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Description	

Space-Time Turbo Equalization and Symbol Detection in Frequency Selective MIMO Channels

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Abstract— **A computationally efficient Space-Time Turbo equalization algorithm is derived for frequency selective Multiple-Input Multiple-Output (MIMO) channels. The algorithm is an extension of the iterative equalization algorithm by Reynolds and Wang for frequency selective fading channels [1]. This paper's proposed equalizer performs MIMO channel estimation, multiple users' signal detection, and decoding, jointly all in an iterative manner. The iterative channel estimator achieves high accuracy in estimating channel parameters even if only relatively small number of unique word symbols are available. The Multiple Users' Signal Detector (MUSD) consists of soft interference cancellers for the multiple users, each followed by a Minimum Mean Square Error (MMSE) filter. With this simple structure, the proposed detector achieves drastic reduction in computational complexity compared with conventional trellis-based Turbo equalizers. Simulation results show the proposed equalizer can properly separate multiple users' signals in frequency selective MIMO channels.**

I. Introduction

Multiple- Input Multiple- Output (MIMO) communication systems allow multiple users to transmit signals sharing the same time- and frequency-slots, and hence should satisfy the capacity demands for broadband mobile communication systems [2]. Since Inter-Symbol Interference (ISI) and Multiple Access Interference (MAI) are the major causes of hostility in the MIMO communication scenario, MIMO receiver have to adequately detect the multiple users' signals in the presence of ISI and MAI severities.

ISI equalization techniques based on Turbo concept have attracted much attention these days. In [3], a Turbo equalization algorithm utilizing trellis diagram of the channel is derived. Although the trellis-based equalizers can achieve optimum performance, the computational complexity becomes prohibitive for large channel memory length since the number of states in the trellis diagram grows exponentially with the channel memory length. Reynolds and Wang recently proposed a new computationally efficient Turbo equalization algorithm for single user ISI channels[1]. Tüchler, *et al.* have derived several new algorithms based on [1]'s concept [4]. Reference [1]'s iterative equalizer utilizes signal detector which is comprised of soft interference canceller, followed by Minimum Mean Square Error (MMSE) filter, which offers much lower computational complexity compared with the trellis-based equalizers.

This paper expands Reynolds and Wang's concept to derive low complexity Turbo equalization algorithm for frequency selective MIMO channels. The derived equalizer performs MIMO channel estimation, multiple users' signal detection, and decoding for each user, jointly all in an iterative manner. The iterative MIMO channel estimator achieves high accuracy in estimating channel parameters even if only relatively small number of unique word symbols are available since the estimator uses as signal references the multiple users' unique word sequences as well as information symbols having Log Likelihood Ratios (LLRs) larger than a certain value. The Multiple Users Signal Detector (MUSD) consists of soft interference cancellers for the multiple users, each followed by a Minimum Mean Square Error (MMSE) filter. Assuming a MIMO channel with N users, each suffering from independent L-path Propagation, and M receive antennas, the MUSD requires an order $N(ML)^3$ computational complexity, in contrast to that the trellis-based detector's computational complexity increases exponentially with $N \times L$. This is a drastic reduction in computational burden when considering N and L values being considerably large with broadband and high capacity systems.

This paper is organized as follows: Section II describes the MIMO communication channel model used in this paper. A mathematical Space-Time representation of the channel is given. Section III proposes a new MIMO Turbo equalization algorithm based on [1]. Section IV proposes a new iterative channel estimation technique. Section V shows results of computer simulations conducted to verify the effectiveness of the proposed algorithm.

