

Title	様相論理における不完全性への代数的アプローチ
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Citation	
Issue Date	2005-09
Type	Thesis or Dissertation
Text version	author
URL	<a href="http://hdl.handle.net/10119/977">http://hdl.handle.net/10119/977</a>
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Description	Supervisor:小野 寛晰, 情報科学研究科, 博士

# An algebraic approach to incompleteness in modal logic

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July 8, 2005

## Abstract

The thesis studies semantics of propositional modal logic. We are concerned with algebraically motivated notions of completeness weaker than Kripke completeness. In particular, we study completeness with respect to atomic algebras ( $\mathcal{A}$ -BAOs), completely additive algebras ( $\mathcal{V}$ -BAOs), algebras which admit residuals ( $\mathcal{T}$ -BAOs), lattice-complete ( $\mathcal{C}$ -BAOs) and  $\omega$ -complete algebras ( $\omega\mathcal{C}$ -BAOs) and algebras with various combinations of the above properties. For example,  $\mathcal{AV}$ -BAOs are atomic and completely additive algebras and  $\mathcal{CA}$ -BAOs are complete and atomic algebras, duals of so-called (normal) neighbourhood frames.  $\mathcal{CAV}$ -BAOs are known to be duals of Kripke frames.

Our main discovery is that all the above notions of completeness are non-equivalent and for most of them the so-called Blok Alternative can be generalized. Many of those completeness notions can be characterized alternatively as conservativity of various minimal extensions with new syntactic objects, axioms and inference rules. The second-order character of Kripke semantics turns out to be the consequence of lattice-completeness of their dual algebras. In particular, the Löb logic — a notorious example of a non-compact logic — fails to be strongly  $\omega\mathcal{C}$ -complete. This observation strengthens all existing results concerning non-complexity, non-compactness or non-canonicity of this system.

The notion of  $\mathcal{AV}$ -completeness turns out to be the most well-behaved one. We show that this notion has many alternative characterizations as, e.g., conservativity of minimal hybrid extensions or completeness with respect to discrete frames. In addition, there is no gap between weak and strong global  $\mathcal{AV}$ -completeness. The class of  $\mathcal{AV}$ -BAOs in a given variety is determined by its subdirectly irreducible members. We characterize modally definable classes of discrete frames (duals of  $\mathcal{AV}$ -BAOs); our characterization is more general than the Goldblatt-Thomason theorem for Kripke frames. These positive results for  $\mathcal{AV}$ -BAOs are contrasted with negative results for other classes of algebras.

Our thesis contains also a case study: investigation of the lattice of tense logics of linear time flows. The reason why those logics deserved our special attention is that all those logics are  $\mathcal{AV}$ -complete, even though Kripke incompleteness is a common phenomenon in this lattice. It turns out that many positive results are provable: in particular, all finitely axiomatizable logics in this lattice have NP-complete satisfiability problem.

Results concerning discrete frames have been obtained jointly with Balder ten Cate. Results concerning tense logics of linear time flows have been obtained jointly with Frank Wolter.