fact, the reason for this is obviously because MinSumMSE aims optimize its precoders based on the point-to-point scenario even though the system itself is multipoint-to-point. Therefore, the convergence characteristics of MinSumMSE becomes nearly the same as without precoding. However, with MaxRate the low SNR value dominates the behavior of iterative waterfilling. Therefore, partial knowledge

[3]Due to space limitations details are omitted.

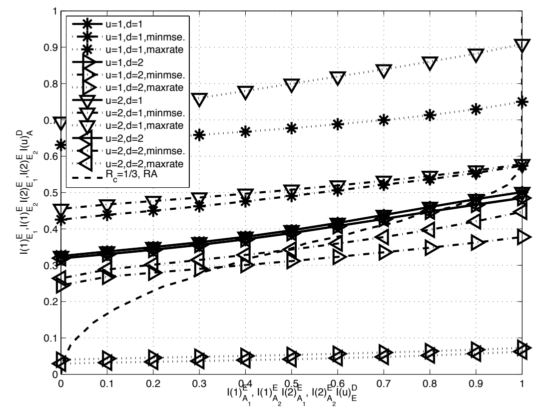


(a) $T = R = 2, D = 2$, SNR$= -1$dB .

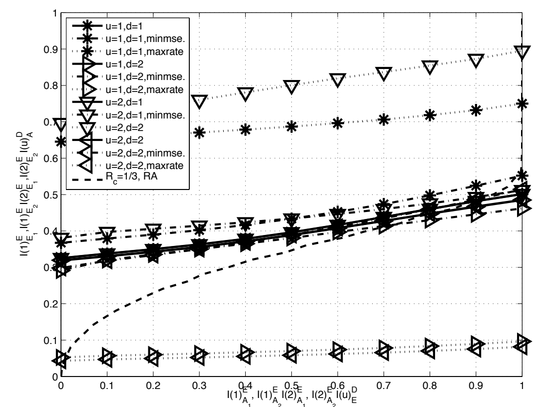


(b) $T = R = 4, D = 4$, SNR$= -2$dB

Fig. 2: EXIT projection with and without precoding, $U = 1$, SNR$= 1$. (dotted lines = max. rate, solid lines = no precoding, dashed dot = MinSumMSE)



(a) Centralized design.



(b) De-centralized design.

Fig. 3: EXIT projection, $U = 2, T = 2, R = 4, D = 2$, SNR$=$ -2dB. (dotted lines = max. rate, solid lines = no precoding, dashed dot = MinSumMSE)

of CSI does not make any significant change in convergence properties of iterative equalizer with MaxRate precoding.

Figures 4a and 4b show for centralized and decentralized design, respectively, the EXIT curves of the equalizers and the decoder with and without precoding for $U = 3, T = 2, R = 2$ and SNR$= 1$dB. It is worth noting that MinSumMSE can not work properly in overloaded scenarios. The reason for this is that MinSumMSE is based on linear transceiver assumption. Thus, we do not investigate the performance of MinSumMSE in these scenarios. Now, it can be observed that the starting and ending points of EXIT curve of equalizer without precoding stay nearly the same. Moreover, the EXIT curve of equalizer without precoding intersects with the RA code EXIT curve at the point close to left corner of the EXIT chart. However, with a MaxRate precoding it is clearly seen that each user can have only one stream which can converge into relatively high mutual information value. Correspondingly, the equalizer EXIT curves for the rest of the streams intersect with the RA code EXIT curve at a point close to the left bottom corner of EXIT chart. Hence, high mutual information values can not be reached. The reason for this is, again, the same as explained previously; the effect of low SNR values to the behavior of iterative waterfilling algorithm dominates the tendency. Moreover, making comparison between figures 4a and 4b, it can be observed that the EXIT characteristics of iterative equalizer with MaxRate are quite similarly in the both centralized and decentralized.

V. Conclusion

In this paper impact of minimum sum MSE and maximum mutual information rate optimized linear precoding strategies on the convergence of iterative MMSE-based multiuser MIMO detector has been discussed. Based on the multidimensional EXIT analysis, the exchange of extrinsic information between iterative equalizer and the channel decoders was analyzed and visualized. It has been shown that precoding enhances the convergence of iterative equalizer, especially, in the multipoint-to-point MIMO's case. Moreover, it has been demonstrated that the use of the linear precoding enhances the separability of the EXIT curves of the simultaneous streams over without precoding; This invokes the idea that different code rates be allocated to the each transmitted streams at the transmitter.