II. MIMO Channel Model

Figure 1 shows a MIMO communication channel model; there are N users, and the receiver is equipped with M antennas. All N users transmit information symbols at the same time- and frequency-slots without spreading their signals in the frequency domain. This paper assumes a coded MIMO system as shown in Figure 2. The information symbols $c_n(i)$s are first encoded by each user's channel encoder where i and n denotes the symbol and user indices, respectively. The coded symbols are then interleaved and modulated according to the modulation format used. The modulated symbols $b_n(k)s$ are then transmitted over frequency selective channels. k denotes the index of

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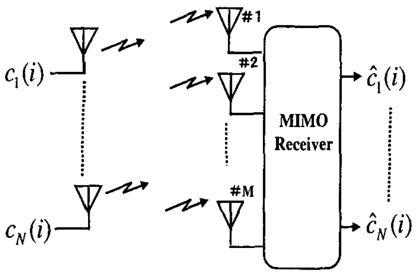

Fig. 1. MIMO Channel Model

modulated symbols. At the receiver, discrete time measurement at the m-th antenna yields the sampled value series, $r_m(k)$, of the antenna output as

$$r_m(k) = \sum_{l=0}^{L-1} \sum_{n=1}^{N} h_{mn}(l) b_n(k-l) + v_m(k), \tag{1}$$

where L is the channel memory length[1]. Without loss of generality, the channel memory length is assumed to be identical for all N users. $h_{mn}(l)$ is a discrete time representation of the channel between the n-th user and the m-th receiver antenna, and $v_m(k)$ is additive white Gaussian noise (AWGN).

Stacking those measurements into a vector form, which is equivalent to sampling in the space domain, results in

$$\begin{aligned} \boldsymbol{r}(k) &\equiv [r_1(k), r_2(k) \dots r_M(k)]^T & (2) \\ &= \sum_{l=0}^{L-1} \boldsymbol{H}(l) \boldsymbol{b}(k-l) + \boldsymbol{v}(k), & (3) \end{aligned}$$

where

$$\boldsymbol{H}(l) = \begin{bmatrix} h_{11}(l) & \dots & h_{1N}(l) \\ \vdots & \ddots & \vdots \\ h_{M1}(l) & \dots & h_{MN}(l) \end{bmatrix}, \tag{4}$$

$$\boldsymbol{b}(k-l) = [b_1(k-l), b_2(k-l) \dots b_N(k-l)]^T, \tag{5}$$

and

$$\boldsymbol{v}(k) = [v_1(k), v_2(k) \dots v_M(k)]^T. \tag{6}$$

Finally, temporal sampling to capture the multipath signals for diversity combining yields the following space-time representation of the received signal $\boldsymbol{y}(k)$

$$\begin{aligned} \boldsymbol{y}(k) &\equiv [\boldsymbol{r}^T(k+L-1), \boldsymbol{r}^T(k+L-2) \dots \boldsymbol{r}^T(k)]^T & (7) \\ &= \boldsymbol{H} \cdot \boldsymbol{u}(k) + \boldsymbol{n}(k), & (8) \end{aligned}$$

where

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}(0) & \dots & \boldsymbol{H}(L-1) & & \boldsymbol{O} \\ & \ddots & & \ddots & \\ \boldsymbol{O} & & \boldsymbol{H}(0) & \dots & \boldsymbol{H}(L-1) \end{bmatrix} \tag{9}$$

is the channel matrix with $\boldsymbol{u}(k)$ and $\boldsymbol{n}(k)$ being

$$\boldsymbol{u}(k) = [\boldsymbol{b}^T(k+L-1) \dots \boldsymbol{b}^T(k) \dots \boldsymbol{b}^T(k-L+1)]^T \tag{10}$$

and

$$\boldsymbol{n}(k) = [\boldsymbol{v}^T(k+L-1) \dots \boldsymbol{v}^T(k) \dots \boldsymbol{v}^T(k-L+1)]^T, \tag{11}$$

respectively.

III. Space-Time Turbo Equalization

A. System Model

Figure 2 shows a block diagram of the Space-Time Turbo equalizer for frequency selective MIMO channels. The equalizer is comprised of a Soft-Input and Soft-Output (SISO) MMSE detector and SISO channel decoders. The basic concept of this configuration follows [1]; our aim is to extend their concept to cover frequency selective MIMO channels.

