A Semantical Study of Relevant Modal Logics

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Abstract

This thesis deals with semantics of relevant modal logics. Originally, modal logics have been developed over classical logic. Recently, the studies of modal logics based on non-classical logic have also been developed. These include relevant modal logics, but many of basic problems on them remain open. In this thesis, we develop a semantical study and show Kripke completeness for a wide class of relevant modal logics in a systematic way.

In general discussion of relevant modal logics, we assume that modal operators $\Box$ and $\Diamond$ are independent. Further, we take regular logics in our sense for basic modal logics.

In this thesis, first we show completeness of our basic relevant logics in terms of both Routley-Meyer frames, Kripke-style semantics, and relevant modal matrices, algebraic semantics. Any regular relevant modal logic is complete with respect to a class of relevant modal matrices, while it is not necessary complete with respect to a class of Routley-Meyer frames. To make any regular relevant modal logic complete by using Routley-Meyer frames, we introduce general frames. Also, we investigate the relationship between general frames and relevant modal matrices as R.Goldblatt developed for classical modal logic.

Our main result in this thesis is a Sahlqvist theorem for relevant modal logics, that is, Kripke completeness of relevant modal logics with Sahlqvist formulas. To obtain it, we show the frame postulate written by a first order sentence corresponding to a given Sahlqvist formula. Also, it is shown that usual Sahlqvist theorem for classical modal logics can be obtained as a special case of our Sahlqvist theorem.

Key Words: relevant modal logics, completeness, Sahlqvist theorem