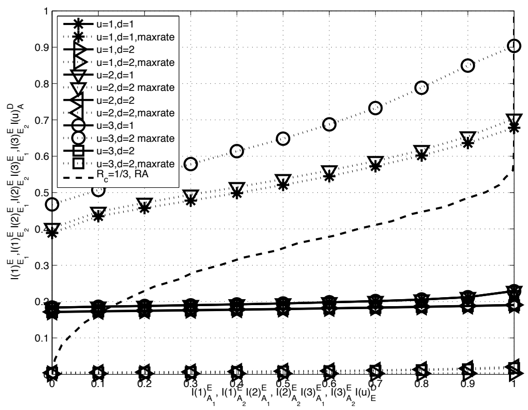
Appendix

A. Soft-cancellation

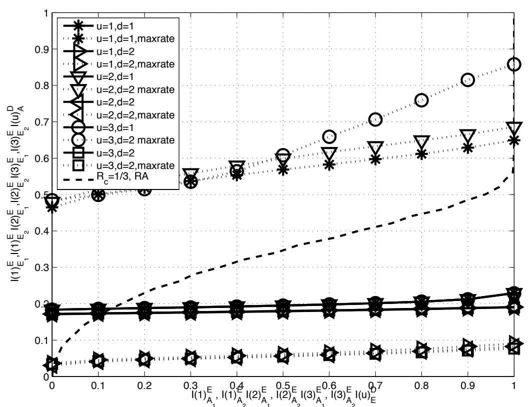
The frequency domain residual interference, $\hat{\mathbf{r}} \in \mathbb{C}^{RK_B \times 1}$, after the cancellation of signal components to be detected from the received signal is given by

$$\hat{\mathbf{r}} = \widehat{\mathbf{A}}_u^{\dagger} \mathbf{F}_R \mathbf{r} - \tilde{\mathbf{\Gamma}} \mathbf{T} \mathbf{F} \tilde{\mathbf{b}}, \tag{28}$$

where $\tilde{\mathbf{\Gamma}} = \widehat{\mathbf{A}}_u^{\dagger} \mathbf{\Gamma} \mathbf{A} \in \mathbb{C}^{RK_S \times UTK_S}$ is the effective channel matrix with corresponding frequency bins. $\tilde{\mathbf{b}} \in \mathbb{C}^{UDK_S \times 1}$ represents the soft-estimate of the multiple user's transmitted signal vector $\tilde{\mathbf{b}} = [\tilde{\mathbf{b}}^1, ..., \tilde{\mathbf{b}}^u, ..., \tilde{\mathbf{b}}^U]^{\dagger}$ with $\tilde{\mathbf{b}}^u \in \mathbb{C}^{DK_S \times 1}$ being the u^{th} user soft estimate of the transmitted layers $\tilde{\mathbf{b}}^u =$



(a) Centralized design.



(b) De-centralized design.

Fig. 4: EXIT projection, $U = 3, T = 2, R = 2, D = 2$, SNR= 1dB.

$[\tilde{\mathbf{b}}^{u,1}, ..., \tilde{\mathbf{b}}^{u,d}, ..., \tilde{\mathbf{b}}^{u,D}]^{\dagger}$. $\tilde{\mathbf{b}}^{u,d} \in \mathbb{C}^{K_S \times 1}$ is given by $\tilde{\mathbf{b}}^{u,d} = \left[\tilde{b}_1^{u,d} \dots \tilde{b}_{k_S}^{u,d} \dots \tilde{b}_{K_S}^{u,d}\right]^{\dagger}$ with $\tilde{b}_{k_S}^{u,d}$ being soft estimate of k_S^{th} transmitted symbol of the u^{th} user's d^{th} stream. Reference [11] describes in detail the first two moments of soft-symbol estimates, $\tilde{b}_{k_S}^{u,d} = E\left\{b_{k_S}^{u,d}\right\}$ and $E\left\{|b_{k_S}^{u,d}|^2\right\}$.

B. Iterative Equalization

In the following derivation it is assumed that all the precoders $\mathbf{T}_u$ to be fixed. It should be also noted that the receivers are independent of other user's receive filters. Therefore, the receive MMSE filters can be optimized independently, user-by-user. By using (10) for each user's channel matrix, the MSE of the uth user using (12) is given by[4]