Binary Phase-Shift Keying (BPSK) is assumed as a modulation scheme used. The detector produces the LLR for each coded bit as

$$\begin{aligned} \Lambda_1[b_n(k)] &= \log \frac{Pr[b_n(k) = +1 | \boldsymbol{y}(k)]}{Pr[b_n(k) = -1 | \boldsymbol{y}(k)]} & (12) \\ &\equiv \lambda_1[b_n(k)] + \lambda_2^p[b_n(k)], & (13) \end{aligned}$$

where $\lambda_1[b_n(k)]$ is the extrinsic information fed to the n-th user's channel decoder following the MMSE detector, and $\lambda_2^p[b_n(k)]$ is the *a priori* information provided by the n-th user's channel decoder.

The channel decoders derive the LLR for each coded bit as

$$\begin{aligned} \Lambda_2[b_n(j)] &= \log \frac{Pr[b_n(j) = +1 | \lambda_1[b_n(j)], j=0, \dots, B-1]}{Pr[b_n(j) = -1 | \lambda_1[b_n(j)], j=0, .., B-1]} & (14) \\ &\equiv \lambda_2[b_n(j)] + \lambda_1^p[b_n(j)], & (15) \end{aligned}$$

where, with j being symbol index after de-interleaving, $\lambda_2[b_n(j)]$ is the extrinsic information fed back to the MMSE detector, and $\lambda_1^p[b_n(j)]$ is the *a priori* information provided by the channel detector. B is the burst length.

Estimates of each user's information symbols can be obtained as

$$\hat{c}_n(i) = sign(\Lambda_2[c_n(i)]) \quad (n = 1, \dots, N) \tag{16}$$

after sufficient times of iterations, where

$$\Lambda_2[c_n(i)] = \log \frac{Pr[c_n(i) = +1 | \lambda_1[b_n(j)], j=0, \dots, B-1]}{Pr[c_n(i) = -1 | \lambda_1[b_n(j)], j=0, .., B-1]}. \tag{17}$$

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[1] Equation (1) is valid regardless of the synchronization in symbol timing among the users. In fact, impulse responses of the channel and filters in the transmitter and the receiver can be folded into $h_{mn}(l)$. See [5].

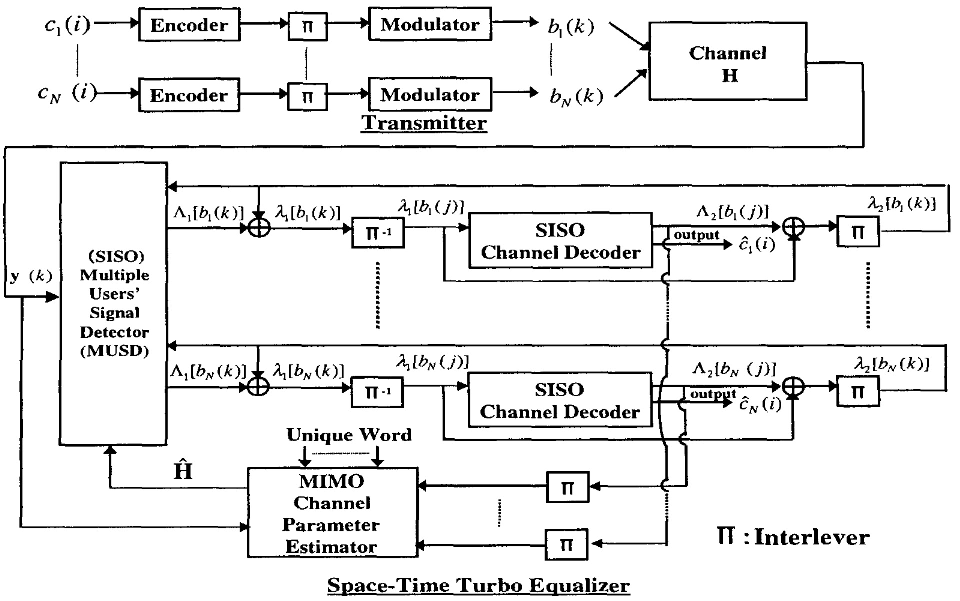

Fig. 2. System Model

B. SISO MMSE MUSD

A block diagram of the MUSD is shown in Fig.3. The MUSD consists of N independent detectors. In the following, we assume the 1-st user is the user of interest. The same algorithm should apply to signal detection of the other $N-1$ users. Utilizing the extrinsic information provided by the n-th user's channel decoder, the MMSE detector first forms soft estimates of all N users' k-th symbol as

