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Enumeration and generalization of the Hoffman puzzle

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“Packing”is an inportant act in the society. The example includes the efficient packing of luggage into container, truck, etc. However, the volume is often limited in general. Therefore, efficient packing of luggage into the limited volume is reguired. Such a problem is called a packing problem.

The packing problem is a problem of packing more objects into a container. General packing problem is known as one of the NP hard problems. Hence there is no hope to solve such a general packing problem efficiently unless P $\neq$ NP. The Hoffman puzzle is a special example of the packing problem.

The Hoffman puzzle is a packing puzzle that consists of a box and 27 blocks. Hoffman introduced it in 1978. It aims for packing all 27 blocks into the box. The block is a rectangular parallelepiped. The lengths of three sides a, b, and c of rectangular parallelepiped are all different. All blocks are congruent boxes of size $a \times b \times c$, and the box has its size of $(a+b+c) \times (a+b+c) \times (a+b+c)$. In the Hoffman puzzle, these three lengths a, b, and c satisfy the following condition $\frac{1}{4}(a+b+c) < a < b < c$. Conway and Cutler showed that the number of solutions of the Hoffman puzzle is 21 in 1981.

The Hoffman-Knuth puzzle is a generalized Hoffman puzzle proposed by Knuth in 2004. Knuth considered a special case of the Hoffman puzzle with a condition $\frac{1}{4}(a+b+c) = a < b < c$. We name the puzzle with the Hoffman-Knuth condition *Hoffman-Knuth puzzle*. In the condition, we have $a + b + c = 4a$. That is, the size of a side of the box is equal to four times of the size of the shortest side of a block. Therefore, it is possible to stack four blocks in a line in the Hoffman-Knuth puzzle.

Knuth consider a special case that $(a, b, c) = (3, 4, 5)$. As a result, he showed that 28 blocks can be packed into the box in three ways. Moreover, Ishino considered the case of a block of size $(a, b, c) = (4, 5, 7)$ in 2010 and examined the case. He found all possible arrangements of 28 blocks. In a solution, some blocks are not fixed in the box.

In this paper, we first analyze the Hoffman puzzle. We use the constraint conditions of the Hoffman puzzle to reduce the amount of search space.

Next we turn to the Hoffman Knuth puzzle. Using the similar technique in the Hoffman puzzle, we obtain all the solutions of the Hoffman Knuth puzzle that aims for packing 28 blocks into the box. The solutions consist of essentially different three patterns, that produce 20 different solutions by swapping and rotation.

In the Hoffman Knuth puzzle, we found that we need additional condition to the original condition $\frac{(a+b+c)}{4} = a < b < c$. More precisely, they also required to satisfy $a < b < \frac{4a}{3}$, $\frac{5a}{3} < c < 2a$, and $(\frac{3a}{2} - b) = (c - \frac{3a}{2})$ to have a solution. That is, the Hoffman Knuth puzzle does not always have a solution. For example, it has a solition if $(a, b, c) = (3, 4, 5), (4, 5, 7), (5, 6, 9)$, but it does not have a solution if $(a, b, c) = (5, 7, 8), (7, 10, 11), (8, 11, 13)$. Moreover, we also showed that we cannot pack the 29th block into the box. Thus these 20 solutions are all the solutions of the puzzle. That is, we succeed to show not only all the solutions of the Hoffman Knuth puzzle, but also the necessary and sufficient condtions to have a solution.