$$\begin{aligned}\mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u) = \; & Tr\{\mathbf{\Sigma}_{b_u}\} - Tr\{\mathbf{\Sigma}_{b_u}\mathbf{S}(n)\mathbf{F}_D^{\ddagger}\mathbf{T}_u^{\ddagger}\tilde{\mathbf{\Gamma}}_u^{\ddagger}\mathbf{\Omega}_u\mathbf{F}_D\} \\ & - \; Tr\{\mathbf{F}_D^{\ddagger}\mathbf{\Omega}_u^{\ddagger}\tilde{\mathbf{\Gamma}}_u\mathbf{T}_u\mathbf{F}_D\mathbf{S}(n)^{\ddagger}\mathbf{\Sigma}_{b_u}^{\ddagger}\} \\ & + \; Tr\{\mathbf{F}_D^{\ddagger}\mathbf{\Omega}_u^{\ddagger}\mathbf{\Sigma}_{\hat{r}_u}\mathbf{\Omega}_u\mathbf{F}_D\} \qquad (29)\end{aligned}$$

where $\mathbf{\Sigma}_{b_u} = E\{\mathbf{b}_u\mathbf{b}_u^{\ddagger}\}$ and $\mathbf{\Sigma}_v = E\{\mathbf{v}\mathbf{v}^{\ddagger}\}$ with $\mathbf{\Sigma}_{\hat{r}_u} \in \mathbb{C}^{RK_S \times RK_S}$ being the covariance matrix of the residual and desired signal components, given by $\mathbf{\Sigma}_{\hat{r}_u} = \mathbf{\Sigma}_{\hat{r}} + \tilde{\mathbf{\Gamma}}_u\mathbf{T}_u\mathbf{F}_D\mathbf{S}(n)_u\breve{\mathbf{\Lambda}}_u\mathbf{S}(n)_u^{\ddagger}\mathbf{F}_D^{\ddagger}\mathbf{T}_u^{\ddagger}\tilde{\mathbf{\Gamma}}_u^{\ddagger}$. The covariance matrix of the residual $\mathbf{\Sigma}_{\hat{r}} \in \mathbb{C}^{RK_S \times RK_S}$ is given by

$$\mathbf{\Sigma}_{\hat{r}} = \tilde{\mathbf{\Gamma}}\mathbf{T}\mathbf{A}\mathbf{F}\mathbf{\Lambda}\mathbf{F}^{\ddagger}\mathbf{A}^{\ddagger}\mathbf{T}^{\ddagger}\tilde{\mathbf{\Gamma}}^{\ddagger} + \sigma^2\mathbf{I}, \qquad (30)$$

where $\tilde{\mathbf{\Gamma}} \in \mathbb{C}^{RK_S \times UTK_S}$ is determined in Appendix A.

For the given set of precoders $\mathbf{T}_u$ the MSE for uth user is convex respect to $\mathbf{\Omega}_u$. Therefore, the standard optimal Wiener solution for the receive filters is determined as

$$\mathbf{\Omega}_u\mathbf{F}_D \;\; = \;\; \mathbf{\Sigma}_{\hat{r}_u}^{-1}\tilde{\mathbf{\Gamma}}_u\mathbf{T}_u\mathbf{F}_D\mathbf{S}(n)^{\ddagger}\mathbf{\Sigma}_{b_u}^{\ddagger}. \qquad (31)$$

Now, let us write the block circulant Hermitian covariance matrix $\mathbf{\Delta} \in \mathbb{C}^{UDK_S \times UDK_S}$ of the feedback soft estimates by $\mathbf{\Delta} = \mathbf{F}\mathbf{\Lambda}\mathbf{F}^{\dagger}$. Due to the time-invariance of the residual interference energy over the frame and the necessity of using the sampling matrix $\mathbf{S}(n)$ can be now eliminated [11]. Now, the MMSE filter output can be written as [12]

$$\hat{\mathbf{b}}_u = \mathbf{\Xi}_u\mathbf{\Pi}_u(\mathbf{F}_D^{-1}\mathbf{T}_u^{\ddagger}\tilde{\mathbf{\Gamma}}_u^{\ddagger}\mathbf{\Sigma}_{\hat{r}}^{-1}\hat{\mathbf{r}} + \mathbf{\Upsilon}_u\tilde{\mathbf{b}}^u). \qquad (32)$$

Because of the limited space, we do not provide detailed derivation process in this paper. The detailed derivation exactly follows [12]. Moreover, the detailed expressions of the following intermediate matrices can be also found from [12]: $\mathbf{\Xi}_u \in \mathbb{C}^{DK_S \times DK_S}$, $\mathbf{\Upsilon}_u \in \mathbb{C}^{DK_S \times DK_S}$ and $\mathbf{\Pi}_u \in \mathbb{C}^{DK_S \times DK_S}$.

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[4] $Tr\{\mathbf{AB}\} = Tr\{\mathbf{BA}\}$