$$\tilde{b}_n(k) = tanh[\frac{\lambda_2[b_n(k)]}{2}] \quad (n = 1, ..., N) \tag{18}$$

which are used to form the *soft replica* $\boldsymbol{H} \cdot \tilde{\boldsymbol{u}}(k)$ of the MAI and ISI components. The *soft replica* is subtracted from the received signal vector $\boldsymbol{y}(k)$ to produce the 1-st user's signal estimate vector as

$$\tilde{\boldsymbol{y}}_1(k) = \boldsymbol{y}(k) - \boldsymbol{H} \cdot \tilde{\boldsymbol{u}}_1(k), \tag{19}$$

where

$$\tilde{\boldsymbol{u}}_1(k) \equiv [\tilde{\boldsymbol{b}}^T(k+L-1) \ldots \tilde{\boldsymbol{b}}^T(k) \ldots \tilde{\boldsymbol{b}}^T(k-L+1)]^T \tag{20}$$

with

$$\tilde{\boldsymbol{b}}(k+l) = [\tilde{b}_1(k+l) \ldots \tilde{b}_N(k+l)]^T. \tag{21}$$

If $l = 0$,

$$\tilde{\boldsymbol{b}}(k) = [0, \tilde{b}_2(k) \ldots \tilde{b}_N(k)]^T \tag{22}$$

with the 1-st element being zero. Equation (19) yields *soft interference cancellation*. Note that [1]'s equalizer aims to soft-cancel only ISI components, whereas the MIMO detector aims to soft-cancel both ISI and MAI components.

The objective of the rest of the algorithm is to suppress the ISI and MAI residuals left after *soft interference cancellation*. An adaptive linear filter is used for this purpose: the $M \times L$-vector $\boldsymbol{w}_1(k)$ of the filter taps is determined so that the MSE between the filter output and the signal point corresponding to the detected desired user's symbol is minimized as

$$\boldsymbol{w}_1(k) = arg \min_{\boldsymbol{w}_1(k)} \|\boldsymbol{w}_1^H(k)\tilde{\boldsymbol{y}}_1(k) - b_1(k)\|^2. \tag{23}$$

Since the derivation of the optimum vector $\boldsymbol{w}_1(k)$ follows [1], only the results are shown below:

$$\boldsymbol{w}_1(k) = [\boldsymbol{H}\boldsymbol{\Lambda}_1(k)\boldsymbol{H}^H + \sigma^2\boldsymbol{I}]^{-1}\boldsymbol{h}_1, \tag{24}$$

where

$$\boldsymbol{h}_1 \equiv [h_{11}(L-1)...h_{M1}(L-1)...h_{11}(0)...h_{M1}(0)]^T \tag{25}$$

and

$$\boldsymbol{\Lambda}_1(k) = diag[\boldsymbol{D}(k+L-1)...\boldsymbol{D}(k)...\boldsymbol{D}(k-L+1)] \tag{26}$$

with

$$\boldsymbol{D}(k+l) = diag[1-\tilde{b}_1^2(k+l)...1-\tilde{b}_n^2(k+l)...1-\tilde{b}_N^2(k+l)]. \tag{27}$$

For $l = 0$,

$$\boldsymbol{D}(k) = diag[1, 1-\tilde{b}_2^2(k), ..., 1-\tilde{b}_N^2(k)] \tag{28}$$

with the (1,1)-element being one. By approximating error at the MMSE filter output by a Gaussian process [1], the extrinsic information of the 1-st user to be delivered to

