Proof–Search in Modal and Temporal Logics

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Abstract

This thesis investigates proof–search procedures for modal and temporal logics. A proof–search procedure is a decision procedure which gives us a proof of a give formula if it is provable. Main target of the present thesis is to give a proof–search procedure for the temporal logic $\mathbf{K_t}$. Since the study of temporal logics is now applied to various branches of computer science including software engineering and artificial intelligence, to find an efficient proof–search procedure for $\mathbf{K_t}$ will be an important problem as $\mathbf{K_t}$ is the most basic one among them. Our study will be a prototype of further studies for logicians who utilize temporal logics for their researches.

In proof–search, we usually need to check whether there are repetitions of the same sequents (or formula sets) or not in proofs. This is called loop–checkings. Naturally, loop–checking causes inefficiency. The most desirable way of avoiding loop–checkings is to introduce such a proof system that loops never occur in its proofs. In standard systems for $\mathbf{K_t}$, several kinds of loops will occur. To get a proof–search procedure for $\mathbf{K_t}$, we begin with finding one for each of modal logics **S4**, **KB** and **K4B**. For, the modal operator $\Box$ of each of these logics behaves like tense operators $[F]$ and $[P]$ in $\mathbf{K_t}$. Our study goes along as follows:

1. proof system for **S4**,
2. proof system for **KB**,
3. proof system for **K4B**,
4. proof system for $\mathbf{K_t}$.

Techniques of getting loop–free proof systems for first three modal logics will be applied to $\mathbf{K_t}$, and thus we can get a loop–free proof for it. To avoid several kinds of loop, we introduce an auxiliary modal operator $\blacksquare$ and *histories*, which are pairs of sets of $\Box$–formulas, to standard proof systems for **S4**, **KB**, **K4B** and $\mathbf{K_t}$. We will see that $\blacksquare$ and histories enable us to avoid loop–checkings. Then, we show that our proof–search procedure using each of these proof systems terminates eventually, and gives a loop–free proof if our proof–search of a given sequent is successful.

On the other hand, when our proof–search procedure fails to find a proof of a given formula, we will give a way of constructing a (finite) counter–model for it. In order to construct counter–models, we introduce finite Kripke frames called *model graphs* which facilitate construction of counter–models. We can easily get a counter–model from a model graph when we construct it. From this, both completeness and the finite model property of each of these four systems follow.

Key Words: **modal logics, temporal logics, proof–search procedures, loop–free proof systems, model graph**