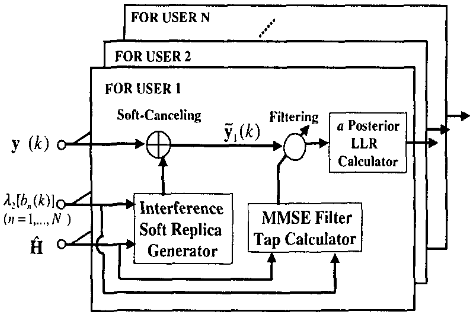

Fig. 3. Multiple Users' Signal Detector (MUSD)

the channel decoder can be derived as,

$$\lambda_1[b_1(k)] = \log\frac{Pr[\boldsymbol{y}(k)|b_1(k)=+1]}{Pr[\boldsymbol{y}(k)|b_1(k)=-1]} \quad (29)$$

$$= \frac{4Re[z_1(k)]}{1-\mu_1(k)}, \quad (30)$$

where $z_1(k)$ is the filter output

$$z_1(k) = \boldsymbol{w}_1^H(k)\tilde{\boldsymbol{y}}_1(k), \quad (31)$$

and

$$\mu_1(k) = \boldsymbol{h}_1^H[\boldsymbol{H}\boldsymbol{\Lambda}_1(k)\boldsymbol{H}^H + \sigma^2\boldsymbol{I}]^{-1}\boldsymbol{h}_1. \quad (32)$$

IV. Iterative Channel Estimation

The sensitivity of Turbo equalization performance to channel estimation error was reported in [6]. It is obvious that increasing the accuracy of channel estimation enhances the performance. In deriving the algorithm described in Section III, all elements of channel matrix H were assumed to be known. This section proposes a new channel estimation scheme that effectively utilizes the iterative mechanism of the MIMO Turbo equalizer.

It is assumed that each of the N users' information sequences is headed by a unique word sequence whose waveform and timing are known to the receiver. Prior to the first iteration for the MIMO equalization, the N users' channel impulse responses are estimated by using the unique word sequences as signal references. Assuming that all the N users' unique word sequences are uncorrelated with each other, the Recursive Least Square (RLS) algorithm may be used to estimate the N users' channel impulse responses. Initial estimates of the N users' channel impulse responses are obtained at the end of the unique word period. The receiver then runs the first iteration of the MIMO equalization algorithm described in Section III. The first iteration produces initial soft estimates of the N users' transmitted symbols given by

$$\tilde{b}_n(k) = tanh[\frac{\lambda_2[b_n(k)]}{2}]. \quad (n=1,...,N) \quad (33)$$

Obviously, the larger the $|\tilde{b}_n(k)|$'s value, the more reliable it is, which suggests that the hard decisions of $\tilde{b}_n(k)$s' having relatively large $|\tilde{b}_n(k)|$ values can be used as additional signal references for channel estimation. Thresholding may properly identify the reliable soft estimates. Additional signal references are then given as signal points corresponding to the hard decision results for the symbols identified as being reliable.

Prior to the second iteration, the RLS parameter estimation algorithm is run again using as signal references both the unique word waveform and the information symbols identified as being reliable. The estimates of the channel impulse responses are then updated. The receiver runs the second iteration for the MIMO equalization using the updated channel estimates. This process is repeated. Because of the Turbo principle, the $|\tilde{b}_n(k)|$ values increase with the iteration number, thereby yielding more additional reference signals. This results in better estimates of the channel impulse responses.

V. Simulation Results

This section presents results of computer simulations conducted to evaluate performances of the proposed MIMO Turbo equalizer. All simulations assumed that channel frequency selectivity is due to an L-path propagation scenario with each path experiencing frequency flat Rayleigh fading, and each of L path components has identical average power. The normalized maximum Doppler frequency f_dT_s, normalized by the symbol duration T_s, was set at 1/20000. It is also assumed that the received signal power of the N users are identical. A rate 1/2 non-systematic convolutional code with the constraint length of 3 and generators $[G_1, G_2] = [5,7]_{oct}$ was used. One burst has 900 coded symbols, headed by a 50-symbol unique word sequence for channel estimation. The RLS algorithm was used for channel estimation. The threshold value for iterative channel estimation was set at 0.5. All the N users were assumed to be symbol- and frame-synchronized for simplicity. A random interleaver was assumed. The size of the interleaver is same as the frame length. The Max-Log-MAP algorithm was used in the SISO channel decoders.

Figure 4 shows for $N=2$, $L=5$ and $M=2$ bit error rate (BER) performance curves of the MIMO proposed equalizer with iterative channel estimation. Each curve was obtained by averaging over all users' BERs. E_b is defined as the average per-information bit energy of each user's signal received by one antenna element. Figure 4 also shows as a reference the BER performance curve of an order-10 (=$L \times M$) Maximum Ratio Combining (MRC) diversity followed by Soft-Input Viterbi channel decoding. To obtain the reference BER curve, channel was assumed to be known. The MRC BER curve corresponds to the case where for each of 2 users, the derived MIMO equalizer can fully exploit the order-10 diversity gain while completely eliminating the MAI and ISI components from the

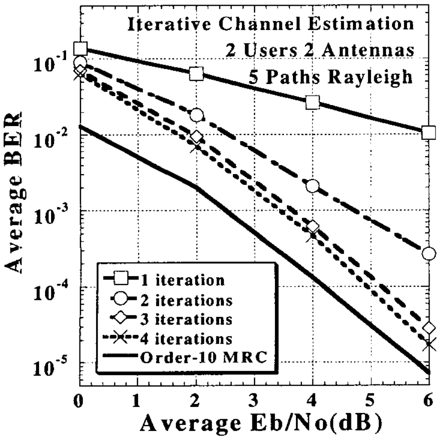

Fig. 4. BER Performance: $N = 2$, $L = 5$, and $M = 2$, with Iterative Channel Estimation

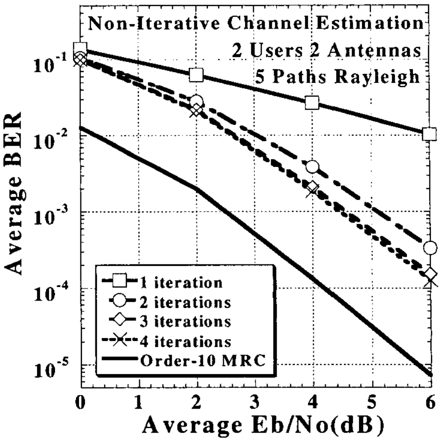

Fig. 5. BER Performance: $N = 2$, $L = 5$, and $M = 2$, without Iterative Channel Estimation

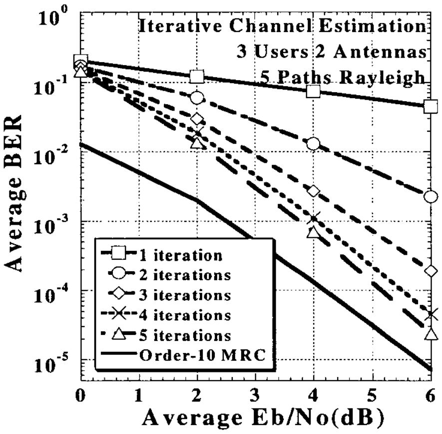

Fig. 6. BER Performance: $N = 3$, $L = 5$, and $M = 2$, with Iterative Channel Estimation

received signal $y(k)$. Results show that the derived MIMO equalizer offers substantial iteration gains, and achieves BER performances close to the MRC BER curve.

Figures 5 shows for $N = 2$, $L = 5$ and $M = 2$ BER performance curves without iterative channel estimation where channel is estimated only once prior to the 1-st iteration using the 50-symbol unique word sequence. It is found that the BER performance is degraded compared to that shown in Fig.4 due to channel estimation error. Hence, the proposed iterative channel estimation technique is effective in improving the equalization performance.

Figure 6 shows for $N = 3$, $L = 5$ and $M = 2$ BER performance curves of the proposed equalizer with iterative channel estimation. The result show that the proposed equalizer can properly separate 3 users.

VI. Conclusion

We have extended Reynolds and Wang's iterative ISI equalizer to derive a Space-Time Turbo equalizer for frequency selective MIMO channels. A new iterative channel estimation technique is also proposed. The derived equalizer performs MIMO channel estimation, multiple users' signal detection, and decoding, jointly all in an iterative manner. The MUSD achieves drastic reduction in computational complexity compared with conventional trellis-based equalizers. Computer simulation results show that the proposed equalizer can fully exploit the diversity gain in frequency selective MIMO channels, while effectively mitigating both ISI and MAI effects to separate multiple users.

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